

Debt Dilution and Firm Investment

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Big question: Do financial frictions matter for firm investment?

Standard models: *short-term debt* only

Empirically, most firm debt is *long-term debt*:

- ▶ for the average U.S. corporation, 67% of total debt does not mature within the next year

This paper:

- ▶ introduces *long-term debt* (and a maturity choice) into a standard model of firm financing and investment

Main result:

- ▶ firms with outstanding long-term debt do not internalize all potential costs of default
- ▶ they increase leverage and default risk
 - ⇒ **"Debt Dilution"**
- ▶ **debt dilution reduces investment and output**

We show this:

- ▶ analytically (2-period model)
- ▶ quantitatively (dynamic model)
- ▶ empirically (using firm-level Compustat data)

Outline

1. Introduction
2. 2-period Model
3. Dynamic Model
4. Empirical Results

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2-period Model: Setup

2 periods: $t = 0, 1$

A firm owned by risk-neutral shareholders:

- ▶ earnings in $t = 1$:

$$f(k) - \delta k + \varepsilon k$$

- ▶ $f(k)$ concave \Rightarrow diminishing returns
- ▶ capital k set in $t = 0$:
 - ▶ idiosyncratic earnings shock ε uncertain
 - ▶ $\mathbb{E}[\varepsilon] = 0$

2-period Model: Debt

Definition

Debt: A bond is a promise to pay one unit of the numéraire good together with a coupon payment c at the end of $t = 1$.

- ▶ firm can raise funds in $t = 0$ by selling a number Δ_b of new bonds at market price p
- ▶ total funds raised in $t = 0$ on the bond market: $p\Delta_b$

Assume that there is an *exogenous* amount b of bonds outstanding \Rightarrow “Long-term” debt

- ▶ these bonds are otherwise identical to the one-period bonds sold in $t = 0$ and due in $t = 1$
- ▶ total stock of debt in $t = 1$: $b + \Delta_b \equiv \tilde{b}$

2-period Model: Debt & Capital

Firm chooses capital k in $t = 0$:

- ▶ firm sells new bonds and gets $\Delta_b p$
- ▶ shareholders inject equity e

$$k = e + p \Delta_b$$

Benefit of debt:

- ▶ total stock of debt in $t = 1$: $\tilde{b} = b + \Delta_b$
- ▶ coupon payments $\tilde{b}c$ are tax-deductible

Shareholder net worth q at the end of $t = 1$:

$$q = k - \tilde{b} + (1 - \tau)[f(k) - \delta k + \varepsilon k - c\tilde{b}]$$

- ▶ debt lowers tax payment by $\tau c\tilde{b}$

2-period Model: Limited Liability & Timing

Definition

Limited Liability: Shareholders are free to default in $t = 1$ and leave the firm to lenders for liquidation. A fraction ξ of firm assets is lost in this case.

Timing:

$t=0$ Given b , the firm chooses k , e , and $\tilde{b} = b + \Delta_b$

$t=1$ ε is realized.

This determines net worth q .

The firm decides whether to default.

2-period Model: Firm Problem

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$t = 1$: **Default threshold** $\bar{\varepsilon}$: $q = 0$

$$\Leftrightarrow k - \tilde{b} + (1 - \tau)[f(k) - \delta k + \bar{\varepsilon}k - c\tilde{b}] = 0$$

2-period Model: Firm Problem

$t = 1$: **Default threshold** $\bar{\varepsilon}$: $q = 0$

$$\Leftrightarrow k - \tilde{b} + (1 - \tau)[f(k) - \delta k + \bar{\varepsilon}k - c\tilde{b}] = 0$$

