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Self-fulfilling Crises and Country Solidarity

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Abstract

Sovereign risk premia reflect investors' beliefs for the equilibrium and off –equilibrium actions of international agents. This paper investigates the international dimension of self-fulfilling sovereign debt crises and characterizes self-interested bailouts (solidarity) and contagion. A credible bailout guarantee by a partner country or international agency can lower a debtor country's borrowing costs and reduce the probability of belief driven default. However, time consistency undermines an international agent's ability to commit to intervention. Investors internalise the probability that a bailout never materializes and this endogenously increases its cost. Hence solidarity will generally be insufficient to rule out non-fundamental equilibria, explaining why high sovereign debt yields can persist despite guarantees. When countries are heavily indebted, expectations of default in one country's debt market can result in the default of its economic partner. Moreover, while large international agents are able to resolve the coordination failure, in contrast to the market, they internalise spillover costs of default and cannot credibly enforce repayment. Introducing information asymmetries results in novel on-equilibrium debt dynamics.

Keywords: Sovereign risk and default, Self-fulfilling crises, Bailouts

JEL Codes: E61, F34, F41, H63

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I. Introduction

Sovereign debt crises arise due to an unfortunate combination of poor economic fundamentals and expectations of default on behalf of international investors. During the European debt crisis, sovereign debt yields proved very sensitive to market sentiments. The European Central Bank (ECB) believed that countries in the Union were caught in a “*bad equilibrium*”.¹ Draghi emphasized that such expectations “*do not concern only the specific countries, but the Euro area as a whole*”. In addition, despite continued efforts by the European community and institutions to restore market confidence and mitigate expectations of default, GIIPS countries continue to pay a risk premium on their debt.² I present a tractable, multi-country model of sovereign debt crises and bailouts which can account for these facts.

Since the seminal work of Calvo (1988) and more recently Cole and Kehoe (2000), a large literature has emerged and has placed emphasis on the management of expectations as an indispensable axis for debt sustainability. A government’s inability to commit to a repayment policy or spending path leaves it vulnerable to self-fulfilling investor beliefs, resulting in equilibrium multiplicity. In one equilibrium, investors offer a positive price for new debt issuances, allowing the government to roll over its debt whereas in the other equilibrium new debt auctions are unsuccessful and default is optimal. Equilibrium multiplicity arises for intermediate levels of fundamentals and in this region investors’ expectations of default are self-fulfilling. In this framework, persistent risk premia and large sudden swings in sovereign yields can arise. This paper extends the literature by analysing how international cooperation, in the form of self-interested bailouts, can mitigate or amplify the macroeconomic effects of the underlying multiplicity in debt markets.

Empirical evidence in favour of self-fulfilling crises has proven elusive. De Grauwe and Ji (2013) show that the rise in spreads for peripheral EU countries was disconnected from fundamentals and they find a large and time varying residual component. Aguiar et al.

¹Mario Draghi verbatim remarks of speech at Global Investment Conference in London on July 26 2012: “The assessment of the Governing Council is that we are in a situation now where you have large parts of the Euro area in what we call a ‘bad equilibrium’, namely an equilibrium where you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios.”

²The European Stability Mechanism (ESM) has committed to lending to Greece at a rate of approximately 1% but Greek 10Y bond yields exceed 7%, evidence that markets do not deem the forward commitment to be credible. Alternatively, this could reflect insolvency concerns, yet the ESM has judged that Greek debt is sustainable.

(2016), using a sample of emerging market and European bonds, document that sovereign yields are only weakly correlated to output growth. Bahaj (2014) employs a narrative identification strategy and finds that 40–60% of trough-to-peak moves in European bond yields can be explained by foreign events. Events are chosen such that they do not contribute to the release of new information on fundamentals, but often reveal changes in the behaviour of international agencies and fluctuations in foreign and system-wide political risk. For example, the study features the agreement to bailouts for other countries, statements by finance ministers, and the outcome of the Greek elections and referenda.

During the European debt crisis, there were successful examples of international cooperation aiming to reduce debt market exposure to market panics. Following the announcement of the ECB's Outright Monetary Transaction (OMT) program, the dispersion of spreads across member states fell, despite the fact that no purchases were actually made. The European Stability Mechanism (ESM), established in 2012, has assumed the role of a permanent firewall for Eurozone member states. There have also been examples of bilateral loans between close economic partners, such as a £3.2 million loan from the UK to Ireland in 2010 and a €2.5 million loan from Russia to Cyprus in 2011 that was later restructured. However, fear of contagion was also widespread in Europe. Expectations of default in one country may have resulted in the deterioration of market conditions and the default of an economic partner. The banking and debt crisis in Cyprus can be largely attributed to the haircut of over 70% on local banks' holdings of Greek debt, amounting to €2.3 billion.

