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Changes in the inflation regime*

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Abstract

Typically inequality is not taken into account to assess the effects of policy changes over aggregate variables. We study the decline of expected inflation in a closed economy, and show that the efficiency effect of the decline is larger than what is usually reported even when inequality is considered. Here, the exogenous joint distribution of the households' characteristics is an important factor behind the magnitude of the change of the aggregates in the economy. In addition, we show that the decline of inflation improves equality of welfare across households for any calibration that is consistent with the actual joint distribution of the households' characteristics in a developed economy.

1 Introduction

The decline in the average inflation rate, between the early 80's and the late 90's was arguably the most sustained, widespread and important among all the economic policy regime changes in recent history. Between the early 80's and the late 90's the average inflation declined by more than 10 percentage

*Please do not quote.

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points across the developed economies and stayed stable since. This paper develops a model to evaluate the effects in welfare and inequality of this change in the inflation regime.

The majority of the empirical evidence finds a positive correlation between average inflation and measures of income inequality. Albanesi (2007) finds a strong correlation between inflation and the Gini coefficient for pre-tax income for 51 industrialized and developing countries between 1966 and 1990. Romer and Romer (1998) have quantitatively similar results obtained by regressing inequality on inflation. Easterly and Fisher (2001) present indirect evidence on the distributional effects of inflation, using household pooling data on 38 countries, find that the low income households perceive inflation as more costly than high income households. Others, like Beetsma and Van DerPloeg (1996), Bulir (1998), Crowe (2006), Dolmas et al. (2000) and Ghossouba and Reed (2017) also find a negative correlation between inflation and income inequality.¹

On the other hand, unanticipated inflation is positively correlated with inequality. Coibion et al. (2012) show that contractionary monetary policy shocks by the Federal Reserve have historically been followed by persistent increases in income and consumption inequality. Doepke and Schneider (2006) who are concerned with how unanticipated inflation shocks affect nominal asset holdings find that the main losers with inflation are the rich families. Richer households lose the most, as they tend to be net savers with deposits and short-term denominated debt. The group that would experience larger net wealth increases is middle-aged, middle-class households because they tend to hold long-term nominally denominated debt in the form of fixed-rate mortgages. Thus, there is a clear difference between unanticipated and anticipated monetary policy, in terms of its effects on inequality.²

This paper aims at understanding how anticipated monetary policy in an economy with structural heterogeneity across households impacts on the aggregate variables. The novelty is that we discipline the model with the empirical facts that characterize the cross section of portfolios used for transactions in most developed economies. One of the goals is to establish how far the results are from the ones obtained in a representative agent economy and evaluate how reasonable is the assumption of aggregation that typically

¹In contrast, Maestri and Roventini (2012) find that for a few countries the empirical correlation between inflation and various inequality series is inconclusive.

²Pedro Amaral (2017) contains a survey of the theoretical channels between monetary policy and income and wealth inequality, as well as a survey of the empirical evidence.

is made in these type of studies. We also want to understand the mechanisms by which different agents are differently impacted by the change in the inflation regime.

There is a large theoretical literature that studies the aggregate implications of inflation in a representative agent economy. There is significantly less research on how the efficiency gain of a lower inflation regime depends on the heterogeneity in the economy, and how this regime change affects inequality. The relatively few existing papers on this subject differ in many ways. They differ in their motivation for money (cash-in-advance restrictions, transactions costs, market timing frictions, trading constraints, and so on), in the (un)availability of assets other than money and production possibilities available. The reported findings often differ in several dimensions and the differences are associated with the type of model adopted. For instance, Akyol (2004) studies optimal risk-sharing in a pure exchange economy where bonds and money are held only for precautionary purposes. Positive inflation in this model ensures maximum risk-sharing, redistributing surplus to low-income agents. In a random matching model Molico (2006) shows that some inflation can improve social welfare because higher inflation can reduce wealth and price dispersion. Boel and Camera (2011) have a model based on Lagos and Wright (2005), where they show that the impact of inflation depends on the financial sophistication of the economy. If money is the only asset, then inflation hurts more the wealthier and more productive agents, while those poorer and less productive may even benefit from inflation. In a more sophisticated financial environment where agents can insure against consumption risk, with other assets, the opposite result holds. Erosa and Ventura (2002) in a transactions cost model show that at the aggregate level, inflation affects the economy as in the standard monetary growth model with a representative agent, but that inflation is effectively a regressive consumption tax. As such a decline in inflation benefits relatively more the poor households.

Our paper is closely related to Erosa and Ventura (2002). We have a similar model, with a transaction technology, bonds and production. Like they, we perform a revenue neutral experiment so that another distortionary tax is adjusted when inflation changes to satisfy the government budget constraint. We address the same two questions: whether the aggregate macro variables depend on the existing degree of exogenous inequality in the economy, and whether the decline in inflation can decrease inequality. There are differences between the two papers. We assume that households' preferences

besides depending on the consumption level are also a function of the labor hours. Thus, unlike in their model, in our model inflation, as well as the alternative tax, affects the intra marginal rate of substitution between consumption and leisure. Another difference is that they consider two types of households: one with high productivity and another with low productivity shocks. In contrast we consider more types of households, that as in the data differ with respect to wealth and productivity.

To consume the household has to buy transactions services. The transactions services are produced according to a transactions technology that uses as inputs credit and cash.³ When the transactions technology is constant returns to scale we are able to obtain a closed form equilibrium solution, using the methods described in Correia (1999), which allow the equity ranking of policies to be done by resorting to a simple relative differential concept. However, this type of technology is not compatible with the cross section facts on transactions.⁴ To conform with these facts we consider an increasing returns to scale technology. However, once the transaction technology has increasing returns to scale we are not able anymore to obtain a closed form equilibrium solution. Instead, we have to solve numerically the equilibrium conditions to obtain the equilibrium.

The paper has novel results. In the context of a constant returns to scale transactions technology a simple condition that establishes the relationship between inflation and welfare inequality is obtained. It is possible to compute the equilibrium prices of the economy as if households were identical, and to determine, in a simple way, the qualitative effect on equity of the decline of inflation. In this framework efficiency increases always with a change to a lower inflation regime. For the GHH preferences the size of the gain does not depend on the distribution of the productivity across the households.⁵ This means that for these preferences we are able to maintain the assumption of a representative agent, or that we have Gorman aggregation.

³It is irrelevant for the results to assume a market for credit or in the household production of credit, since the tax on labor is not the alternative instrument in the model. If that was the case then it would make a difference. The market provision of liquidity can be interpreted as being done by the financial sector.

⁴First, high income individuals use cash and cash plus checks for a smaller fraction of their transactions than low income individuals. Second, the fraction of household wealth held in liquid assets decreases with income and wealth. And third, a nontrivial fraction of households does not own a checking account and/or do not use credit cards to perform transactions.

⁵These preferences are described in Greenwood, Hercowitz and Huffman (1988).

Although the economy is populated with heterogeneous households, the aggregate macroeconomic variables do not depend on the specific distribution of the households' wealth or efficiency levels. On the other hand, in general the effect over welfare inequality depends on the characteristics of the wealth and productivity distributions as well as the functional form of the utility function. For the GHH preferences and for any reasonable joint distribution of wealth and efficiency the welfare inequality decreases with the change to a lower inflation regime.

In the context of an increasing returns to scale transactions technology, contrary to Erosa and Ventura (2002), for a given policy the aggregate effect depends on the exogenous heterogeneity. We consider a model economy that is able to replicate the cross section data on the means of payments used in transactions and calibrate it to the U.S. wealth and income distribution quintiles. It is found that the difference to the representative agent case is significant. Thus, the effect of the decline on efficiency is dependent on the existing heterogeneity. Another result is that the effects on equity tend to reinforce the effects on efficiency. There is no trade-off: a lower inflation increases efficiency and equity. Finally, the paper confirms Erosa and Ventura (2002) result that the impact of a moderate inflation varies noticeably across segments of society.

The remainder of the paper is organized as follows. Section 2 presents the model economy. Section 3 discusses the case where the transaction technology is constant returns to scale. Although this simple model can display the observed cross section characteristics of consumption and labor, it is not able to replicate the cross section evidence on wealth composition and transaction patterns. Section 4 takes care of this by extending the model to an increasing returns to scale technology. With this extension the model can replicate the cross section facts on payments patterns but it becomes a non-aggregable heterogeneous household model, and the equilibria associated with the different levels of inflation can only be computed by resorting to the traditional numerical computation methods. Section 5 discusses the solution method, the calibration and the results for this case. Finally, Section 6 concludes.

2 The model

The set-up is a flexible price cash in advance economy real business cycle model with heterogeneous households in which, as in Correia and Teles

(1996), a transactions technology generates an endogenous size of the credit good sector. As we are interested in studying the optimal level of inflation in the long run we assume there is no uncertainty and that inflation is fully anticipated by firms and households. As such, without loss of generality we restrict our focus to the different stationary equilibria associated with the different inflation rates. Given stationarity, the real interest rate is constant across policies and there is a one to one relationship between changes in inflation and changes in the nominal interest rate. The monetary policy regime is therefore characterized either by the nominal interest rate or by the inflation rate.

In order to consume households must pay the price of the good but must also incur in transactions costs, associated with the use of credit services and cash. As households have the same utility function, if the transactions function has constant returns to scale then the intratemporal marginal rate of substitution between consumption and leisure, per units of efficiency, is equated among all households. However, if the transactions technology is not of the constant returns type then these marginal rates of substitution will not be equated across households. Thus, an increasing returns to scale transactions technology, which is suggested by the data, together with heterogenous households generates a friction in the economy.

In the model firms are profit maximizers. The production technology uses capital, K_{t-1} , and labor, N_t , to produce the period t good according to a Cobb-Douglas production function, $F(K_{t-1}, N_t) = zK_{t-1}^\alpha N_t^{1-\alpha}$, with $z > 0$ and $0 < \alpha < 1$. The markets for the factors of production are competitive so that the wage rate, w_t , is equal to the marginal physical productivity of labor,

$$w_t = F_N(K_{t-1}, N_t) = (1 - \alpha) z \left(\frac{K_{t-1}}{N_t} \right)^\alpha, \quad (1)$$

and the rent on capital, r_t^K , is equal to the marginal physical productivity of capital,

$$r_t^K = F_K(K_{t-1}, N_t) = \alpha z \left(\frac{N_t}{K_{t-1}} \right)^{1-\alpha}. \quad (2)$$

The households take decisions over consumption, labor, capital as well as over the means of payment. Households hold money because it is an alternative means of payment to costly credit. Credit services are produced by a production function that uses only labor as input. Like in Erosa and Ventura (2002), the production function of this service uses one (efficiency)

unit of labor per unit of service produced. Competition guarantees that financial intermediaries will make zero profits and charge a price equal to their marginal cost.

The transaction technology is

$$s_{i,t} = l(m_{i,t-1}, C_{i,t}),$$

where $s_{i,t}$, $m_{i,t-1}$, and $C_{i,t}$ represent, credit services, real balances and consumption of household i , respectively. Function l is decreasing in $m_{i,t-1}$ and increasing in $C_{i,t}$. In the literature, see for instance Correia and Teles (1996), it is usually assumed that $s_{i,t}$ is time spent in transactions, which is not traded in the market. Here, because it is more realistic and simplifies the analysis, we take it as being traded in the market.

The households are heterogeneous in two dimensions. They are differentiated by their efficiency level, and their initial real wealth, represented by E_i and A_i . The efficiency levels have a positive support but the initial real wealth can be negative for some households. Each household sells hours, $N_{i,t}$, in the labor market. The market real wage for each unit of $E_i N_{i,t}$ is w_t .

Each household maximizes the discounted sum of future momentary utility levels, where the discount parameter is β , with $0 < \beta < 1$. That is, household i maximizes

$$\sum_{t=0}^{\infty} \beta^t u_{i,t}, \quad (3)$$

where $u_{i,t}$ is the momentary utility function. Function $u_{i,t}$ is the same across households, $u_{i,t} = u(C_{i,t}, N_{i,t})$.

The sequence of budget constraints of household i is:

$$\begin{aligned} P_t(1 + \tau_c)C_{i,t} + w_t P_t s_{i,t} + M_{i,t} + B_{i,t} + P_t K_{i,t} &\leq w_t P_t E_i N_{i,t} + \\ M_{i,t-1} + (1 + R)B_{i,t-1} + P_t K_{i,t-1} (r_t^K + 1 - \delta), &\text{ for } t = 0, 1, 2, \dots \end{aligned} \quad (4)$$

where P_t is the price of the good at date t , τ_c is the tax on consumption, R is the net nominal interest rate, δ is the depreciation rate and r_t^K is the rental rate of capital. The initial nominal wealth level of household i , $P_0 A_i$, is $M_{i,-1} + (1 + R)B_{i,-1} + P_0 K_{i,-1} (r_0^K + 1 - \delta)$, where $M_{i,-1}$ is the initial nominal money holdings, $B_{i,-1}$ is the initial nominal bonds, and $K_{i,-1}$ is the initial capital.