$t = 0$: **Firm problem** given b :

$$\max_{k, e, \Delta_b, \tilde{b}, \bar{\varepsilon}} -e + \frac{1}{1+r} \int_{\bar{\varepsilon}}^{\infty} [k - \tilde{b} + (1 - \tau)[f(k) - \delta k + \varepsilon k - c\tilde{b}] \varphi(\varepsilon) d\varepsilon$$

$$\text{s.t.: } \bar{\varepsilon}: k - \tilde{b} + (1 - \tau)[f(k) - \delta k + \bar{\varepsilon}k - c\tilde{b}] = 0$$

$$k = e + p\Delta_b$$

$$\tilde{b} = b + \Delta_b$$

2-period Model: Creditors' Problem

We have assumed that fraction ξ of firm assets is lost in case of default

Here: $\xi = 1 \Rightarrow$ liquidation value of the firm is zero

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$t = 0$: Risk-neutral lenders break even on expectation:

$$p = \frac{1}{1+r} [1 - \Phi(\bar{\varepsilon})] (1 + c)$$

2-period Model: Equilibrium

$t = 0$: Firm maximizes shareholder value subject to creditors' break even condition:

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$$k = e + p(\tilde{b} - b) \Rightarrow e = k - p(\tilde{b} - b)$$

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\Rightarrow This problem can be re-written in terms of k and $\bar{\varepsilon}$

2-period Model: Equilibrium

Simplification: $c = r$

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Consolidated problem in $t = 0$ given b :

$$\max_{k, \bar{\varepsilon}} -k + \underbrace{[1 - \Phi(\bar{\varepsilon})]}_P \underbrace{(\tilde{b} - b)}_{\Delta_b} + \frac{1 - \tau}{1 + r} k \int_{\bar{\varepsilon}}^{\infty} [\varepsilon - \bar{\varepsilon}] \varphi(\varepsilon) d\varepsilon$$

subject to: $\tilde{b} = G(\bar{\varepsilon}, k)$

2-period Model: First Order Conditions

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Capital k :

$$\underbrace{-1}_{\text{Marginal cost of capital}} + \underbrace{[1 - \Phi(\bar{\varepsilon})] \frac{\partial G(\bar{\varepsilon}, k)}{\partial k}}_{\text{Marginal increase in value of debt}} + \underbrace{\frac{1 - \tau}{1 + r} \int_{\bar{\varepsilon}}^{\infty} [\varepsilon - \bar{\varepsilon}] \varphi(\varepsilon) d\varepsilon}_{\text{Marginal increase in expected dividend}} = 0$$

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Threshold value $\bar{\varepsilon}$:

$$\underbrace{[1 - \Phi(\bar{\varepsilon})] (1 - \tau) k \frac{\tau c}{1 + (1 - \tau) c}}_{\text{Marginal tax benefit of } \bar{\varepsilon}} - \underbrace{\varphi(\bar{\varepsilon}) (1 + c) (\tilde{b} - b)}_{\text{Marginal increase in expected costs of default internalized by the firm}} = 0$$

2-period Model: Debt Dilution

Choice of threshold value $\bar{\varepsilon}$:

- ▶ marginal increase in total expected costs of default

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- ▶ marginal increase in expected costs of default **internalized by the firm**

$$\varphi(\bar{\varepsilon})(1 + c) \underbrace{(\tilde{b} - b)}_{\Delta_b}$$

- ▶ firm disregards that by increasing $\bar{\varepsilon}$ it also reduces ("*dilutes*") the value of previously issued bonds b

Proposition

The default rate $\Phi(\bar{\epsilon})$ is increasing in b .

- ▶ the higher is b , the lower is the fraction of total default costs internalized by the firm

2-period Model: Debt Dilution

Proposition

For $b > \bar{b}$, capital k is falling in b .

$$\bar{b} = \frac{(1 - \tau)k \left[\frac{f(k)}{k} - f'(k) \right]}{1 + (1 - \tau)c}.$$

For $b < \bar{b}$, capital k is increasing in b .

- ▶ the higher is b , the higher is \bar{e}
- ▶ ambiguous effect of higher \bar{e} on capital:
 - ▶ lower effective tax rate \Rightarrow higher capital
 - ▶ lower bond price \Rightarrow higher cost of capital \Rightarrow lower capital
- ▶ for $b > \bar{b}$, the second effect dominates

2-period Model: Debt Dilution

Proposition

If k is falling in b , leverage \tilde{b}/k is increasing in b .