This paper provides a theoretical framework to investigate how international cooperation can rein in high yields on sovereign debt and their cross-country dispersion. I build a two country model where each country borrows from a continuum of atomistic investors and is subject to self-fulfilling crises, à la Cole and Kehoe (2000). Investors form distinct expectations of default for each sovereign. A country's default inflicts spillover costs on the other country through trade disruptions, political costs and portfolio exposures as in Tirole (2015). A credible bailout promise can invalidate investors' expectations of default and lower government borrowing costs at no cost to taxpayers.³ The reduction in borrowing costs following a credible bailout guarantee, is equal to the difference in investors' expectations of joint default as opposed to national default. A key contribution of this paper is to relax the assumption that a large international agent can commit to a given policy. A bailout policy is not credible if it suffers from time inconsistency as defined in

³Calvo (1988) was the first to point out that self-fulfilling crises can be prevented by "*an act of belief*".

Lucas and Stokey (1983) and Chari and Kehoe (1990). Ex-ante, a promise to bailout is optimal but ex-post the country in good credit standing may prefer to let its partner default rather than undertake the costly bailout. Although expectations of national default can be invalidated through off-equilibrium transfers, there exists uninsurable systemic risk in the form of system-wide default expectations, limiting both the effectiveness and the credibility of the bailout guarantee. Such risk can reflect political risk in the Eurozone due to expectations of dissolution of the Union.⁴ For sufficiently high levels of debt, in the absence of a credible bailout guarantee, an expectations-driven default in one country can push its partner to fundamental default (contagion).

The model seeks to highlight the incentives driving the agents underlying a complex institutional infrastructure. I analyse key features of sovereign debt markets such as strategic uncertainty, moral hazard, enforceability of sanctions and information asymmetry. Furthermore, I extend the literature on self-fulfilling crises to allow for market panic of arbitrary duration, which proves to be critical in determining the size of the required bailout. I show that the cost of bailouts is higher under persistent expectations of default as opposed to short-lived market panic. The model features an endogenous propagation mechanism as shocks to fundamentals and expectations induce changes in investor expectations for the equilibrium and off-equilibrium behaviour of international agents. I show that, despite popular opinion, bailout guarantees under limited commitment can incentivize governments to reduce their outstanding stock of debt to sustainable levels. Since a bailout guarantee is credible only for sufficiently low levels of debt, a borrowing government may choose to reduce its debt level so that investors believe it will be bailed out in the case of market panic. I show that in the presence of spillover costs of default, a country acting as lender of last resort for its partner cannot credibly enforce sanctions to recover debt payments, unless it resells debt to the market. This hypothesis is consistent with data presented in Schlegl, Trebesch, and Wright (2016), who find that bilateral loans are defaulted upon almost twice as often as market held bonds. I therefore model bailouts as transfers and not loans. Finally, I show that the existence of multiple equilibria, under asymmetric information, gives rise to a region for debt (pooling equilibrium) where market panics no longer

⁴Tirole (2015) points out that it is puzzling from an optimal risk sharing perspective that the relevant insurance pool for European countries is only the Eurozone. An effective risk sharing arrangement would seek to spread risk broadly geographically and politically such as the IMF's Flexible Credit Line. The IMF was initially the lender of last resort during the European crisis but its importance diminished with the introduction of the ESM. The solution lies in the incentives of economic and political partner to bear the risk.

necessarily imply default and bailouts are not always successful. In contrast to the existing literature, bailouts are observed on the equilibrium path.

Related Literature: Sovereign debt models with an endogenous default choice were first studied in the seminal work of Eaton and Gersovitz (1981). Arellano (2008) analyzes government policy under income fluctuations and shows that defaults are optimal when income is low. These models did not feature multiple equilibria. Self-fulfilling crises arising from static coordination failures amongst international investors have been studied in the influential work of Cole and Kehoe (2000), Lorenzoni and Werning (2013) and Corsetti and Dedola (2016). Cole and Kehoe (2000) consider the government's optimal time consistent policy in a dynamic stochastic general equilibrium model. Lorenzoni and Werning (2013) investigate multiplicity arising due to a dynamic coordination failure between investors who fear future default. In both models, there is a role for a large agent to coordinate expectations through purchases and prevent self-fulfilling crises. Corsetti and Dedola (2016) evaluate the Central Bank's ability to prevent self-fulfilling crises via debt debasement and balance sheet policies.

This paper also relates to a growing literature on international sovereign debt crises. Tirole (2015), in a framework with output risk, shows that spillover costs can be used as collateral when a country borrows from the market. I extend this result to a dynamic setting with self-fulfilling crises and show that spillover costs act as *off-equilibrium collateral*, eliciting a bailout from the partner country when the market refuses to lend. The potential for future sharing of strategic uncertainty acts as an additional incentive for cooperation. Arellano and Bai (2013) consider a two country model, where linkages arise because countries can simultaneously bargain with the market in the case of default, to extract a larger surplus. They incorporate the importance of market structure to sovereign debt models and highlight the possibility of a coordination failure between borrowers. The literature on bailouts that aim to eliminate roll over risk is small and assumes a third-party large agent who can commit to an intervention. Roch and Uhlig (2014) and Conesa and Kehoe (2014) consider debt purchases from bailout agencies committed to restoring a fundamental equilibrium. They find that expectations-driven defaults are only avoided at the cost of increased fundamental defaults, due to increased borrowing and a higher debt burden. I analyse the credibility of a bailout guarantee as an equilibrium object by considering a symmetric environment where countries can choose to undertake a bailout for one another. Cooper (2012) extends Calvo (1988) and considers a two-period model where countries

belong to a federation and will collectively undertake a bailout to smooth consumption across countries.