In a stationary equilibrium the intertemporal marginal rate of substitution for consumption implies:

$$\frac{P_{t+1}}{P_t(1+R)} = \beta. \quad (5)$$

The non arbitrage condition between bonds and capital requires that

$$(1+R) = \frac{(r_t^K + 1 - \delta) P_{t+1}}{P_t}. \quad (6)$$

From (5) and (6) we obtain that r_t^K is constant across policies and time. Moreover, from (2) the ratio of total hours to capital, $\frac{N_t}{K_{t-1}}$, is constant across policies and time. Therefore, the real wage, from (1), will be constant across policies and time too.

From (4) and (6) we can write the intertemporal budget constraint of household i as

$$\sum_{t=0}^{\infty} Q_t [M_t \mathcal{R} + P_t(1 + \tau_c)C_{it} + P_t w s_{i,t} - P_t w E_i N_{i,t}] = P_0 A_i, \quad (7)$$

where $Q_t = \frac{1}{(1+R)^t}$, $\mathcal{R} \equiv \frac{R}{1+R}$ and $A_i \equiv \frac{M_{i,-1} + (1+R)B_{i,-1}}{P_0} + K_{i,-1} (r_0^K + 1 - \delta)$.

Dividing (7) by P_0 and using the fact that in a stationary equilibrium condition (5) is satisfied we arrive at the stationary budget constraint of household i ,

$$(1 + \tau_c)C_i + w l(m_i, C_i) + \mathcal{R} m_i = w E_i N_i + (1 - \beta) A_i, \quad (8)$$

where we have dropped the index t to simplify the notation.

The optimal choice of money holdings satisfies:

$$-w l_{m_i} = \mathcal{R}, \text{ for } \frac{m_i}{C_i} < 1. \quad (9)$$

This equation says that the choice of real money is such that the cost of one additional unit of money, \mathcal{R} , should equalize the benefit in reducing the transaction costs, which is the decline in hours spent with credit transactions that that additional unit of money enables times the wage, $w l_{m_i}(m_i, C_i)$.

Clearing in the market for the good implies,

$$F(N, K) = \int_S C_i di + w \int_S l(m_i, C_i) di + K\delta + G, \quad (10)$$

where S is the set of households. In equation (10) we used the clearing condition in the labor market, $N = \int_S E_i N_i di$, and the clearing condition in the market for capital, $K = \int_S K_i di$.

The government collects revenues, from the inflation tax and from the tax on consumption, to finance an exogenous constant public consumption, G , and payments on the initial public liabilities,

$$A_g \equiv \frac{1}{P'_0} \left(\int_S M_{i,-1} di + (1 + R) \int_S B_{i,-1} di \right).$$

The initial public liabilities include the initial stock of money and the initial stock of public bonds.

The stationary budget constraint of the government can be obtained from (8) and (10),

$$\mathcal{R} \int_S m_i di + \tau_c \int_S C_i di = G + (1 - \beta) A_g. \quad (11)$$

The first term on the left hand side of equation (11) is the seigniorage and the second term is the consumption tax revenue. The first term on the right hand side of equation (11) is the government consumption, G , and the second term the payments on the public liabilities, A_g . Thus, when a change in the inflation regime is contemplated, the change in the interest rate implies an associated change in the consumption tax, so that the government is able to finance the same public consumption and debt payments. In particular, if the economy is on the "efficient" side of the Laffer curve a drop in the interest rate implies an increase in the consumption tax.

3 Constant returns to scale in transactions

In this section function l is homogeneous of degree 1, i.e. constant returns to scale CRS, and so it can be written as $l(m_i, C_i) = L(m_i/C_i)C_i$. Function L is characterized by $L' < 0$ and $L'' > 0$, so that an increase in the real quantity of money decreases the time spent with transactions at a decreasing rate. For a given ratio of money to consumption, $\frac{m_i}{C_i}$, the marginal and the average labor productivity on transactions do not depend on the level of consumption.⁶ An

⁶We consider this case as the benchmark case. Later, in accordance with the evidence, it will be assumed that the transactions technology has increasing returns.

example of such a technology is:

$$l(m_i, C_i) = k \left(1 - \frac{m_i}{C_i}\right)^2 C_i. \quad (12)$$

In this case condition (9) implies

$$\frac{m_i}{C_i} = \left(1 - \frac{\mathcal{R}}{2kw}\right) \leq 1, \quad (= 1 \text{ for } R = 0). \quad (13)$$

This expression has the basic money demand properties, namely that money demand increases with the amount of transactions, C_i , and declines with the opportunity cost of money, \mathcal{R} .

For a general CRS function, condition (9) implies

$$-L' \left(\frac{m_i}{C_i}\right) = \frac{\mathcal{R}}{w}, \quad (14)$$

or

$$\frac{m_i}{C_i} = \frac{m}{C} = -(L')^{-1} \left(\frac{\mathcal{R}}{w_t}\right), \text{ for all } i \in S. \quad (15)$$

The proposition follows:

Proposition 1: When the transaction technology is CRS, $\frac{m_i}{c_i}$ is identical across households. The quantity of money that a rich household maintains, as a fraction of his transactions, is the same as the one held by a poor household.

For the transaction technology (12), the budget constraint can be rewritten as

$$\mathcal{P}C_i = wE_iN_i + (1 - \beta)A_i, \quad (16)$$

where $\mathcal{P} \equiv (1 + \tau_c) + wk \left(1 - \frac{m_i}{C_i}\right)^2 + \mathcal{R} \frac{m_i}{C_i}$ is the effective price of consumption, and $\frac{m_i}{C_i}$ is given by equation (15). For a general CRS function, the effective price of consumption is

$$\mathcal{P} \equiv (1 + \tau_c) + wL \left(\frac{m_i}{C_i}\right) + \mathcal{R} \frac{m_i}{C_i}. \quad (17)$$

It follows that \mathcal{P} is constant across households, and includes the direct tax on consumption, τ_c , and the indirect cost associated with the means of

payment, $wL + \mathcal{R}\frac{m}{C}$. This indirect cost depends on the opportunity cost of holding cash, \mathcal{R} , the unitary cost of labor used in credit, w , and the transactions technology, L .

The problem of household i can be rewritten as maximizing $u(C_i, N_i)$ subject to (16). Among the first order conditions we have

$$\frac{\partial u}{\partial C_i} = \lambda \mathcal{P} \text{ and } \frac{\partial u}{\partial N_i} = -\lambda w E_i, \quad (18)$$

where λ is the Lagrange multiplier of (16). These two conditions imply that the intratemporal marginal rate of substitution between leisure and consumption is equal to the relative price of leisure

$$-\frac{u_{N_i}}{u_{C_i}} = \frac{w E_i}{\mathcal{P}}. \quad (19)$$

Inflation imposes two types of welfare costs: the cost of misallocation of resources from the good sector to the credit sector and a distortion between the relative price of leisure and its intratemporal marginal rate of substitution with consumption. A decrease in inflation diminishes these two costs, while a decrease in the consumption tax only reduces the distortion between the relative price of leisure and its intratemporal marginal rate of substitution with consumption.

We now show that a government revenue neutral decrease in inflation decreases the effective price of consumption. More specifically, we prove that the value of \mathcal{P} that keeps the revenue of government constant is decreasing in \mathcal{R} . Consider the experiment of increasing the consumption tax in the amount $\Delta\tau_c$, and decreasing the nominal interest rate by $\Delta\mathcal{R}$, so that

$$\Delta\tau_c = -\frac{m}{C}\Delta\mathcal{R} > 0. \quad (20)$$

The change in \mathcal{P} , $\Delta\mathcal{P}$, associated with (20) can be computed from (17)

$$\Delta\mathcal{P} = \Delta\tau_c + \frac{m}{C}\Delta\mathcal{R} + wL' \frac{\partial \left(\frac{m}{C}\right)}{\partial \mathcal{R}} \Delta\mathcal{R} + \mathcal{R} \frac{\partial \left(\frac{m}{C}\right)}{\partial \mathcal{R}} \Delta\mathcal{R}. \quad (21)$$

By taking into account that $L' = -\frac{\mathcal{R}}{w}$, from (14), we obtain that the sum of the last two terms in (21) is zero. If τ_c and \mathcal{R} change according to (20) then $\Delta\mathcal{P} = 0$. From (19) we get that N and C do not change since $\Delta\mathcal{P} = 0$. On the other hand, in this experiment the change in the revenue of the

government, $(\tau_c + \mathcal{R}\frac{m}{C})C$, is equal to $\mathcal{R}C\Delta\left(\frac{m}{C}\right)$, which is positive because $\Delta\left(\frac{m}{C}\right) > 0$. Since all the functions involved are continuous, it is possible to decrease \mathcal{P} and maintain the revenue of the government, if τ_c is increased and \mathcal{R} decreased in a smaller proportion than in (20).

Below we prove that in this environment a strong version of the Friedman rule holds. This result is novel because it generalizes the Friedman rule to an environment where we do not need to have a representative household. Moreover, it shows that the welfare effects of inflation can be summarized by their effects on the effective price of consumption, \mathcal{P} , which clarifies the Erosa and Ventura (2002) results.

For those households with positive initial wealth the intuition is clear: When \mathcal{P} drops the real effective wage of all households increases, i.e. $\frac{wE_i}{\mathcal{P}}$ goes up, which benefits all households, additionally, if an household has $A_i > 0$ then a drop in \mathcal{P} is advantageous too, since it increases his real effective initial wealth, $\frac{wE_i}{\mathcal{P}}$. However, if an household has $A_i < 0$ then the drop in \mathcal{P} is harmful, as it increases his real effective initial debt.⁷

It is convenient to introduce some definitions that will help proving a strong version of the Friedman rule. Define the optimal choices of household i as

$$\{C^*(\mathcal{P}, wE_i, A_i), N^*(\mathcal{P}, wE_i, A_i)\} = \arg \max \{u(C_i, N_i) \text{ s.t. (16)}\}.$$

It follows that the household i 's flow indirect utility is

$$v_i \equiv v(\mathcal{P}, wE_i, A_i) \equiv u(C_i^*, N_i^*). \quad (22)$$

Differentiation of (22) gives

$$\frac{\partial v_i}{\partial \mathcal{P}} = u_{C_i} \frac{\partial C_i^*}{\partial \mathcal{P}} + u_{N_i} \frac{\partial N_i^*}{\partial \mathcal{P}}. \quad (23)$$

The differentiation of (16) with respect to \mathcal{P} gives

$$C_i + \mathcal{P} \frac{\partial C_i^*}{\partial \mathcal{P}} - wE_i \frac{\partial N_i^*}{\partial \mathcal{P}} = 0. \quad (24)$$

Using (23), (24) and (18):

$$\frac{\partial v_i}{\partial \mathcal{P}} = -u_{C_i} \frac{C_i}{\mathcal{P}} < 0. \quad (25)$$

⁷Since consumption, C_i , is positive and equal to $\frac{wE_i}{\mathcal{P}}N_i + (1 - \beta)\frac{A_i}{\mathcal{P}}$ there is a lower bound on A_i .

Since a lower inflation is associated with a lower effective price Proposition 2 follows:

Proposition 2: A decline in inflation compensated by a revenue neutral variation in the consumption tax is a Pareto movement. Even when lump-sum taxes are not available and there is no representative household everyone is better off if money is not taxed. The government should follow the Friedman rule and set the nominal interest rate to zero.

Next, we turn our attention to how inflation affects inequality in the economy. More formally, we want to determine how the ratio of any pair of utilities, $\frac{v_i}{v_j}$, changes with inflation when both the numerator and denominator are positive and the numerator is smaller than the denominator. Does this ratio increase with a decrease in inflation? If the answer is affirmative then household i and household j , become less distant (in terms of utility) from each other, i.e. a decrease in inflation leads to a reduction in the welfare inequality. We prove that in general the answer depends on the instantaneous utility function, and the distributions of wealth and efficiency levels across households.

The sign of the change in the relative welfare caused by a change in the inflation level is given by the sign of the derivative $\left(\partial \frac{v_i}{v_j} / \partial \mathcal{P}\right)$,

$$\text{sign} \left(\partial \frac{v_i}{v_j} / \partial \mathcal{P} \right) = \text{sign} \left(\frac{\partial \log v_i}{\partial \log \mathcal{P}} - \frac{\partial \log v_j}{\partial \log \mathcal{P}} \right),$$

and

$$\frac{\partial \log v_i}{\partial \log \mathcal{P}} - \frac{\partial \log v_j}{\partial \log \mathcal{P}} = \frac{\partial v_i}{\partial \mathcal{P}} \frac{\mathcal{P}}{v_i} - \frac{\partial v_j}{\partial \mathcal{P}} \frac{\mathcal{P}}{v_j} = \frac{u_{C_j} C_j}{v_j} - \frac{u_{C_i} C_i}{v_i}. \quad (26)$$

This is the general result linking relative welfare changes to the elasticity of utility to consumption. This result is summarized in Proposition 3.