- ▶ if k is falling in b , higher $\bar{\epsilon}$ implies higher debt \tilde{b} and therefore higher leverage
- ▶ if k is increasing in b , this may or may not hold

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Definition

Short-term Debt: In period t , the firm can sell a short-term bond. This is a promise to pay $1 + c$ in period $t + 1$.

t : firm receives $p^S \tilde{b}^S$

$t+1$: firm pays $(1 + c) \tilde{b}^S$

Definition

Long-term Debt: In period t , the firm can sell a long-term bond. A fraction γ of this bond matures each period. This is a promise to pay $\gamma + c$ in period $t + 1$, $(1 - \gamma)(\gamma + c)$ in period $t + 2$, $(1 - \gamma)^2(\gamma + c)$ in period $t + 3$, etc. ...

t : firm receives $p^L \tilde{b}^L$

$t+1$: firm pays $(\gamma + c)\tilde{b}^L$

$t+2$: firm pays $(1 - \gamma)(\gamma + c)\tilde{b}^L$

$t+3$: firm pays $(1 - \gamma)^2(\gamma + c)\tilde{b}^L$

$t+4$: etc.

Definition

Floatation cost on the bond market:

$$\eta [\tilde{b}_t^S + (\tilde{b}_t^L - b_t)]$$

The firm pays η for each bond sold on the bond market

Dynamic Model: Equilibrium

Dynamic Model: Equilibrium

Firm maximizes shareholder value subject to creditors' break even condition:

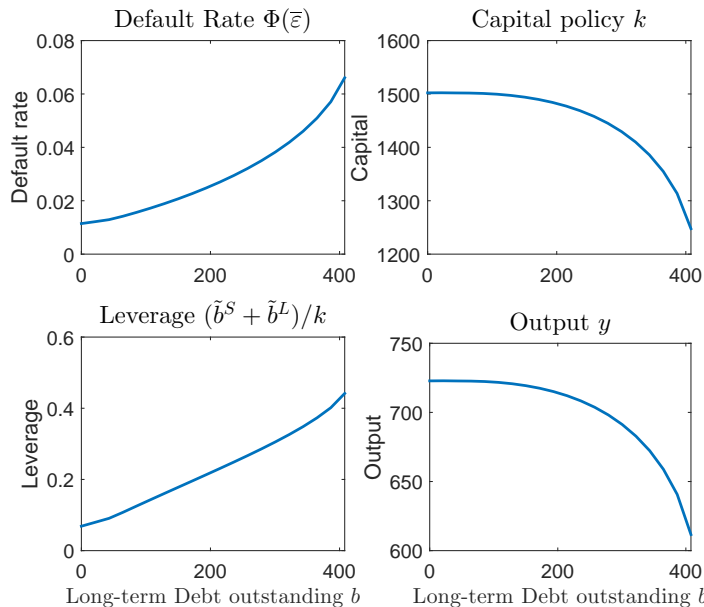
- ▶ firm cannot commit to future actions
- ▶ firm must take future firm policy as given
- ▶ time-consistent policy
- ▶ Markov Perfect equilibrium

▶ Equilibrium

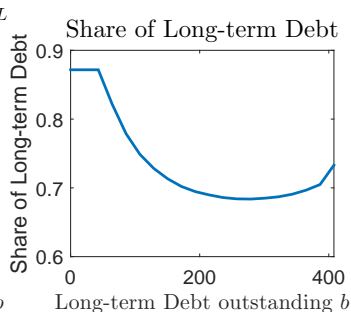
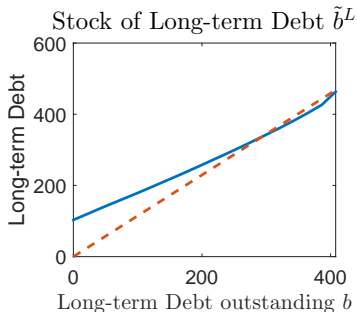
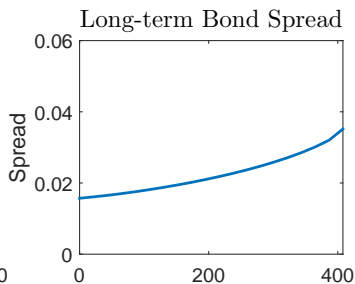
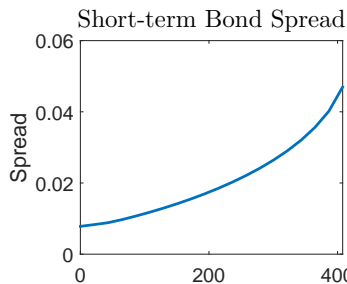
Dynamic Model: Parametrization