Section 2 outlines the basic framework in a one country model of strategic default. Section 3 tackles the two-country model with self-interested bailouts and contagion. Section 4 looks at the effects of bailout guarantees on ex-ante optimal borrowing policy. Section 5 discusses the enforceability of repayment by the official sector. Section 6 introduces information asymmetry, and section 7 concludes.

II. A basic self-fulfilling crises framework

A. Full commitment

I consider an infinite horizon model building on the real framework in Aguiar et al. (2015a). I begin by presenting the consumption smoothing problem faced by a government. The government's utility is given by,

$$U = \int_0^{\infty} e^{-\rho t} u(c(t)) dt$$

I require standard assumptions of smoothness and concavity of the utility function and $\lim_{c \downarrow 0} u'(c) = \bar{u}$, $\lim_{c \uparrow \infty} u'(c) = 0$, $\bar{u} < \infty$. I define $V : \bar{\Omega} \rightarrow \mathbb{R}$ as the value function for the utility of repayment. $\bar{\Omega}$ is the set of debt levels b for which the government wants to participate in debt markets. The government's consumption function is given by $C : \bar{\Omega} \rightarrow \mathbb{R}_{++}$.

The government borrows by issuing non-contingent bonds to a continuum of risk-neutral, atomistic investors who constitute the international market. I denote the stock of outstanding debt by $b(t)$. Investors can invest at the world interest rate r^* and discount the future at the same rate as the government. The total wealth commanded by these investors is large. As in Cole and Kehoe (2000), investors will purchase any amount of bonds as long as their expected rate of return is r^* . The interest rate schedule is a function of the outstanding stock of debt, $r : \bar{\Omega} \rightarrow \mathbb{R}_+$.

Bonds must be continuously rolled over and their price is normalised to 1. Debt issuances contribute to servicing the primary deficit. The government receives a constant endowment

flow y and consumes $c(t)$. The controlled process for debt is given by,

$$\begin{cases} \dot{b}(t) = c(t) - y + r(b(t))b(t) \\ b(0) = b_0 \end{cases}$$

The government chooses a consumption level and whether or not to default. The utility of servicing debt is given by,

$$V(b) = \max_{c(t)} \int_0^{\infty} e^{-\rho t} u(c(t)) dt \quad (1)$$

$$s.t \quad \dot{b}(t) = c(t) - y + r^*b(t) \quad (2)$$

The government can, at any point in time, choose to default on its outstanding stock of debt. Default is punished by a direct cost proportional to the endowment αy and exclusion from debt markets.⁵ Default is an absorbing state and yields pay-off \underline{V} given by,

$$\underline{V} = \frac{u((1 - \alpha)y)}{\rho} \quad (3)$$

An equilibrium is a value function $V(b)$ and an associated policy function C , such that (1) is maximized subject to (2), given the interest rate r^* . Assuming $\rho = r^*$, if repayment is optimal, the government consumes $c(t) = y - r^*b(t)$ and $\dot{b}(t) = 0$. The government will choose to default if $V(b) < \underline{V}$. Since, $V(b)$ is a decreasing function, a country will never default for levels of debt in $\bar{\Omega} = [0, \frac{\alpha y}{r^*}]$.

B. Limited commitment

If the government is unable to commit to a consumption path or default policy and this leaves it vulnerable to self-fulfilling crises à la Cole and Kehoe (2000). Investors decide whether or not to purchase new debt simultaneously. For intermediate levels of debt, the government will repay its debt only if investors roll over debt. At the heart of self-fulfilling crises is a static coordination failure amongst investors which gives rise to a strategic complementarity. In this section, I extend the literature to allow for an arbitrary period of

⁵Direct costs of default can reflect trade disruptions, reputational damage and political costs. As is standard in the literature, they are assumed independent of the stock of defaulted debt. Yue (2010) allows for a recovery rate on debt and Bi (2008) allows renegotiation and re-entry after default.

market exclusion for the government, motivated as a persistent coordination failure between investors. I argue that this has important implications for quantitative studies.

In the case of market exclusion, I assume the government is required to repay its outstanding stock of debt at rate χ , such that $\dot{b}(t) + \chi b(t) \leq 0$. This assumption serves as a reduced form of maturity, or can be interpreted as an ad-hoc restructuring protocol.⁶ This results in a front loaded debt repayment schedule, a feature common in recent sovereign debt crises.⁷ The model highlights that debt sustainability is high dimensional object with debt-to-GDP being an insufficient statistic.