Proposition 3: When $v_j > v_i > 0$ a decline in inflation decreases inequality if $\frac{u_{C_j} C_j}{v_j} < \frac{u_{C_i} C_i}{v_i}$ for all $C_j > C_i$, and when $v_i < v_j < 0$ a decline in inflation decreases inequality if $\frac{u_{C_j} C_j}{v_j} > \frac{u_{C_i} C_i}{v_i}$ for all $C_j > C_i$.

For some momentary utility functions like the isoelastic, i.e. $u(C_i, N_i) = \frac{[C_i(1-N_i)]^{1-\eta}}{1-\eta}$, it is easy to determine the effects of inflation on inequality. For this utility function the expression (26) is zero and it follows from proposition 3 that the relative welfare does not change with inflation. For other

momentary utility functions, like the one we will be using later on in the computational exercises, it is less trivial to establish whether expression (26) holds. Consider the momentary utility function, u , which belongs to the GHH class proposed by Greenwood, Hercowitz and Huffman (1988),

$$u(C_i, N_i) = \frac{1}{1 - \sigma} \left(C_i - \epsilon \frac{N_i^{1+\chi}}{1 + \chi} \right)^{1-\sigma}, \quad \chi, \epsilon > 0, \sigma \geq 0, \text{ and } \sigma \neq 1, \quad (27)$$

where σ is the curvature parameter, ϵ determines relative importance of leisure, $1 - N_i$, and consumption, C_i and $1/\chi$ is the Frisch elasticity of labor supply. For this utility function the expression (26) is equal to

$$\frac{u_{C_j} C_j}{v_j} - \frac{u_{C_i} C_i}{v_i} = (1 - \sigma) * \text{sign}(\Phi_{i,j}), \quad (28)$$

where

$$\Phi_{i,j} = \left[\frac{C_j}{C_j - \epsilon \frac{N_j^{1+\chi}}{1+\chi}} - \frac{C_i}{C_i - \epsilon \frac{N_i^{1+\chi}}{1+\chi}} \right].$$

We assume that in equilibrium $C_i - \epsilon \frac{N_i^{1+\chi}}{1+\chi} > 0$, for all i , which implies that $v_i < v_j < 0$ for $\sigma < 1$ and $v_j > v_i > 0$ for $\sigma > 1$. In order to have a reduction in inequality after a decrease in inflation we must have, according to proposition 3, $\Phi_{i,j} < 0$. Thus, the change in relative welfare is determined solely by the sign of the difference between the gross-to-net consumption ratios, $\Phi_{i,j}$. The term $\Phi_{i,j}$ is negative if $\frac{N_j^{1+\chi}}{C_j} < \frac{N_i^{1+\chi}}{C_i}$. For these preferences the competitive equilibrium condition for the intra marginal rate of substitution between leisure and consumption, (19), implies

$$\epsilon N_j^{1+\chi} = \frac{w E_j N_j}{\mathcal{P}}.$$

Thus,

$$\frac{N_j^{1+\chi}}{C_j} \propto \frac{w E_j N_j}{w E_j N_j + (1 - \beta) A_j} \propto \frac{1}{1 + (1 - \beta) \frac{A_j}{w E_j N_j}},$$

and the sign of the term $\Phi_{i,j}$ depends on the initial wealth to labor income ratios $\frac{A_k}{w E_k N_k} \propto \frac{A_k}{E_k^{\frac{1+\chi}{\chi}}}$, for $k = i, j$, or

$$\text{sign}(\Phi_{i,j}) = \text{sign} \left((E_j)^{\frac{1+\chi}{\chi}} A_i - (E_i)^{\frac{1+\chi}{\chi}} A_j \right).$$

The condition for the sign of $\Phi_{i,j}$ to be negative is that

$$\frac{A_i}{A_j} < \left(\frac{E_i}{E_j}\right)^{\frac{1+\chi}{\chi}}, \text{ for } i \text{ and } j \text{ s.t. } v_i < v_j, A_j > 0, \text{ or} \quad (29)$$

$$\frac{A_i}{A_j} > \left(\frac{E_i}{E_j}\right)^{\frac{1+\chi}{\chi}}, \text{ for } i \text{ and } j \text{ s.t. } v_i < v_j, A_j < 0. \quad (30)$$

We can summarize the conditions under which a decrease in inflation reduces relative welfare:

Proposition 4: For the momentary utility function, u , given by (27), a decline in inflation compensated by a revenue neutral increase in the consumption tax rate improves the welfare distribution when either (29) or (30) is met.

There are simple corollaries to Proposition 3:

Corollary 1: If all households have the same positive initial wealth, $A_i = A_j > 0$ for all i and j , whatever the distribution of efficiency, a revenue neutral decrease of inflation would increase inequality.

It is easy to verify this corollary. If $A_i = A_j$ and household j is better off than household i then $E_j > E_i$. In this case condition (29) is not met:

$$1 > \left(\frac{E_i}{E_j}\right)^{\frac{1+\chi}{\chi}}.$$

Corollary 2: If all households have the same productivity, i.e. $E_i = E_j > 0$, for all i and j , whatever the distribution of wealth, a revenue neutral decrease of inflation would reduce inequality.

Notice that if $A_j > 0$ then condition (29) is met since $\frac{A_i}{A_j} < 1$; and if $A_j < 0$ then condition (30) is met since $\frac{A_i}{A_j} > 1$.

According to proposition 4, in general a decline in inflation does not imply an automatic welfare inequality reduction. It depends on whether condition (29) or condition (30) is satisfied. Conditions (29) and (30) depend on the labor supply elasticity, $\frac{1}{\chi}$, the distribution of wealth, and the distribution of efficiency levels. The literature has considered values for $1/\chi \in [0.1, 1]$ (see Dyrda et al (2012)). Among developed countries the data indicates that households with more wealth are also more productive, i.e. the two characteristics are positively correlated, and that wealth is more concentrated

than labor income.⁸ We consider the five representative households associated with the quintiles of the wealth distribution corresponding to the 1998 Survey of Consumer Finances (SCF).⁹ According to Budria et al. (2002) the wealth quintiles of the shares on the total wealth for the 1998 SCF sample are from the lowest quintile to the upper quintile: -0.3% , 1.3% , 5% , 12.2% and 81.7% . And the share in labor earnings for these wealth quintiles are 8% , 13% , 16.6% , 19.9% and 42.5% , respectively. Using these numbers it is trivial to check that condition (29) is verified even for the most extreme case, i.e. when $\frac{1}{\chi} = 1$. Thus, for the US the sign of $\Phi_{i,j}$ is negative and according to proposition 4 a decrease in inflation decreases inequality.

Next, we study how the aggregate variables react to changes in the distribution of wealth and labor productivities for the case of the utility function given by (27). We show that changes in the distribution of wealth or labor productivities, that do not affect the aggregate wealth or the aggregate productivity, do not affect the aggregate macroeconomic variables. Later, we will see that this property does not hold if the transactions technology is not of the constant returns to scale type.

The problem of maximizing (27) subject to (16) implies:

$$N_i = \left[\frac{wE_i}{\epsilon\mathcal{P}} \right]^{\frac{1}{\chi}}, \quad (31)$$

and

$$C_i = \frac{wE_i}{\mathcal{P}} N_i + \frac{(1-\beta)}{\mathcal{P}} A_i. \quad (32)$$

Equations (31) and (32) show that the labor supply of household i is independent of his wealth, A_i , and the aggregate consumption is a function of the aggregate wealth, $\int_S A_i di$. Thus, changes in the distribution of the A_i , that do not change $\int_S A_i di$, do not affect the aggregate variables. It is more demanding to prove that aggregate output and consumption do not depend on the distribution of the labor productivity.

⁸For instance, Díaz-Giménez et al. (1997) report Gini indices for labor earnings, income (inclusive of transfers) and wealth in 1992 of 0.63, 0.57, and 0.78, respectively, while for 1995 Budria et al. (2002) report values 0.61, 0.55 and 0.80. Moreover, it is well known that economic models have had difficulties in quantitatively generating the observed degree of wealth concentration from the observed income inequality, see Cagetti and De Nardi (2008).

⁹In section 4 we have a brief discussion for this choice of the base year.

Two different heterogeneous economies with the same "effective" productivity of the representative economy, $\int_S E_i^{\frac{1+\chi}{\chi}} di$, the same aggregate wealth, $\int_S A_i di$, the same public consumption, g , the same initial public debt, A_g , and the same nominal interest rate, R , have the same aggregate consumption and output. It does not matter the dispersion of the E_i 's, as long as $\int_S E_i^{\frac{1+\chi}{\chi}} di$ is invariant, because from (31) and (32) we obtain that aggregate consumption is

$$\int_S C_i di = \left[\frac{1}{\epsilon} \right]^{\frac{1}{\chi}} \left[\frac{w}{\mathcal{P}} \right]^{\frac{1+\chi}{\chi}} \int_S E_i^{\frac{1+\chi}{\chi}} di + \frac{(1-\beta)}{\mathcal{P}} \int_S A_i di, \quad (33)$$

and the output can be written, using (1) and (31), as

$$F(N, K) = \left(\frac{w^{\frac{1+\chi}{\chi}}}{1-\alpha} \right) \left(\frac{1}{\epsilon \mathcal{P}} \right)^{\frac{1}{\chi}} \int_S E_i^{\frac{1+\chi}{\chi}} di. \quad (34)$$

We summarize this result as a proposition:

Proposition 5: Both the aggregate consumption and aggregate output are invariable to variations in the wealth dispersion and productivity dispersion that keep $\int_S A_i di$ and $\int_S E_i^{\frac{1+\chi}{\chi}} di$ unchanged.¹⁰

In the next section we study what changes when an increasing returns to scale transactions technology is assumed.

¹⁰Notice that, if instead the metric was to keep the average productivity, $\int_S E_i di$, fixed, an increase in the dispersion of the E_i 's would increase $\int_S E_i^{\frac{1+\chi}{\chi}} di$. Everything else equal, an economy with a higher $\int_S E_i^{\frac{1+\chi}{\chi}} di$ would have higher aggregate consumption, and thus would have a lower τ_c , which would imply a lower \mathcal{P} , in order for the government budget constraint to be verified. Therefore, in the economy with more dispersion of the E_i 's, the aggregate consumption, $\int_S C_i di$, which is given by (33), and the output, which is given by (34), would be larger for two reasons: because $\int_S E_i^{\frac{1+\chi}{\chi}} di$ is higher and because the effective price, \mathcal{P} , is lower. The money consumption ratio, $\frac{m}{c}$, does not change as R is invariant too.

4 Economies of scale in transactions

The assumption of a constant returns to scale the transactions technology is at odds with the cross section evidence on payment patterns. High income families are more likely to perform cash management activities that reduce their exposure to the inflation tax per dollar transacted with money. As such we now assume an increasing returns to scale credit technology,

$$l(m_i, C_i) = k \left(1 - \frac{m_i}{C_i}\right)^2 C_i + \left(1 - \frac{m_i}{C_i}\right) \bar{N}, \quad (35)$$

The main difference between (35) and the credit technology (12), used before, is the inclusion of a cost, $\left(1 - \frac{m_i}{C_i}\right) \bar{N}$. This is a fixed cost for a given share of transactions with credit, $\left(1 - \frac{m_i}{C_i}\right)$.

When this transactions technology is used, the first order condition (9), which gives the optimal decision on money holdings is

$$\begin{aligned} \frac{m_i}{C_i} &= 1 - \frac{\mathcal{R}}{2wk} + \frac{\bar{N}}{2kC_i} \text{ if } C_i > \frac{w\bar{N}}{\mathcal{R}}, \text{ and} \\ \frac{m_i}{C_i} &= 1 \text{ if } C_i \leq \frac{w\bar{N}}{\mathcal{R}} \end{aligned} \quad (36)$$

It is immediate to verify that, for any $\bar{N} > 0$, the larger is the level of consumption, C_i , the smaller is the share of transactions realized with cash, $\frac{m_i}{C_i}$.

Proposition 6 follows:

Proposition 6: When transaction technologies are increasing returns to scale, the share of cash used in transactions is not constant across households. Richer households carry a lower share of their transactions with cash than poorer households.

This money demand is in line with the facts on payments. The portion of transactions paid with cash depends on the total volume of transactions carried out by the household. Households with different C_i 's have different $\frac{m_i}{C_i}$'s. The households with $C_i \leq \frac{w\bar{N}}{\mathcal{R}}$ only use cash to pay for their transactions. Define household s to be the richest household that only uses cash to pay for his transactions, i.e. one that has a level of transactions C_s , such that

$C_s = \frac{w\bar{N}}{\mathcal{R}}$. According to our ordering of the households then all households i , such that $i \geq s$, have $C_i \leq C_s$ and $m_i = C_i$. The other subset of the population, $i < s$, has $C_i > \frac{w\bar{N}}{\mathcal{R}}$ and use both cash and credit for payments. However, they use a higher share of credit, the larger are their transactions, that is the richer they are. Therefore, the higher the household's wealth, the lower is the household's cash to wealth ratio. For the group of households such that $i < s$, the money demand is given by $m_i = \left(1 - \frac{\mathcal{R}}{2wk}\right) C_i + \frac{\bar{N}}{2k}$.