Variable	Description	Value	Target/Source
r	riskless rate	0.0309	
δ	depreciation	0.391	Capital-output ratio 2.07
γ	repayment rate	0.1283	Long-term debt share 67.4%
c	debt coupon	r	
τ	tax rate	0.3	<i>Hennessy and Whited (2005)</i>
σ_ε	st.dev. earnings	0.6275	Leverage 27.2%
α	decr. returns	0.9	<i>Blundell and Bond (2000)</i>
η	floatation cost	0.0109	<i>Altinkilic and Hansen (2000)</i>
ξ	default cost	0.62	Credit spread 2.30%

Dynamic Model: Policy Functions



Dynamic Model: Policy Functions



Dynamic Model: Maturity Choice

Trade-off between short-term and long-term debt (LTD):

- ▶ LTD saves **floatation costs** on the bond market
- ▶ but LTD also creates **debt dilution** in the future
⇒ higher default risk in the future

Higher future default risk hurts the firm:

- ▶ lower price of LTD sold today!
- ▶ default risk convex in b
- ▶ incentive to **reduce LTD** as b increases

Higher future default risk also hurts the holders of previously issued LTD b :

- ▶ higher b means less of the total cost of LTD is internalized by the firm!
- ▶ incentive to **increase LTD** as b increases

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One measure of debt dilution is the *OLD*-Share, the ratio of LTD outstanding b to total debt $\tilde{b}^S + \tilde{b}^L$:

$$OLD\text{-Share} = \frac{b}{\tilde{b}^S + \tilde{b}^L}$$

Theoretical prediction: the *OLD*-Share is...

- ▶ ... **positively** correlated with leverage and default risk
- ▶ ... **negatively** correlated with capital

Empirical test:

- ▶ firm-level data from Compustat 1984-2014
- ▶ Moody's Default & Recovery Database 1988-2014
- ▶ excluding financial firms and utilities

Convert the panel into a **cross-section** of firms: for firm j we use...

- ▶ **average** of firm j 's *OLD-Share* in year $t, t + 1, t + 2, \dots$
- ▶ **average** of firm j 's *leverage* in year $t, t + 1, t + 2, \dots$
- ▶ etc.

Empirical Results: Leverage

OLS (Industry FE)	Leverage	Leverage	Leverage (low Z-score)	Leverage (high Z-score)
<i>OLD</i> -Share		0.0470*** (3.83)	0.0654** (3.31)	0.0222* (2.12)
Tobin's q	0.0273*** (6.42)	0.0278*** (6.64)	0.0456*** (7.46)	0.0148 (1.70)
Profitability	-0.171*** (-8.20)	-0.172*** (-8.43)	-0.0796** (-2.77)	-0.0291 (-0.76)
Tangibility	0.253*** (6.59)	0.243*** (6.19)	0.282*** (5.47)	0.118*** (4.01)
Firm age	-0.00413*** (-7.66)	-0.00411*** (-7.75)	-0.00323*** (-3.38)	-0.00280*** (-5.60)
log Sales	0.0146*** (7.51)	0.0121*** (6.30)	0.0112** (3.04)	0.0156*** (6.81)
adj. R^2	0.2524	0.2557	0.2344	0.3025
N	5118	5115	2556	2559