The coordination failure arises as follows: each individual investor, prior to lending, considers a scenario where all other investors refuse to lend to the government. The individual investor commands measure zero wealth, therefore, his lending does not relax the governments financing needs. Should the government prefer to repay, absent new debt auction revenue, then each investor will find it profitable to individually deviate and purchase new debt. I present a model where market exclusion persists for a duration δ , stretching over several debt auctions.⁸ This allows me to distinguish between short market panics and longer intervals of market exclusion. In reality, the duration of market exclusion will depends on institutional features such as the format and frequency of debt auctions, and on the persistence of default expectations. The repayment schedule χ and duration of coordination failures δ are key determinants whether governments are vulnerable to self-fulfilling crises.

A government faced with market exclusion (roll-over crisis) undertakes the following opti-

⁶Note that the maturity structure of debt is inconsequential if the government is able to commit to not default. Chatterjee and Eyigungor (2012) argue that short and long term debt will be equivalent even in the presence of default risk if the sovereign agrees to compensate investors via a transfer if the price of debt falls due to debt dilution and investors agree to compensate the sovereign if bond prices rise.

⁷According to data from the Greek Public Debt Management Agency, the Greek repayment schedule is significantly frontloaded. In emerging countries, Bai, Kim, and Mihalache (2014) rationalise the issuance of short term but back loaded liabilities.

⁸This resembles a forgiving grim trigger strategy. Once investors anticipate the possibility of default, they do not lend for δ .

mization,

$$V^{\text{off-eq}}(b) = \max_{c(t)} \int_0^\delta e^{-\rho t} u(c(t)) dt + e^{-\rho \delta} V(b(\delta)) \quad (4)$$

$$\begin{aligned} \text{s.t. } \dot{b}(t) &= c(t) - y + r^* b(t), & \dot{b}(t) &\leq -\chi b(t) \\ & & b(t) &\in \bar{\Omega} \end{aligned} \quad (5)$$

A self-fulfilling crisis can occur in equilibrium if $V^{\text{off-eq}}(b) < \underline{V}$.

Lemma 1 (Optimal deleveraging policy)

A government excluded from markets will find it optimal to repay its debt at the minimum rate possible $\dot{b}(t) = -\chi b(t)$, for any duration of exclusion δ .

Proof. Provided in the Appendix A1. ■

While a government is excluded from markets, I show that the deleveraging constraint binds for all $\delta > 0$. At δ , consumption jumps and the government pursues a constant consumption and debt path, derived as the optimal control to (1-2). Investors demand interest r^* on existing debt entering the period of market exclusion, reflecting the *restored equilibrium* interest rate.⁹

Lemma 2 (Sustainable level of debt)

The level of debt \underline{b} below which a government is not vulnerable to self-fulfilling crises is given by,

$$\underline{b} = \max \left\{ b(t) \leq \frac{y}{\chi + r^*} : V^{\text{off-eq}}(b) \geq \underline{V} \right\} \quad (6)$$

As $\delta \rightarrow 0$ or $\chi \rightarrow 0$, a self-fulfilling crisis is not possible for $b \in \bar{\Omega}$.

Proof. Provided in the Appendix A2. ■

Lemma 1 implies $V^{\text{off-eq}}$ is decreasing function in δ , since, as the exclusion period grows, a greater portion of debt will need to be retired.¹⁰ The sustainable level of debt \underline{b} is therefore decreasing in δ , a dimension overlooked by the existing literature. Conesa and Kehoe (2014)

⁹If we consider (χ, δ) to be a clause in the debt contract, the government will never choose to trigger it if there is no roll over crisis. This was pointed out in Aguiar et al. (2013) for a fixed χ but generalises immediately.

¹⁰Any δ can be part of an equilibrium. If all other investors share a horizon of δ , no individual investor has an incentive to deviate from this. The exclusion period can be derived to reflect a required recovery rate of debt off-equilibrium in which case $\delta(\chi)$ will be an increasing function. Allowing the repayment schedule to affect the horizon at which investors assess credibility reduces the benefits to long maturity debt by leading to a prolonged coordination failure amongst investors.

and Roch and Uhlig (2014) assume market exclusion off-equilibrium lasts one period only ($\delta = 1$) whereas Cole and Kehoe (2000) assume perpetual exclusion ($\delta = \infty$).

High gross financing needs in any given period will prompt a government to default, even if the total stock of debt that needs to be retired is small. Extending the debt repayment schedule ($\chi \uparrow$) reduces the financing needs of the government for all t and leads to much higher levels of sustainable debt. On the other hand, shortening the duration of the exclusion period ($\delta \downarrow$) is less effective since the financing needs of the government in the early periods remain high, leading to non-linear comparative statics.