The stationary budget constraint of household i , where $i \in S$, can be written as

$$\mathcal{P}_i C_i + w \left(1 - \frac{m_i}{C_i}\right) \bar{N} = wE_i N_i + (1 - \beta) A_i, \quad (37)$$

where

$$\mathcal{P}_i \equiv (1 + \tau_c) + \mathcal{R} \frac{m_i}{C_i} + wk \left(1 - \frac{m_i}{C_i}\right)^2. \quad (38)$$

The effective price of consumption, \mathcal{P}_i , is now specific to each household, includes a cost of holding money, $\mathcal{R} \frac{m_i}{C_i}$, and a cost of using credit, $wk \left(1 - \frac{m_i}{C_i}\right)^2$.

The transactions technology considered is appealing. The heterogeneity of the effective price of consumption across households is only a function of the share of cash payments done by the households, which, as stated in proposition 6, is now different across them. The higher the fraction of payments with cash the higher is the effective price of consumption, since

$$\frac{d\mathcal{P}_i}{d\frac{m_i}{C_i}} = \mathcal{R} - 2wk \left(1 - \frac{m_i}{C_i}\right) = \frac{w\bar{N}}{C_i} > 0. \quad (39)$$

It follows that:

Proposition 7: With economies of scale in the credit technologies the households with a higher $\frac{m_i}{C_i}$ ratio, i.e. the poorer agents consume less, face a higher effective price of consumption.

This result is quite important to understand the relation between inflation and inequality. It says that when the monetary model economy is able to replicate the payments facts, households react to inflation differently, some decide not to use credit, and those that use it choose different intensities. For an invariant tax rate on consumption, the existence of increasing returns to scale in the use of credit implies, an effective price of consumption that

decreases with the volume of transactions done. The richer the household is the lower is his effective price of consumption.

According to proposition 7, the effective consumption price for the poorer households is higher than for the richer ones, i.e. $\frac{\mathcal{P}_i}{\mathcal{P}_j} > 1$ for $i > j$ (according to the ordering $C_i < C_j$). When inflation increases all households face a higher price but because the richer households can substitute cash for credit at a lower cost, the price faced by the richer households increases by less than the one faced by the poorer households. The relative price of consumption across agents, i.e. the relative price $\frac{\mathcal{P}_i}{\mathcal{P}_j}$ for all $i > j$, increases with the inflation level. Thus, the effect on equity of an inflation regime change is amplified since inflation amplifies the exogenous inequality.

This can be shown formally. Using equation (38) the sign of the direct effect of a marginal change in inflation on the ratio $\frac{\mathcal{P}_i}{\mathcal{P}_j}$, i.e. $\frac{\partial \frac{\mathcal{P}_i}{\mathcal{P}_j}}{\partial \mathcal{R}}$, is given by

$$\mathcal{P}_j \frac{m_i}{C_i} - \mathcal{P}_i \frac{m_j}{C_j} = \left[(1 + \tau_c) + wk \left(1 - \frac{m_j}{C_j} \frac{m_i}{C_i} \right) \right] \left(\frac{m_i}{C_i} - \frac{m_j}{C_j} \right) > 0 \quad (40)$$

as $1 \geq \frac{m_i}{C_i} > \frac{m_j}{C_j}$. Thus, inflation has a direct effect on the relative effective price of consumption that is positive, i.e. $\frac{\partial \frac{\mathcal{P}_i}{\mathcal{P}_j}}{\partial \mathcal{R}} > 0$. Proposition 8 follows:

Proposition 8: With economies of scale in the transactions technology, the direct effect of inflation is regressive. Higher inflation increases the consumption effective price of the poorer households more than the consumption effective price of the richer ones.

We saw in the previous section that inflation is an additional source of inequality when the credit technology is a constant returns to scale technology. For the relevant labor supply elasticities and distributions of wealth and income, a decrease in inflation improves the welfare distribution, according with proposition 4. Instead, with increasing returns to scale, besides being another source of inequality, the direct effect of inflation is regressive also. Thus, we should expect that a decline of inflation would reduce inequality further, as it would have effects similar to the ones associated with the implementation of a more progressive fiscal policy.

With increasing returns to scale the effect over seigniorage can be different too. The richer households, may pay more seigniorage when inflation decreases. The change in the seigniorage paid by household i due to a change

in \mathcal{R} is $\Delta\left(\mathcal{R}\frac{m_i}{C_i}C_i\right) = \left(\frac{m_i}{C_i} - \frac{\mathcal{R}}{2kw}\right)C_i\Delta\mathcal{R} + \mathcal{R}\frac{m_i}{C_i}\frac{\partial C_i}{\partial R}\Delta\mathcal{R}$. Assuming that for a rich household the second term is sufficiently small, which happens when $\frac{m_i}{C_i}$ is relatively small and $\frac{\mathcal{R}}{2kw}$ is relatively high according with (36), then it is possible that seigniorage paid by that household may increase with a drop in \mathcal{R} . That is clearly the case for the richer households that did not use cash, i.e. for which $\frac{m_i}{C_i} = 0$, they will start paying seigniorage if the decrease in inflation is sufficiently large, for them to start using cash.

A necessary condition for aggregation is that the prices faced by the different agents be identical, but we have just seen that is not the case with increasing returns, since the effective price of consumption is specific to the household. Below, we calibrate and compute numerically the stationary equilibria associated with different levels of inflation in this non-aggregable heterogeneous household model. As always, when this type of methodology is used, the results are conditional on the particular calibration chosen. The calibration includes both values for the parameters that determine the aggregate behavior and a joint distribution of characteristics across households.

5 Calibration and findings

In this section, we describe the numerical solution, the parameterization of the economy and present the main findings. We are able to replicate the aggregate qualitative effects of inflation referred by the literature and report the distributive effects of inflation. The results show that is important to consider heterogeneity to assess the impact of inflation.

5.1 The numerical solution

Although the model is an heterogeneous non-aggregable model, the computation of the equilibrium is quite simple. It amounts to solving a system of equilibrium static equations for a given policy. In this section we describe these equilibrium conditions.

Define function $f(C_i)$ as being the optimal ratio between money holdings and consumption for household i , which is given by (36), and function $h(C_i)$ as the effective consumption price for household i , which is given by (38). We repeat both expressions here for convenience,

$$f(C_i) \equiv 1 - \frac{\mathcal{R}}{2wk} + \frac{\bar{N}}{2kC_i} = \frac{m_i}{C_i}, \quad (41)$$

and

$$h(C_i) \equiv (1 + \tau_c) + \mathcal{R} \frac{m_i}{C_i} + wk \left(1 - \frac{m_i}{C_i}\right)^2 = \mathcal{P}_i. \quad (42)$$

The budget constraint (37) of each household i can be rewritten as

$$h(C_i)C_i + w(1 - f(C_i))\bar{N} = wE_iN_i + (1 - \beta)A_i. \quad (43)$$

A first order condition of the problem of maximizing (27) subject to (43) is the intratemporal decision of household i

$$N_i = \left(\frac{1}{\epsilon} \frac{wE_i}{h(C_i) + h'(C_i)C_i - w\bar{N}f'(C_i)} \right)^{\frac{1}{\chi}} = \left(\frac{1}{\epsilon} \frac{wE_i}{\mathcal{P}_i} \right)^{\frac{1}{\chi}}. \quad (44)$$

Production is a function of aggregate efficiency hours, $N = \int_S E_i N_i di$, and aggregate capital, $K = \int_S K_i di$. Firms behave as perfect competitors. The equilibrium real wage, w , is equal to the marginal productivity of labor,

$$w = (1 - \alpha)z \left(\frac{K}{N} \right)^\alpha. \quad (45)$$

The capital labor ratio $\frac{K}{N}$ is obtained from conditions (2), (5) and (6):

$$\frac{1}{\beta} - 1 + \delta = \alpha z \left(\frac{N}{K} \right)^{1-\alpha}. \quad (46)$$

The government chooses the interest rate, \mathcal{R} , and the consumption tax, τ_c , that satisfy the budget constraint, (11), which we rewrite again here

$$\mathcal{R} \int_S m_i di + \tau_c \int_S C_i di = g + (1 - \beta)A_g. \quad (47)$$

Given the parameters β , α , δ , and z , equations (46) and (45) determine the real wage w . Given w and the parameters β , ϵ , χ , k , and \bar{N} , the initial wealth levels, A_i 's, the initial public debt, A_g , the interest rate, \mathcal{R} , the efficiency levels, E_i 's, and the public consumption, g , the four equations (41)-(44), one for each household, and equation (47) determine simultaneously the equilibrium variables $\{C_i, N_i, \frac{m_i}{C_i}, \mathcal{P}_i\}$, for every agent $i \in S$ and the tax on consumption, τ_c .

Instead of solving a big system of equations our code takes an initial guess solution for τ_c^0 and solves the various sets of four equations, (41)-(44), one

set per household, for that τ_c^0 , obtaining the values $\{C_i^0, N_i^0, \left(\frac{m_i}{C_i}\right)^0, \mathcal{P}_i^0\}_{i \in S}$. Equation (47) is verified for these values. If the left hand side of equation (47) is larger than the right hand side then the subsequent guess for the consumption tax, τ_c^1 is lower than τ_c^0 . If the left hand side of equation (47) is smaller than the right hand side then the next guess for the consumption tax, τ_c^1 is higher than τ_c^0 . For this new guess τ_c^1 we obtain $\{C_i^1, N_i^1, \left(\frac{m_i}{C_i}\right)^1, \mathcal{P}_i^1\}_{i \in S}$ and again check if equation (47) is verified for these values. The iterations stop when the left hand side and right hand side are equal up to an infinitesimal value specified by the user.

5.2 Calibration

The real net worth of US households and non-profit organizations can be obtained from the Federal Reserve's "Flow of Funds" reports, which go back to 1945. The net worth of the US households and non-profit organizations has remained relatively consistent over time. The wealth-to-GDP ratio in the period from 1945 and 1994 did not change much, fluctuated between, 3.04 to 3.72, with a historical average of 3.45 for the 50 years from 1945 – 94. After 1994 this ratio increased substantially achieving its highest value, 4.9, in 2006, and decreasing thereafter to 3.98 in 2011. The historical average for the period 1945-2011 is 3.6. Piketty and Zucman (2014) also document that the wealth income ratio for the US gradually risen over the past four decades, from about 300% in 1970 to 400% in 2010.¹¹

Typically in the data wealth or net worth includes all assets held by the households (real estate, financial wealth, vehicles) net of all liabilities (mortgages and other debts); it is a comprehensive measure of most marketable wealth. The key facts about the distribution of wealth have been highlighted in a large number of studies, among others in Wolff (2012) and Kennickell (2009). Wealth is extremely concentrated, and much more so than earnings and income, as shown by Diaz-Gimenez et al. (1997), Budria et al. (2002), and Diaz-Gimenez et al (2011). For instance in the US, in 1998 the households in the top 1% of the wealth distribution held around one third of the total wealth in the economy, and those in the top 5% held about 60 percent.

¹¹According to Piketty and Zucman (2014) the top eight developed countries over the past four decades, saw their wealth to income ratio rise from about 200–300% in 1970 to 400–600% in 2010. Italy achieved the highest value at 700%.

At the other extreme, the bottom 40 percent had little or no assets at all.

For the calibration of the A_i 's and E_i 's we chose the wealth distribution of 1998 for two reasons: it corresponds to the middle point of the 9 available data sets from the SCF and displays a level of inequality that is roughly the average of the nine levels of inequality associated with those 9 distributions of wealth. We took a conservative approach by adopting the values in Budria et al (2002) which imply a more homogeneous distribution of wealth than others, like Wolff (2012) or Kennickell (2009). As we referred earlier, according with Budria et. al (2002) the wealth quintiles of the shares on the total wealth for the 1998 SCF sample were from the lowest quintile to the upper quintile: -0.3% , 1.3% , 5% , $12, 2\%$ and 81.7% . And in 1998 the share in labor earnings for the wealth quintiles considered were: 8% , 13% , 16.6% , 19.9% and 42.5% .¹²

The α and δ were assumed to be $1/3$ and 0.1 , respectively. These are typical values in the literature for these parameters and imply a steady state capital output ratio of 2.6 annually, and a steady state investment equal to 26% of the GDP. Assuming that the total public debt is 95% percent of the annual GDP, which was approximately the value in 2011, and the monetary base is 5% of the annual GDP, which was approximately the value between 1984 to 2007, then the public liabilities, A_g , are 100% of the GDP.¹³ Thus, we took the ratio net wealth over output to be equal 3.6, which is the historical average for the period 1945-2011.