Empirical Results: Default

Logit (Industry FE)	Default	Default	Default (low Z-score)	Default (high Z-score)
<i>OLD</i> -Share		0.529* (2.14)	0.830** (2.76)	-0.345 (-0.66)
Leverage	3.989*** (11.55)	3.922*** (11.45)	3.421*** (8.04)	3.470*** (4.65)
Tobin's q	-1.237*** (-6.29)	-1.238*** (-6.30)	-1.026*** (-4.29)	-1.112*** (-3.52)
Profitability	-1.407*** (-4.04)	-1.528*** (-4.53)	-0.861* (-2.27)	-4.906*** (-5.10)
Firm age	0.00323 (0.35)	0.00408 (0.43)	0.0107 (0.89)	0.0150 (0.85)
log Sales	0.429*** (11.25)	0.410*** (10.39)	0.421*** (9.67)	0.379*** (4.67)
Pseudo R^2	0.2096	0.2114	0.2377	0.1638
N	5118	5115	2556	2559

Empirical Results: Asset Growth

OLS (Industry FE)	$\Delta \log \text{ Assets}$	$\Delta \log \text{ Assets}$	$\Delta \log \text{ Assets}$ (low Z-score)	$\Delta \log \text{ Assets}$ (high Z-score)
<i>OLD</i> -Share		-0.0697*** (-7.43)	-0.0868*** (-4.91)	-0.0524*** (-4.58)
Leverage	-0.0810*** (-5.23)	-0.0725*** (-4.75)	-0.0264 (-1.47)	-0.0442 (-1.52)
Tobin's q	0.0339*** (6.46)	0.0342*** (6.56)	0.0251*** (3.40)	0.0401*** (3.98)
Profitability	0.183*** (7.65)	0.189*** (8.00)	0.148*** (4.92)	0.184** (2.74)
Firm age	-0.00646*** (-9.18)	-0.00638*** (-9.07)	-0.00675*** (-6.64)	-0.00640*** (-6.92)
log Sales	0.00556** (2.73)	0.00871*** (4.47)	0.01000** (3.14)	0.00435 (1.88)
adj. R^2	0.0830	0.0964	0.0299	0.1425
N	5116	5114	2555	2559

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Summary of results:

- ▶ we introduce *long-term debt* (and a maturity choice) into a standard model of firm financing and investment
- ▶ **debt dilution** increases default risk and leverage, and **reduces investment and output**

Room for policy (*work in progress*):

- ▶ debt dilution is a time-inconsistency problem
- ▶ future firm behavior affects price of long-term bonds sold today
- ▶ this is not internalized by the firm in the future
- ▶ policy options: seniority for LTD, ban of LTD, limit for LTD, limit for leverage, ...

Thank you!

Dynamic Model: Equilibrium

Firm maximizes shareholder value subject to creditors' break even condition:

$$V(b) = \max_{k, e', \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon}, p^S, p^L} -e' + \frac{1}{1+r} \left[\int_{\bar{\varepsilon}}^{\infty} [q' + V((1-\gamma)\tilde{b}^L)] \varphi(\varepsilon) d\varepsilon + \Phi(\bar{\varepsilon}) V(0) \right]$$

$$\text{s.t.: } q' = k - \tilde{b}^S - \gamma\tilde{b}^L + (1-\tau)[k^\alpha - \delta k + \varepsilon k - c\tilde{b}^S - c\tilde{b}^L]$$

$$\bar{\varepsilon}: \quad q' + V((1-\gamma)\tilde{b}^L) = V(0)$$

$$k = e' + p^S \tilde{b}^S + p^L (\tilde{b}^L - b) - \eta(\tilde{b}^S + \tilde{b}^L - b)$$

$$p^S = \frac{1}{1+r} \left[[1 - \Phi(\bar{\varepsilon})] (1+c) + \Phi(\bar{\varepsilon}) \frac{(1-\xi)\tilde{q}}{\tilde{b}^S + \tilde{b}^L} \right]$$

$$p^L = \frac{1}{1+r} \left[[1 - \Phi(\bar{\varepsilon})] \left(\gamma + c + (1-\gamma) p^L ((1-\gamma)\tilde{b}^L) \right) + \Phi(\bar{\varepsilon}) \frac{(1-\xi)\tilde{q}}{\tilde{b}^S + \tilde{b}^L} \right]$$

▶ Go back