Following the literature, I define the interval $(b, \bar{b}]$ as the crisis region \mathcal{C} . In this region, equilibrium selection depends on a sunspot which coordinates investor beliefs. The arrival time of default expectations τ is a random variable governed by a Poisson process with intensity λ . The probability of an expectations-driven default in any interval $t \in [0, T]$ is given by,

$$\mathbb{P}(\tau \leq T \mid b \in \mathcal{C}) = 1 - e^{-\lambda T}$$

In anticipation, investors demand a risk premium λ for levels of debt in the crisis region, such that, they earn r^* in expectation. This results in a discontinuous interest rate schedule. The government's participation condition in debt markets is evaluated at $r(b) = r^* + \lambda$, to define the fundamental default threshold $\bar{b} = \max(\bar{\Omega})$.

III. International self-fulfilling crises

I define *solidarity* to be the international cooperation that arises from the optimizing behaviour of governments.¹¹ Sovereign risk premia reflect investor expectations for the equilibrium and off-equilibrium behaviour of large agents, such as other governments and international agencies. I develop the problem faced by a country in good credit standing, should it choose to undertake a bailout for its partner. Solidarity can eliminate self-fulfilling crises and reduce market financing costs, with no purchases necessary on the equilibrium

¹¹A key difference between countries and international agencies is that the governance of the latter is designed to suppress individual incentives in favour of the institutional agenda. The mandates and structures of these agencies are designed to endow the agency with credibility which can be modelled by modifying the utility $U = u(c) - \eta(b)$, where $u(c)$ is utility from own consumption or loans to other countries and $\eta(b)$ is the disutility from a self-fulfilling default in the system which can be a function of the defaulting country's debt. $\eta'(b) < 0$ reflects the degree of discipline.

path, if it is a time consistent policy. For this reason, the ESM publishes its “*forward commitment capacity*” to both inform markets that it is willing to assume to role of lender of last resort, and to convince markets of the credibility of this promise.¹² Uninsurable risk in the system, which can reflect political risk in the Eurozone, undermines both the credibility and effectiveness of solidarity.

Investor beliefs are coordinated by sunspot processes that can dictate the switch to a bad equilibrium. These processes govern news that affect investor expectations of default and is unrelated to fundamentals.¹³ Three independent Poisson processes reflect the arrival of default expectations in each national debt market and for the system as a whole, with constant intensity $\{\gamma_n\}_{n \in \{i,j,z\}}$ respectively.¹⁴ Absent solidarity, national default expectations for country i are formed with intensity $\lambda_i^c = \gamma_z + \gamma_i$ and analogously for j . Joint default expectations are formed with $\lambda^s = \gamma_z$. In the presence of a credible commitment to bailout, a self-fulfilling crisis can only arise following system-wide default expectations.

I focus on a two country model, symmetric in policy functions and fundamentals. The initial symmetry can break down both on and off equilibrium because of heterogeneity in default expectations. Should a country choose to default, it incurs a flow cost of default αy and is excluded from markets. Its partner incurs endowment cost $\hat{\alpha} y$ due to spillover costs of default. These can be motivated by trade disruptions, political costs and financial exposures. In the case of joint default, each country earns utility \hat{V} , incurring a cost of $(\alpha + \hat{\alpha})y$ and experiencing exclusion from bond markets.

A. *Solidarity*

Solidarity in the model results in a bailout game with two-sided lack of commitment. Governments cannot commit to repay their own outstanding debt, nor can they commit to bailout a partner. A country in good credit standing, should its partner face market exclusion, can choose to undertake a bailout b^I to restore a fundamental competitive equilibrium.

¹²Source: <http://www.esm.europa.eu/assistance/FCC/index.htm>

¹³Equilibrium selection may well be the result of a fundamental shock such as a fall in output or a debt shock but the model focuses on the strategic uncertainty in sovereign debt markets. I abstract from direct effects of fundamental shocks and focus on their implications for equilibrium selection. Cole, Neuhann, and Ordoñez (2016), in a model of endogenous information acquisition, show that small fluctuations in fundamentals can induce large changes in risk spreads internationally.

¹⁴This resembles a multivariate exponential model, as discussed in Duffie and Singleton (1999). Arrivals cannot occur simultaneously.

This is a time consistent policy only if the expected discounted cost of the intervention is lower than the cost of partner default. I focus on bailouts aiming to restore a fundamental equilibrium, therefore, I assume no transfer will be made if the borrowing country is subject to fundamental default (solvency).¹⁵ The lower multiplicity threshold given by (6), defines the maximum level of debt below which a government is not subject to equilibrium multiplicity $b^s = \underline{b}$, absent international cooperation.

A bailout programme $\{b^I(t)\}_{t \in [0, \delta]}$ extends over the duration of market exclusion and is designed to satisfy the borrower's participation constraint. This is preferred by the lender to a lump-sum transfer and it is more in line with observed bailout programmes. The sovereign in good credit standing faces contemporaneous and intertemporal incentives to undertake a bailout. It will commit resources to a bailout in $t \in [0, \delta]$ to avoid the collateral damage $\hat{\alpha}$ and to eventually restore a *good* equilibrium at δ , where it will enjoy perpetually higher consumption due to the sharing of strategic uncertainty. The continuation value for the solidarity problem reflects the *restored equilibrium* wherein investors form expectations using the system-wide sunspot but optimally choose to disregard national sunspots. Governments' utility in the restored equilibrium is given by $V^{hom}(b)$ as detailed in Appendix II.