The parameter values of the utility function are $\epsilon = 1$ and $\chi = 2$ (labor elasticity of 0.5), which are within the intervals considered in the the literature. The discount factor is $\beta = 0.97$, which implies a real annual interest rate of about 3%, which with a 12% annual inflation rate, which was a typical value in the 70s and 80s, implies a nominal annual interest rate of 15% (or $\mathcal{R} = 13\%$) and with a 2% annual inflation rate implies a nominal annual interest rate of 5% (or $\mathcal{R} = 4.8\%$).

Credit services related with transactions are a small percentage of output on average. Aiyagari and Eckstein (1995) study the size of the banking sector before and after successful stabilizations of high inflations. The facts suggest that the relative size of the banking sector increases during a period of accel-

¹²Let Y stand for income. We took $A_1 = -0.003Y \cdot 3.6$, $A_2 = 0.013Y \cdot 3.6$, $A_3 = 0.05Y \cdot 3.6$, $A_4 = 0.122Y \cdot 3.6$ and $A_5 = 0.817Y \cdot 3.6$. And for the E_i 's we chose $wE_1N_1 = 0.08Y$, $wE_2N_2 = 0.13Y$, $wE_3N_3 = 0.166Y$, $wE_4N_4 = 0.199Y$ and $wE_5N_5 = 0.425Y$.

¹³After 2007 the ratio monetary base to GDP increased substantially but at the same time the Federal Reserve Bank started to accumulate large amounts of public debt and mortgage debt.

erating inflation and decreases immediately following a successful monetary stabilization. According to Aiyagari and Eckstein (1995) the value added share of credit services related with transactions for high inflation countries can reach 4%. For a low inflation countries that value is smaller, for the US in 1993 Aiyagari et al (1998) estimated that the costs (as a percentage of GDP) of U.S. commercial banks in 1993 was a little over 1%. Silva (2012), for an economy calibrated to the US, estimates the value added share of credit services for a 10% inflation to be about 2% of the GDP, and for a 0% inflation to be about 0.5% of GDP. In comparison with these estimates we were conservative: our calibration for \bar{N} and k delivers values around 1.8% of GDP for an inflation rate of 12% and costs of 0.1% for an inflation rate of 2%. For the ratio of public consumption over output we assumed it to be 0.18, which is not far from its historical value or from the values this ratio assumed in the recent past.

We measure the money supply M as the M2 stock from the Board of Governors of the Federal Reserve System.¹⁴ The model delivers a money velocity of 1.8 when the interest rate is 3% and a value of 2.9 for an interest rate of 15%. The M2 money velocity in the US between 1959-1990 fluctuated between 1.7 and 1.9. After 1990 it increased steadily until 1997 when it achieved the value of 2.2 thereafter it decreased gradually, achieving the value of 1.74 in 2010.

5.3 The change of the inflation tax

Figure 1 shows the effects of an inflation regime change associated with a decline of the nominal interest rate, R , from 15% to 3% when the model is calibrated to the US economy. As the inflation rate decreases the money consumption ratios increase for all quintiles. Except for the high interest rate levels, the decrease in inflation decreases total seigniorage and increases the

¹⁴According to Alvarez et al (2009) there is no substantial difference in the opportunity cost of demand deposits (in M1) and the components of M2 (savings and time deposits), but with respect to other short-term assets the difference is substantial. For instance the foregone interest to hold assets in retail banks relative to short-term Treasury securities U.S. between 1959-2006 was on the order of 2 percentage points. We interpret the government seigniorage as having two parts: the interest on the monetary base plus a tax on the profits of the banks, which is equal to the product between the nominal interest rate and the difference between M2 and the monetary base. Assuming that these profits are completely taxed away then seigniorage is simply the product between the nominal interest rate and M2.

total consumption tax revenue. For the high interest rates the decrease in inflation increases the seigniorage payments of the richer households which leads to an increase in the aggregate seigniorage. With the decline in the inflation rate the effective prices of consumption decrease for all quintiles and the consumption levels and hours increase for all quintiles. The effective prices of consumption converge towards the value $1 + \tau_c$.

It can be seen from figure 1, that as a result of the decrease in the inflation rate all the consumptions increase, but the consumption levels of the poorer households have a higher growth rate. For instance the consumption of the lower quintile grows 7.2% when inflation drops from 13% to 0%, while the consumption of the upper quintile only grows 3.9%. Similarly for hours, they increase for all quintiles but increase by more for the lower quintiles. For example the hours of the lower quintile grow 2.2% when inflation drops from 13% to 0%, while hours of the upper quintile only grow 1.2%. The growth rates of consumption and hours are larger for the lower quintiles because they are the ones that experience higher reductions in the effective price of consumption.

The decline in inflation increases efficiency significantly and monotonically. The cost in terms of output of a 15% nominal interest rate is almost 1.23% with respect to the best scenario, which corresponds to every household choosing a money consumption ratio of one.¹⁵ When the nominal interest rate is 5%, which corresponds to a 2% inflation rate, that cost is only 0.07%. The decline in inflation has a positive effect on equity too. All equity ratios increase monotonically with the decrease in the inflation rate. Thus, a more equal economy can be attained as well as a more efficient one, without having to resort to a pure redistribution of wealth.

To study how heterogeneity affects the results we consider a fictitious economy where all households are equal, which we will refer from now on as the homogeneous economy. Each household has the average productivity, $\left(\frac{1}{5} \int_S E_i^{\frac{1+x}{x}} di\right)^{\frac{x}{1+x}}$, and the average wealth, $\frac{1}{5} \int_S A_i di$, of the benchmark economy. The government consumption and government debt levels are the ones of the benchmark economy. The parameters are unchanged too. Unlike in the benchmark economy, in the homogeneous economy all households have the same intratemporal marginal rate of substitution. As a consequence the aggregate output is larger and the effective price smaller in the homo-

¹⁵A sufficient condition to achieve the best scenario is to follow the Friedman rule.

geneous economy. Figure 2 shows the effects in this economy of decreasing the nominal interest rate. Qualitatively the effects are similar to the ones in the benchmark economy. The sign of the effects does not depend on the heterogeneity of the economy, but the magnitudes of the effects depend on the inequality.

For any interest rate the aggregate output and consumption are larger in the benchmark economy than in homogeneous economy. The differences are significant, aggregate consumption in the benchmark economy is lower than in the homogeneous economy by about 2% and output by about 1.5%. For all interest rates the effective price in the homogeneous economy is smaller than the effective price paid by any of the quintiles in the benchmark economy. The smaller difference between the two effective prices happens when the when inflation is zero, the effective price in the homogeneous economy is 1.356, while the lowest effective price in the benchmark economy is 1.363. The consumption tax in the homogeneous economy is lower than the consumption tax in the benchmark economy, and as inflation increases the difference increases. This explains why the difference between the effective price in the homogeneous economy and the effective lowest price in the benchmark economy increases with inflation.

The decline in inflation increases output in the homogeneous economy too, but by less than in the heterogeneous economy. In the benchmark economy the output of a 15% interest rate is 1.14% smaller than the output associated with the Friedman rule, while in the homogeneous economy the output of a 15% interest rate is 1.03% smaller than the output associated with the Friedman rule, i.e. a difference of 10.7%. When the nominal interest rate is 4% the difference for the Friedman rule are 0% in the homogeneous economy and 0.05% in the heterogenous economy. Thus, the efficiency costs are underestimated in the homogeneous economy.

5.4 Effects of heterogeneity on production and equity

Do the equilibrium variables change substantially when the distribution of wealth in the economy becomes more unequal but everything else in the economy is kept constant? Does production decrease as welfare inequality increases? These are the questions we address in this section with another experiment. In this experiment the parameters, the interest rate, public consumption, public debt, households' productivity and aggregate wealth are invariable but the distribution of wealth in the economy changes. We want

to know what happens when the dispersion of the distribution of wealth increases. More specifically the population is divided in two groups: one group contains one fifth of the population, that becomes richer, while the other group contains the remaining part, four fifths of the population, which become poorer. All the households have the same productivity level, the nominal interest rate is kept at 15%, and the remaining parameters are the ones we considered for the benchmark economy, including the aggregate wealth level. Figure 3 contains the results of this experiment. The variable represented in the horizontal axis is an index of the difference in wealth for the two types. When this index takes the value 1 households are identical. As this index increases the representative household of the poorer group becomes poorer, he consumes less, has a higher ratio of money to consumption and faces a higher effective price of consumption. On the other hand, as the representative household of the richer group has more wealth, he consumes more and has a lower ratio of money to consumption. As the index of inequality increases, initially the effective price faced by the richer decreases as transaction costs decrease, but for high levels of wealth inequality, the consumption tax starts increasing faster, and even the richer households face a higher effective price of consumption.

In figure 3 we consider a large set of wealth distributions: from a situation of no inequality to a situation where the distribution of wealth is such that the representative household of the richer group consumes about ten times the amount consumed by the representative household of the poorer group. For the most unequal distribution of wealth considered the output level is about 1.8% smaller than the output associated with an equal distribution of wealth. Since the equilibrium reacts to the redistribution of wealth across the households, for a constant interest rate, the tax on consumption that satisfies the budget constraint of the government changes too. The consumption tax rate necessary to finance the exogenous government consumption increases always with the inequality. The consumption tax rate increases by almost two percentage points, from a little over 25% to about 27% as we go from an economy without inequalities to an economy with a very unequal distribution of wealth.

In the previous section we observed that decreasing simultaneously the wealth and productivity inequality led to an increase in production. In this section we saw that the same happens when we decrease wealth inequality only, keeping the distribution of productivity constant. Thus, a pure redistribution of wealth across households increases output. Here too, heterogeneity

across households implies aggregate outcomes that are somewhat different from the ones typically obtained when households are assumed homogeneous.

6 Conclusions

In this paper we have shown that, contrary to most heterogeneous agent economies, the aggregate equilibrium is also significantly affected by the heterogeneity. Identical economies in everything, except on the exogenous households' wealth and productivity joint distribution have different aggregate equilibria. While in the literature there is already a connection between inflation and inequality, there is to our knowledge no strong results on how the relation between changes in inflation and the aggregate outcome is influenced by the underlying heterogeneity. The effect of a change in the inflation regime over efficiency and welfare inequality depends on the characteristics of the wealth and productivity joint distribution.

Therefore in order to capture the gains from the decline in inflation, we need to take a position on the joint distribution that describes the households in the economy. Once we introduce in the model a distribution of the households' characteristics in line with the empirical evidence, we get positive effects on equity of an inflation reduction, and obtain in addition results on efficiency that are larger, when compared with the ones usually reported in the literature.

References

- [1] Aiyagari, S. Rao and Zvi Eckstein, 1995. "Interpreting monetary stabilization in a growth model with credit goods production," Working Papers 525, Federal Reserve Bank of Minneapolis
- [2] Aiyagari, S. Rao, R. Anton Braun, and Zvi Eckstein, 1998, Transaction Services, Inflation, and Welfare, *Journal of Political Economy*, Vol. 106, No. 6.
- [3] Akyol, Ahmet, 2004, Optimal Monetary Policy in an Economy with Incomplete Markets and Idiosyncratic Risk. *Journal of Monetary Economics*, 51(6), 1245-1269

- [4] Albanesi, S., 2007, "Inflation and Inequality", *Journal of Monetary Economics*, 54, 1088-114.
- [5] Alvarez, Fernando, Andrew Atkeson, and Chris Edmond (2009). "Sluggish Responses of Prices and Inflation to Monetary Shocks in an Inventory Model of Money Demand." *Quarterly Journal of Economics*, 124(3): 911-967
- [6] Amaral, Pedro, 2017, "Monetary Policy and Inequality", *Economic Commentary*, Federal Reserve Bank of Cleveland
- [7] Beetsma, Roel, and Frederick Van Der Ploeg, 1996, "Does Inequality Cause Inflation? The Political Economy of Inflation, Taxation and Government Debt," *Public Choice*, Vol. 87, pp. 143–62.
- [8] Boel, Paola and Gabriele Camera, 2011, Financial Sophistication and the Distribution of the Welfare Cost of Inflation. *Journal of Monetary Economics*, 56(7), 968-978
- [9] Budria Rodriguez, Santiago, Javier Díaz-Giménez, Vincenzo Quadrini, and José-Victor Rios-Rull, 2002, "Updated facts on the U.S. distributions of earnings, income, and wealth", *Federal Reserve Bank of Minneapolis Quarterly Review* 26, 2–35.
- [10] Bulir, Ales, 1998, "Income Inequality: Does Inflation Matter?" IMF Working Paper 98/7, (Washington: International Monetary Fund).
- [11] Cagetti, Marco and M. De Nardi, 2008, "Wealth inequality: data and models", *Macroeconomic Dynamics*, 12 (Supplement 2), 285–313
- [12] Correia, I., 1999, "On the Efficiency and Equity Trade-off", *Journal of Monetary Economics*, 581-603.
- [13] Correia, I. and P. Teles, 1996, "Is the Friedman Rule optimal when money is an intermediate good?" *Journal of Monetary Economics*, 38.
- [14] Coibion, O., Gorodnichenko, Y., Kueng, L. and Silvia, J. (2016). "Innocent Bystanders? Monetary Policy and Inequality in the U.S.", NBER Working Paper Series, n° 18170.