A country in good credit standing faces the following problem when its partner is excluded from markets,

$$V^{sol}(b) = \max_{c(t), b^I(t)} \int_0^\delta e^{-\rho t - \lambda^c t} [u(c(t)) + \lambda^c \hat{V}] dt + e^{-\rho \delta - \lambda^c \delta} V^{hom}(b(\delta)) \quad (7)$$

$$\begin{aligned} s.t. \quad \dot{b}(t) &= c(t) - y + (r^* + \lambda^c)b(t) + b^I(t) \\ b(t) &\in \bar{\Omega} \end{aligned} \quad (8)$$

The country in good credit standing assumes the role of a bailout agent. While undertaking the bailout, it faces default at rate λ^c which is reflected in its borrowing costs, $r^* + \lambda^c$. This is due to the absence of cooperation and contributes to the cost of the bailout. The bailout agent is itself subject to an endogenous credit ceiling, \bar{b} due to its inability to commit to repaying its own debt. The country in good credit standing will optimally attempt to restore a fundamental equilibrium with the minimum possible bailout.¹⁶ The minimum

¹⁵This aims to reflect the stance of international institutions when lending. To implement this, a debt brake will generally be required as in Tirole (2015).

¹⁶This can be shown to be the outcome of a Nash Bargaining game where the country in good credit

transfer necessary to make a repayment strategy optimal for the recipient country is given by,

$$b^I(b, t) = \min\{\tilde{b} : V^{\text{off-eq}}(b, \tilde{b}, t) > \underline{V}\} \quad (9)$$

The bailout programme $\{b^I(t)\}_{t \in [0, \delta]}$ leaves the borrowing government ϵ better off repaying its debt rather than defaulting at any point in time. The country excluded from the market will need to retire debt held by international investors at the same rate, χ , as in the no cooperation benchmark. However, it can use official financing b^I to smooth its consumption. It now faces the following constraints,

$$\begin{aligned} \dot{b}(t) + b^I(t) &= c(t) - y + (r^* + \lambda^s)b(t) \\ \dot{b}(t) + \chi b(t) &= 0 \end{aligned}$$

The minimum bailout $b^I(b)$ required to restore a lending equilibrium is increasing in the recipient country's debt $b(t)$ therefore $V^{\text{sol}}(b)$ is a decreasing function.¹⁷ Investors demand $r^* + \lambda^s$ on the outstanding debt of the country experiencing exclusion which is the yield in the restored equilibrium. The uninsurable risk λ^s , therefore, endogenously increases the size of the necessary bailout and thus undermines credibility. Bailouts generally fail to restore the fundamental equilibrium because of uninsurable risk, $\lambda^s > 0$. This can also be interpreted as an exogenous probability that the bailout is wasted or lost before it arrives, therefore λ^s can illustrate political risk in the Eurozone.

The size of the bailout is increasing in δ and the bailout schedule is increasingly front-loaded as χ rises. For a given level of outstanding debt, investors deem solidarity to be credible if both countries prefer the intervention to their respective outside options. If this is the case, multiplicity in national debt markets is eliminated, conditional on at least one country maintaining access to the market.

For levels of debt in the solidarity region $b \in \mathcal{S}$, an international self-fulfilling crisis arrives at rate λ^s . In the symmetric expectations case $\lambda_i = \lambda_j$, the solidarity region is defined as,

$$\mathcal{S} = \left\{ b : b \in (b^s, b^c] \right\}, \quad (10)$$

standing has bargaining power of 1 and aims to capture the political dynamics observed during debt crises.

¹⁷The intervention b^I exists since $V^{\text{off-eq}}(b, b^I) > V(b)$ if $b^I = \chi b(t)$, $V^{\text{off-eq}}(b, 0) < V(b)$ for $b > b^s$ and $V(b)$ and $V^{\text{off-eq}}(b)$ are both monotonically decreasing in b .

where b^s is the lower multiplicity threshold given by (6) and b^c , given by,

$$b^c = \max \left\{ b \leq \frac{y - b^I(b)}{r^* + \lambda^c} : V^{\text{sol}}(b) \geq \hat{V}(b) \right\} \quad (11)$$

is the maximum level of debt for which both countries at least weakly prefer the bailout. The solidarity zone is non-empty in the multiplicity region if $\hat{\alpha} > 0$.

The sovereign in good credit standing optimally chooses to pursue a constant consumption path while undertaking the bailout. The consumption policy depends discontinuously on the stock of debt. At δ , the optimal strategy will be either to run down debt to b^s in finite periods or to maintain a constant stock of debt, leading to eventual default. In Appendix B1, I derive the optimal policy and bailout schedule, assuming the country experiencing market exclusion repays at rate χ , as in the no intervention benchmark.