- [15] Crowe, Christopher (2006) "Inflation, Inequality, and Social Conflict" IMF Working Paper 06/158, (Washington: International Monetary Fund).
- [16] Curtin, Richard T., F. Thomas Juster, and James N. Morgan, 1989, Survey estimates of wealth: An assessment of quality. In Robert E. Lipsey and Helen Stone Tice (eds.), *The Measurement of Saving, Investment and Wealth*, Studies in Income and wealth, vol. 52, pp. 473–548. Chicago: University of Chicago Press.
- [17] Díaz-Giménez, Javier, Vincenzo Quadrini, and José-Victor Rios-Rull, 1997, "Dimensions of inequality: Facts on the U.S. distributions of earnings, income and wealth", *Federal Reserve Bank of Minneapolis Quarterly Review* 21, 3–21.
- [18] Díaz-Giménez, Javier, Andy Glover and José-Victor Rios-Rull, 2011, *Facts on the Distributions of Earnings, Income, and Wealth in the United States: 2007 Update* Federal Reserve Bank of Minneapolis Quarterly Review 34, 2–31.
- [19] Doepke, M. and M. Schneider, 2006, "Inflation and the Redistribution of Nominal Wealth", *Journal of Political Economy*, vol. 114, no. 6.
- [20] Dolmas, Jim, Gregory W. Huffman, and Mark A. Wynne, 2000, "Inequality, Inflation and Central Bank Independence," *Canadian Journal of Economics*, Vol. 33, No. 1, pp. 271–87.
- [21] Dyrda, S., Greg Kaplan, and José-Víctor Ríos-Rull, 2012, "Business Cycles and Household Formation: The Micro vs the Macro Labor Elasticity", NBER Working Paper No. 17880
- [22] Easterly, W. and S. Fisher, 2001, "Inflation and the Poor", *Journal of Money, Credit and Banking*, 1, 159-178.
- [23] Erosa, A. and G. Ventura, 2002, "On inflation as a regressive consumption tax", *Journal of Monetary Economics* 49 (4), pp. 761–795.
- [24] Gorman, W. M., 1953, "Community Preference Fields", *Econometrica*, 21, 63-80.

- [25] Ghossouba, V. and Robert R. Reed, 2017, "Financial development, income inequality, and the redistributive effects of monetary policy", *Journal of Development Economics* 126, pp. 167–189.
- [26] Greenwood, J., Z. Hercowitz and G. Huffman, 1988, "Investment, Capacity Utilization and the Real Business Cycle", *American Economic Review*, 78, 402-417.
- [27] Kennickell, Arthur B., 2003, A Rolling Tide: Changes in the Distribution of Wealth in the U.S., 1989–2001, Mimeo, Federal Reserve Board, occasional staff study.
- [28] Kennickell, Arthur B., 2009, Ponds and Streams: Wealth and Income in the U.S., 1989 to 2007, FEDS Working Paper 2009-13, Federal Reserve Board, Washington, DC.
- [29] Kennickell, A., Starr-McCluer, M., 1996. Household Saving and Portfolio Change: Evidence from the 1983–1989 SCF Panel. Finance and Economics Discussion Series 18, Division of Monetary Affairs, Federal Reserve Board, Washington, DC.
- [30] Kennickell, A., Starr-McCluer, M., Sunden, A., 1997. Family finances in the US: recent evidence from the survey of consumer finances. *Federal Reserve Bulletin* 83, 1–24.
- [31] Kessler, D., Wolff, E., 1991. A comparative analysis of household wealth patterns in France and the United States. *Review of Income and Wealth* 37 (3), 249–266.
- [32] Lagos and Wright, 2005, A unified framework for monetary theory and policy analysis *Journal of Political Economy*, 113 (3), pp. 463–484
- [33] Maestri, V. and Roventini, A. (2012). "Inequality and macroeconomic factors : a time series analysis for a set of OECD countries", Working Paper 34, University of Verona
- [34] Molico, Miguel, 2006, The Distribution Of Money And Prices In Search Equilibrium. *International Economic Review*, 47(3), 701-722.
- [35] Silva, Andre C. (2012), Rebalancing Frequency and the Welfare Cost of Inflation, *American Economic Journal: Macroeconomics*, 4(2): 153-183.

- [36] Thomas Piketty and Gabriel Zucman, 2014, Capital is back: wealth-income ratios in rich countries 1700–2010, *Quarterly Journal of Economics*.
- [37] Romer, C.D., Romer, D.H., 1998, Monetary policy and the well-being of the poor, In: *Income Inequality: Issues and Policy Options*. Federal Reserve Bank of Kansas City.
- [38] Wolff, Edward N., 1996, International comparisons in wealth inequality. *Review of Income and Wealth* 42, 433–451.
- [39] Wolff, Edward N., 1998, Recent trends in the size distribution of household wealth, *Journal of Economic Perspectives* 12, 131–150.
- [40] Wolff, Edward N., 1999, The Size Distribution of Wealth in the United States: A Comparison among Recent Household Surveys, in James P. Smith and Robert J. Willis editors, *Wealth, Work, and Health: Innovations in Measurement in the Social Sciences*, pp. 209-232.
- [41] Wolff, Edward N., 2012, The Asset Price Meltdown and the Wealth of the Middle Class, mimeo, August.
- [42] Wolff, Edward N., and Ajit Z., 2009. Household Wealth and the Measurement of Economic Well-Being in the United States. *Journal of Economic Inequality*, 7: 83-115.

Figure 1: Decreasing Nominal Interest Rate

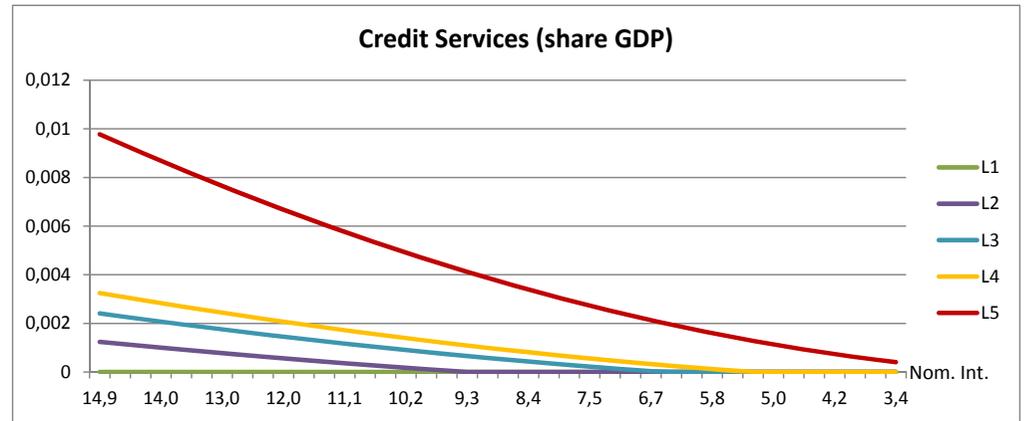
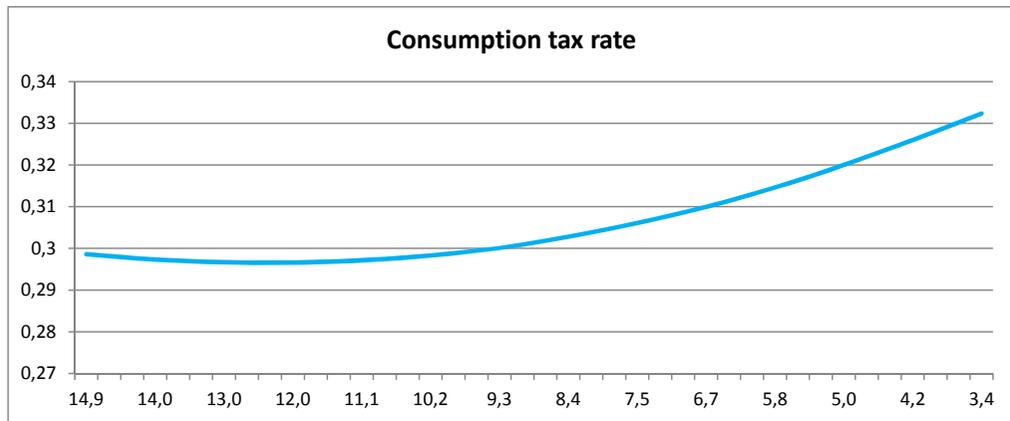
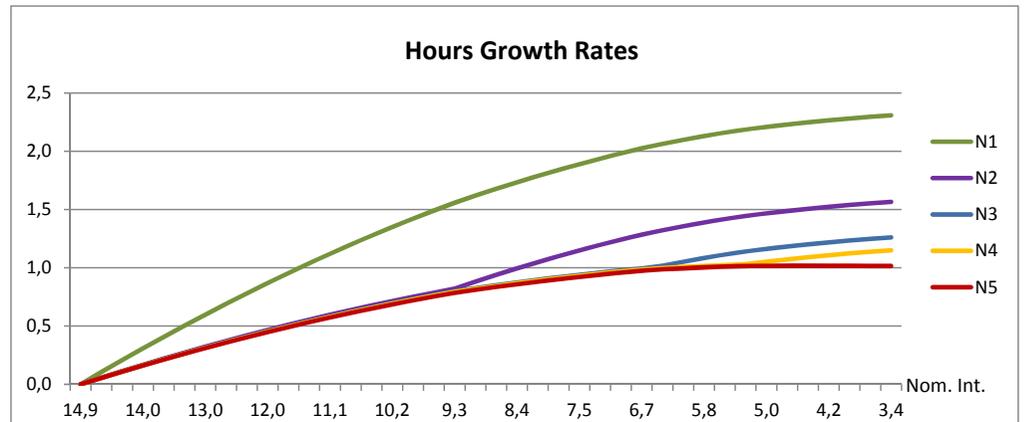
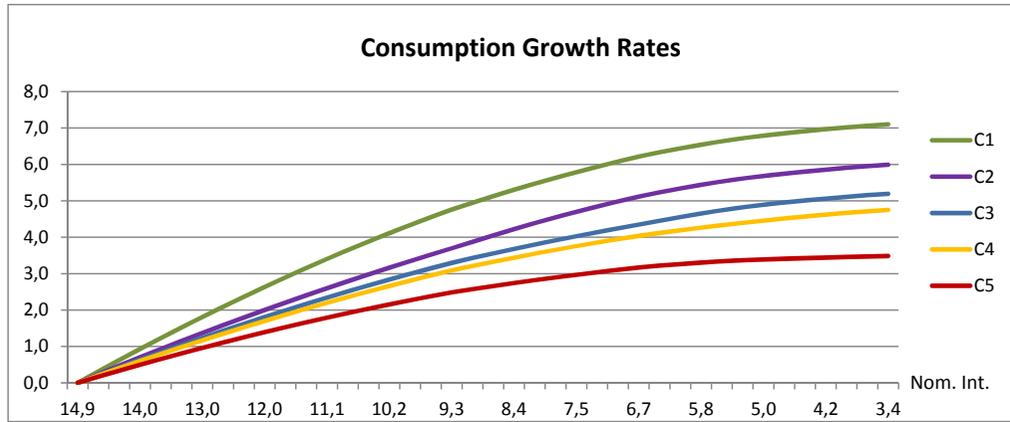
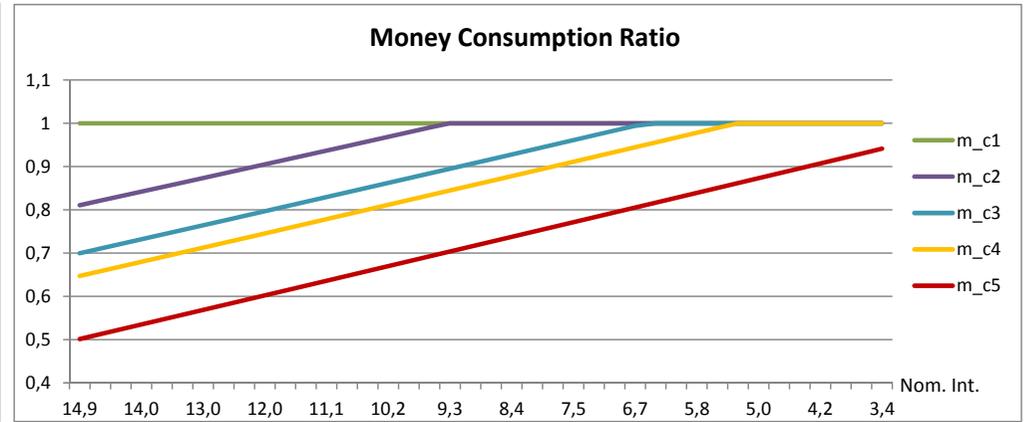
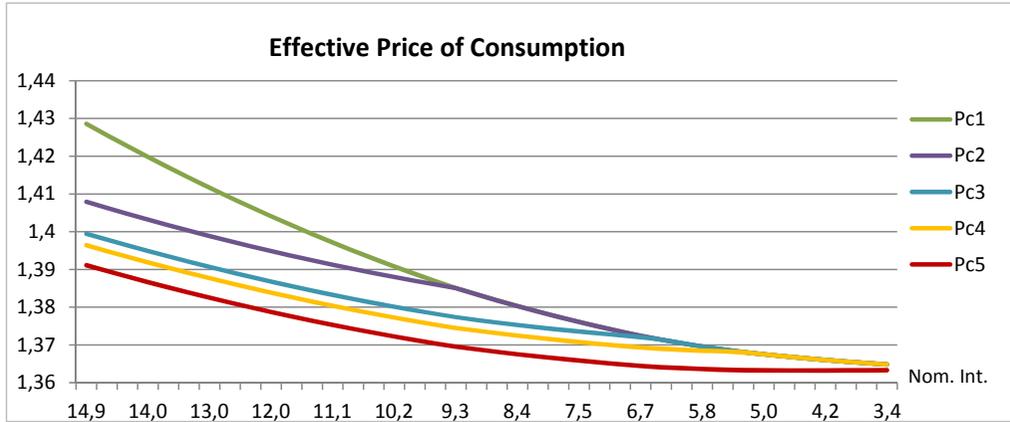


Figure 1: Decreasing Nominal Interest Rate

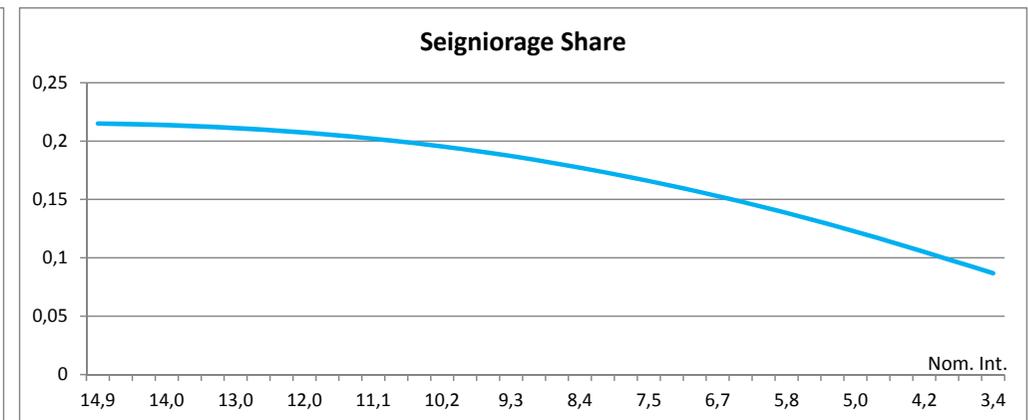
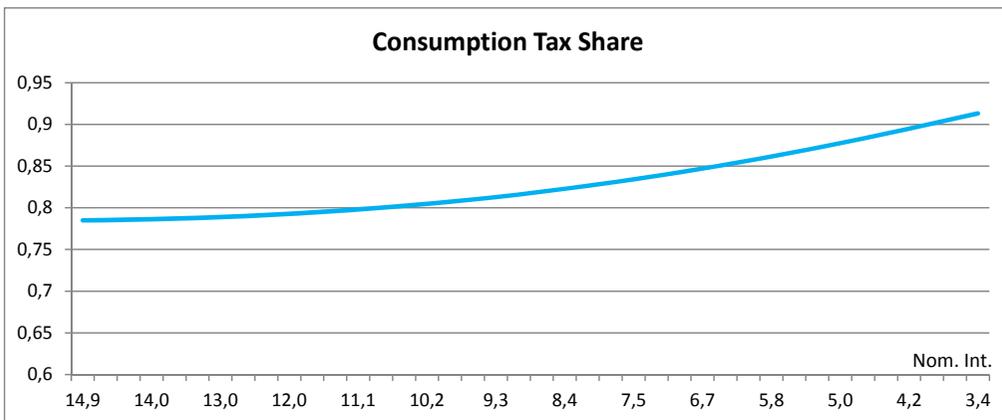
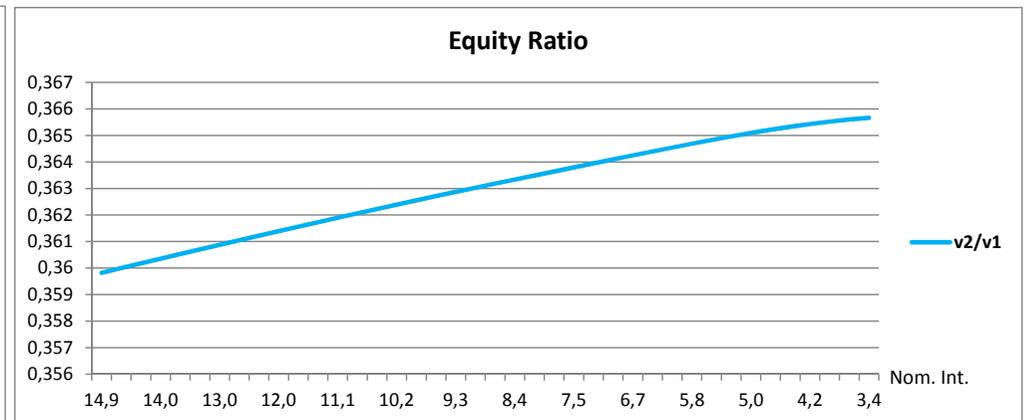
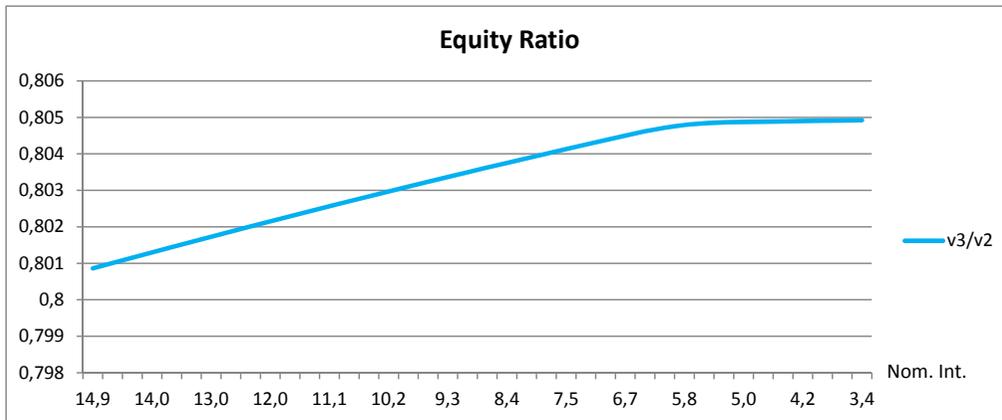
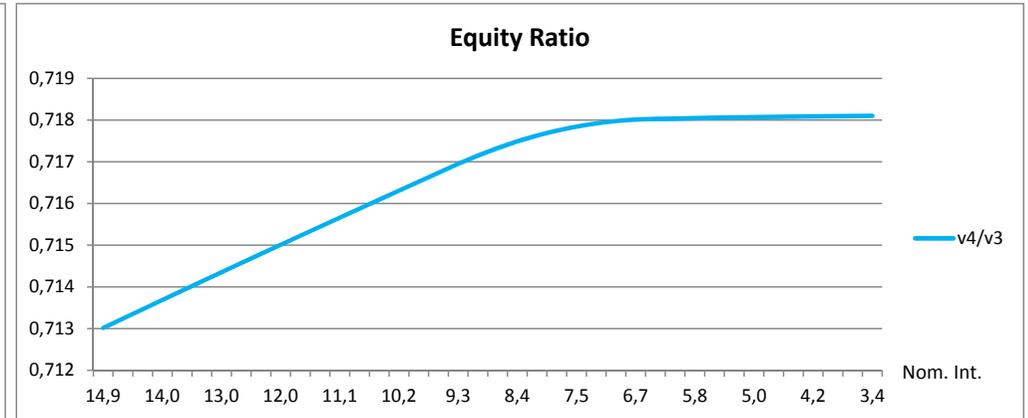
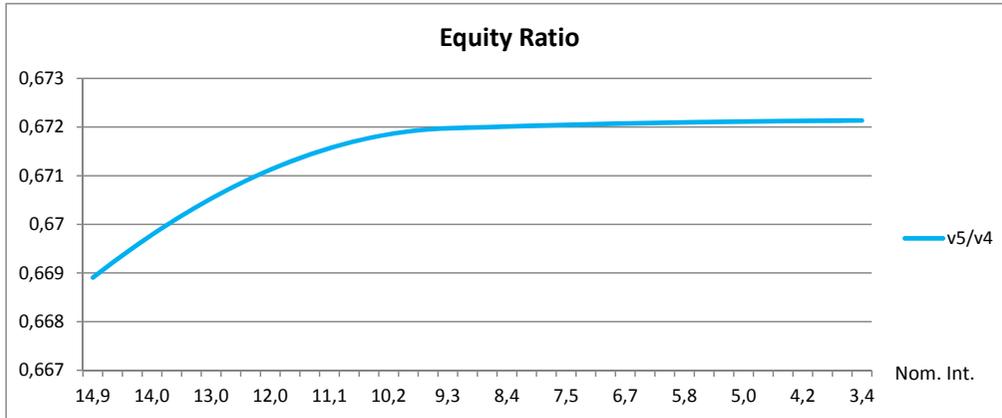