The country in good credit standing will undertake the bailout if the return to intervention is higher than the outside option of letting its partner default $\hat{V}(b)$. In case of partner default, a country incurs collateral damage $\hat{\alpha}y$ but maintains access to markets. Lemma 3 explains when self-fulfilling crises can arise despite solidarity.

Lemma 3 (Failure of solidarity to restore a fundamental equilibrium)

Sovereign debt may trade at a premium, even in the presence of bailout guarantees, because the guarantees are not credible ($b > b^c$) and because bailouts are probabilistic ($\lambda^s > 0$).

Proof. Provided in the Appendix A3. ■

For $b > b^c$, sovereign bond yields are high and a dispersion in yields persists because investors expect no bailout in the case of market panic. For $b \in \mathcal{S}$, national self-fulfilling defaults are ruled out by an off-equilibrium bailout and a self-fulfilling crisis arrives at rate λ^s when there is system-wide panic. As a result, sovereign debt carries a risk premium λ^s in \mathcal{S} .

The model generalises easily to an asymmetric environment. Countries may have different levels of endowment, $y_i \neq y_j$, and face different expectations of default, $\lambda_i \neq \lambda_j$. Spillover costs of default can be represented by an increasing and concave function of the size of the defaulting country, $\hat{\alpha}(y_j)y_i$. A country in good credit standing i , when its partner j is in exclusion, faces utility $V_i^{\text{sol}}(b)$ while undertaking a bailout. It faces default with arrival rate $\lambda_i^c = \lambda^s + \lambda_i$, which is reflected in its borrowing cost. The size of the bailout $b^I(b_j(t))$

will be decreasing in the fundamentals of the country in exclusion, y_j .

If country i is not subject to self-fulfilling default, $\lambda_i^c = 0$, solidarity extends the no crisis region for its partner j to $[0, b_j^c]$. The solidarity threshold for country j , given by b_j^c , is decreasing in the probability of default for country i , λ_i^c . If all strategic uncertainty can be insured away, then there can be no equilibrium default in the solidarity region and countries borrow at the risk free rate r^* .

B. Contagion

The fear of contagion has driven many policy decisions on the management of debt crises. An expectations-driven default in one country can lead to the fundamental default of its economic partner. Default will inflict collateral damage $\hat{\alpha}y$ on the partner country and may lead to an increase in borrowing costs. This can drive the partner country to default if fundamentals are weak. In the symmetric model, a contagion region will always exist, wherein countries face a higher rate of arrival of self-fulfilling crises $\lambda^p = \lambda^s + \lambda_i + \lambda_j$, and investors demand a higher risk premium in anticipation. The bond yield is higher than that of the one country benchmark, since the sovereign debt market is vulnerable to a wider set of sentiments.

Contagion is only possible for levels of debt where solidarity is not deemed credible, $b > b^c$, else spillovers never materialize because expectations of national default are invalidated by an off-equilibrium bailout. To derive the contagion region as an equilibrium object I characterize $\hat{V}(b)$, the value function for a sovereign when its partner has defaulted. The problem resembles a one country model, where the endowment flow is $(1 - \hat{\alpha})y$ due to collateral damage, and there is no scope for international cooperation, as detailed in Appendix II.

For levels of debt in the contagion zone $b \in \mathcal{P}$, conditional on one country experiencing self-fulfilling default, the partner country faces fundamental default. An international self-fulfilling crisis arrives at rate λ^p . The contagion zone \mathcal{P} is defined as,

$$\mathcal{P} = \left\{ b : b \in (\max\{b^c, b^p\}, \bar{b}] \right\}, \quad (12)$$

where b^p is the contagion threshold given by,

$$b^p = \max \left\{ b \leq \frac{(1 - \hat{\alpha})y}{r^* + \lambda^c} : \hat{V}(b) \geq \underline{\hat{V}} \right\} \quad (13)$$

and $\bar{b} = \max(\bar{\Omega})$. The contagion zone is non-empty in the multiplicity region if $\hat{\alpha} > 0$.

Faced with an outside option of joint default due to contagion, a country in good standing will be willing to undertake a larger bailout. Should solidarity remain credible for $b \geq b^p$, a contagion outcome is avoided for levels of debt $b^p < b \leq b^c$. Formally, a solidarity promise is credible against the threat of contagion if the set $(b^p, \bar{b}] \cap \{b : V^{\text{sol}}(b) \geq \underline{\hat{V}}\}$ is non-empty.¹⁸ In this region, if the government excluded from market refuses the bailout then both countries are driven to default.¹⁹

C. Equilibrium with solidarity and self-fulfilling crises

In this section I solve for the symmetric, rational expectations equilibrium. Proposition 1 details the central result of this paper.