Figure 1: Decreasing Nominal Interest Rate

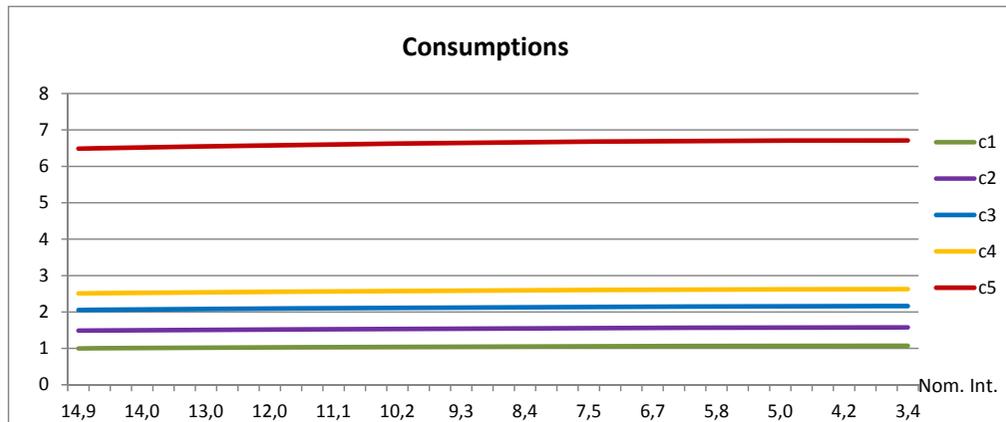
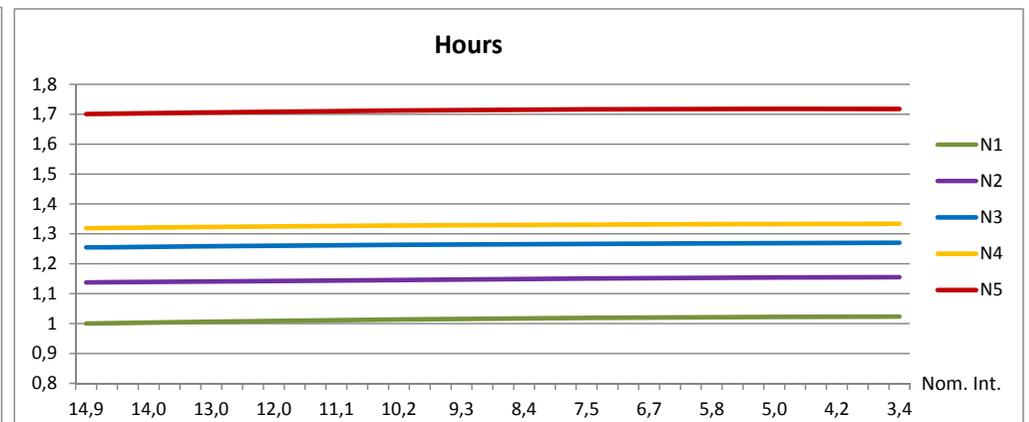
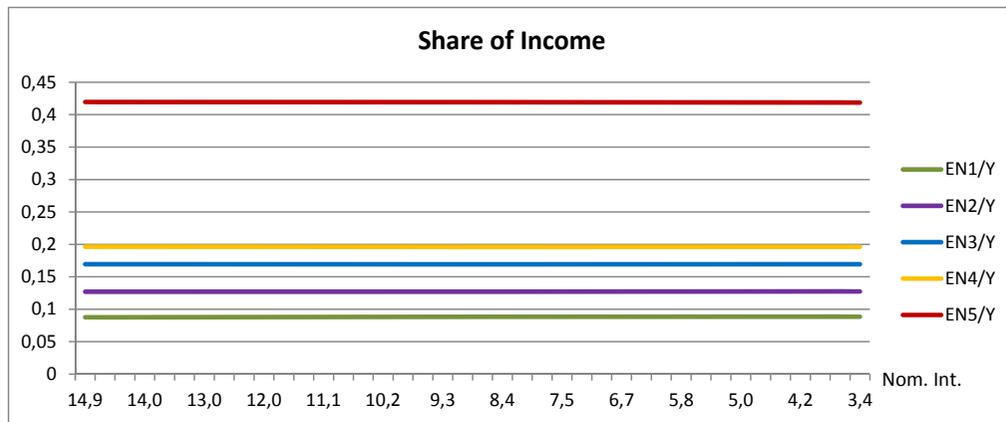
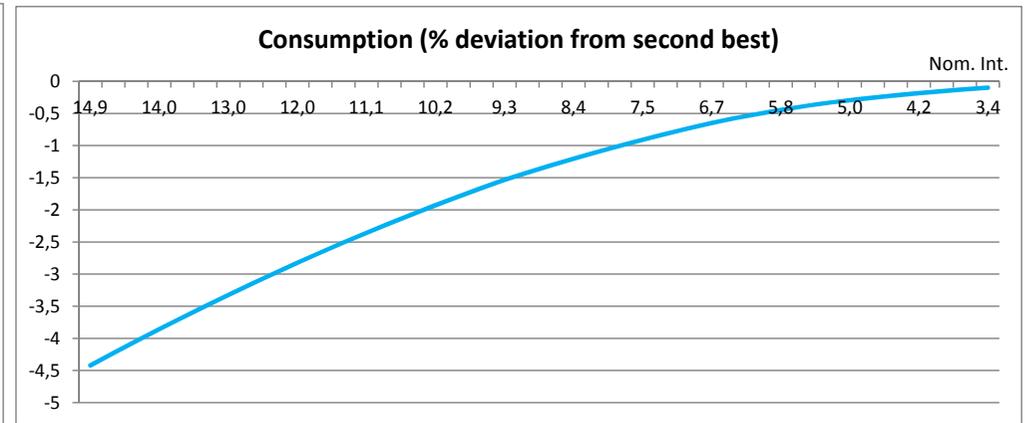
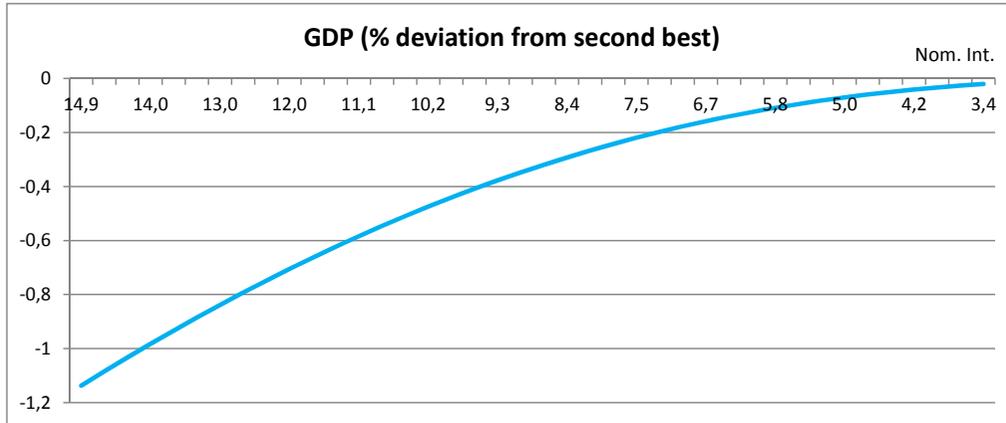
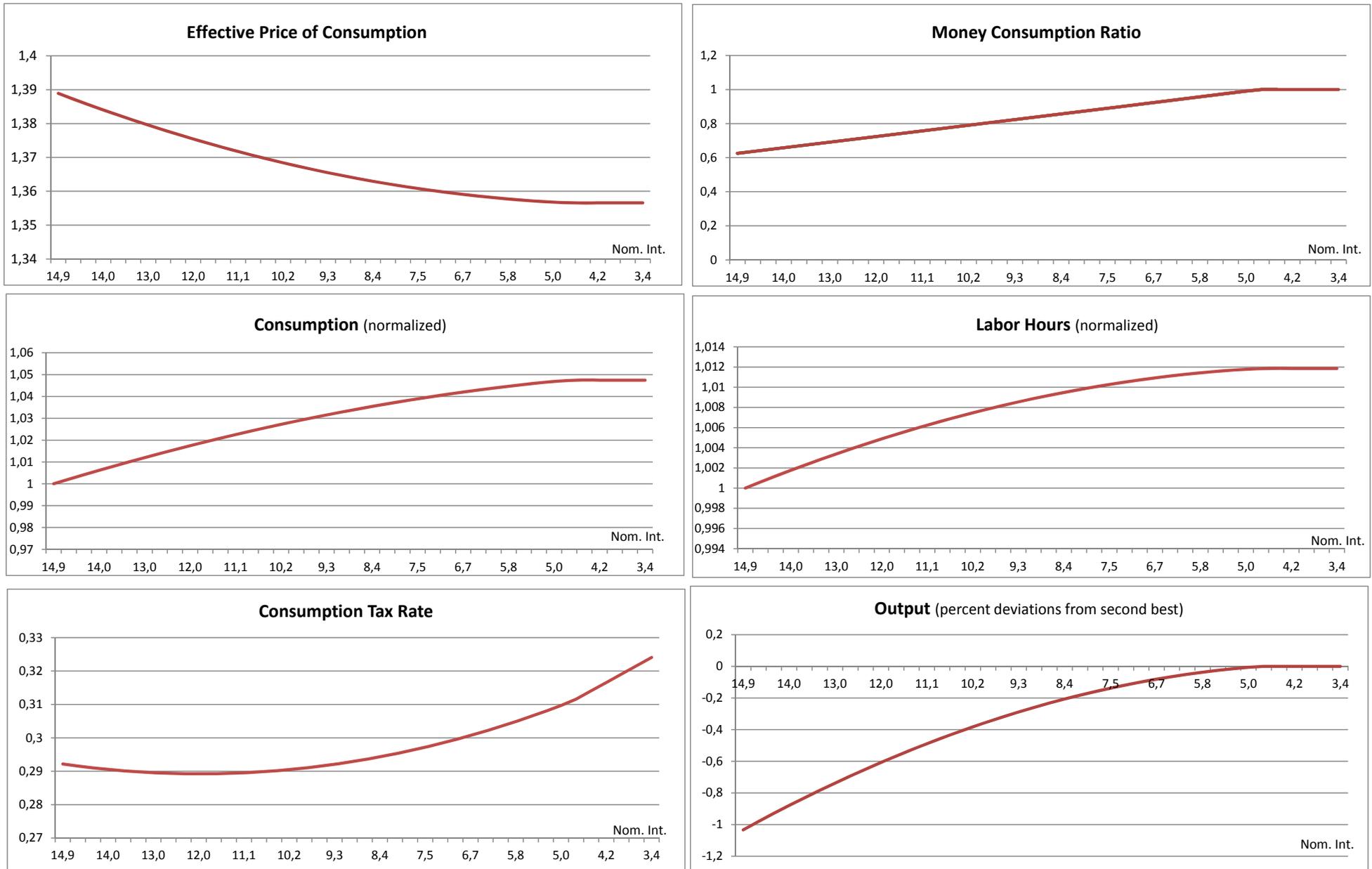
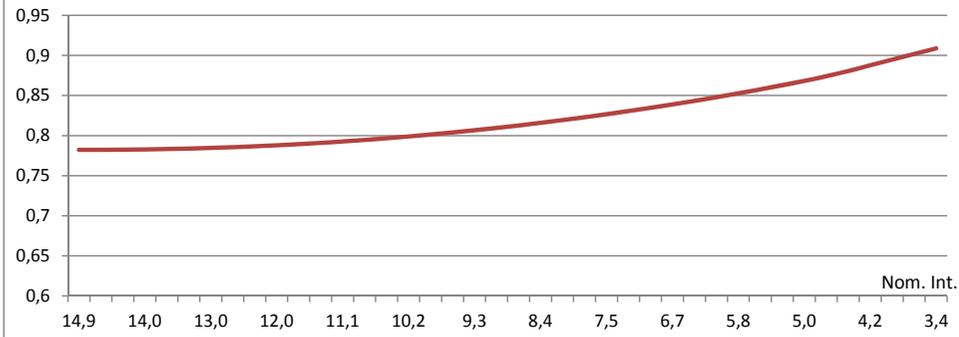


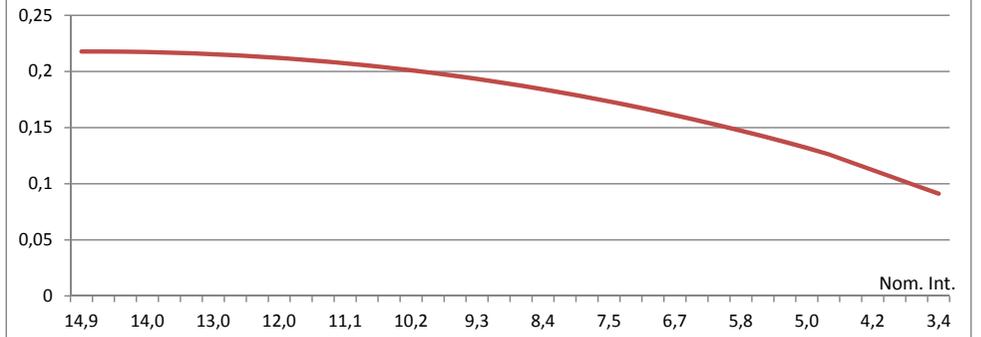
Figure 2: Decreasing Interest Rate in the Homogeneous Economy



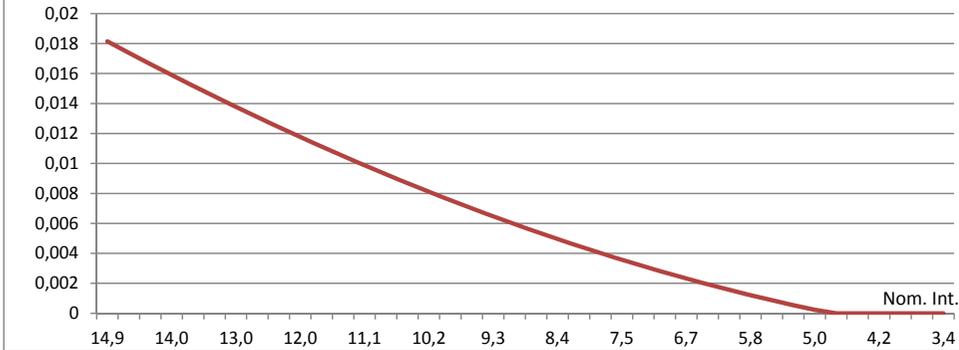
Consumption Tax Share



Seigniorage Share



Credit Services (% of GDP)



Output (% diff. hetero. & homo.)

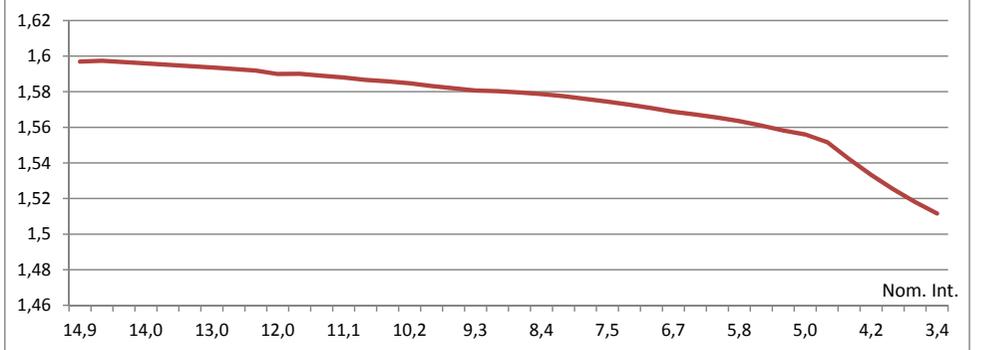


Figure 3: Increasing Wealth Inequality