Proposition 1 (Crisis equilibrium with solidarity)

Sovereign risk premia reflect investor expectations for the equilibrium and off-equilibrium behaviour of other countries and international agencies. In the symmetric, two country economy with spillovers, investors internalize the strategic interaction between governments:

- i. No crisis zone; For $b \leq b^s$, fundamentals are sufficiently strong and self-fulfilling crises cannot arise in equilibrium. Investors demand the risk free rate r^* .*
- ii. Solidarity zone \mathcal{S} ; For $b \in (b^s, b^c]$ solidarity is credible, self-fulfilling crises arrive at rate λ^s and investors demand $r^* + \lambda^s$.*
- iii. Crisis zone \mathcal{C} ; For $b \in (b^c, b^p]$ solidarity is not credible. Self-fulfilling crises arrive at λ^c and investors demand $r^* + \lambda^c$.*
- iv. Contagion zone \mathcal{P} ; For $b \in (\max\{b^c, b^p\}, \bar{b}]$ the expectations-driven default of one country will result in the fundamental default of its partner. Self-fulfilling crises*

¹⁸To see that this is possible, notice that as $\lim_{\delta \rightarrow 0} V^{\text{sol}} = V$. However, if $\hat{\alpha} > 0$ then $\hat{V} < V$ even if $\delta = 0$. As a result, $b^p < b^c < \bar{b}$ is possible.

¹⁹In response to an agreed haircut on Greek debt and a rejected bailout by Cyprus The Economist asks, "Who really holds the gun- the firing squad, or the prisoner?", Small island, big finger, March 23rd 2013.

arrive at λ^p and investors demand $r^* + \lambda^p$.

v. *Fundamental default*; For $b > \bar{b}$, investors expect the government to default and refuse to purchase new debt.

Proof. Provided in the Appendix A4. ■

Investors demand a lower interest rate on new debt issuances if they expect a rescue of the issuing country in the case of market panic. The probability of system-wide default expectations raises sovereign risk premia for the issuing country through two channels. Firstly, this implies a probability that the bailout may fail or never come. Secondly, spillover costs of default give rise to a contagion region. The multiplicity region $b \in (b^s, \bar{b}]$ will be non-empty if the probability of self-fulfilling crises is sufficiently low. Conditional on $b^s < \bar{b}$, the solidarity and contagion regions are non-empty. Although ex-ante, committing to a bailout is always the optimal policy for a country in good credit standing, ex-post the cost of the bailout can be prohibitively high. I now present the definition of the equilibrium and proceed to characterise optimal policy.

Definition 1

A symmetric competitive equilibrium in the two country economy consists of a consumption function for each sovereign $C : \bar{\Omega} \rightarrow \mathbb{R}_+$, value functions $V : \bar{\Omega} \rightarrow \mathbb{R}$, $V^{\text{off-eq}} : \bar{\Omega} \rightarrow \mathbb{R}$, $V^{\text{hom}} : \bar{\Omega} \rightarrow \mathbb{R}$ and $\hat{V} : \bar{\Omega} \rightarrow \mathbb{R}$ and intervals $\mathcal{S}, \mathcal{P} \subset \bar{\Omega}$ such that,

- i. $r \in R(\bar{\Omega})$;
- ii. for $b \in \bar{\Omega}$, given r , $V(b)$ is the value function for the solution of each sovereign's equilibrium problem and C gives the maximizing choice;
- iii. investors earn r^* in expectation $\forall b \in \bar{\Omega}$;
- iv. $V(b_0) \geq \underline{V}$, $\forall b \in \bar{\Omega}$;
- v. $V^{\text{off-eq}}(b) \geq \underline{V}$, $\forall b \leq b^s$;
- vi. $V^{\text{sol}}(b) \geq \hat{V}(b)$, $\forall b \in \mathcal{S}$;
- vii. $\hat{V}(b) < \hat{V}$, $V^{\text{sol}}(b) < \hat{V}$, $\forall b \in \mathcal{P}$

(i) summarises regularity conditions required on the interest rate schedule, as in Aguiar et al. (2015a), which I detail in Appendix II. I restrict attention to equilibria where $r(b)$ is time invariant and monotonic, which ensures that all debt trajectories are steady state or downward trajectories. (ii) and (iii) are the optimality conditions for the governments and the investors respectively. (iv) requires that default is not optimal at $t = 0$. (v–vii) ensure that equilibrium regions detailed in Proposition 2 are consistent with the off-equilibrium

behaviour of international agents, specifying the intervals \mathcal{S} and \mathcal{P} as equilibrium objects. The crisis zone \mathcal{C} is defined as the complement to \mathcal{S} and \mathcal{P} in the multiplicity region $(b^s, \bar{b}]$. The equilibrium is Markov perfect.

I construct the equilibrium value function for a government in a model of self-fulfilling crises, solidarity and contagion. Each government faces the following problem,

$$\begin{aligned}
V(b) = \max_{c(t)} \int_0^\infty e^{-\rho t - \sum_i \lambda^i \int_0^t \mathbb{1}_i(b(s)) ds} & \left[u(c(t)) + \sum_{i \setminus c} \lambda^i \hat{V}_i \mathbb{1}_i(b(t)) + \right. \\
& \left. \left(\lambda^s \underline{V} + (\lambda^c - \lambda^s)(\hat{V}(b) + \underline{V}) \right) \mathbb{1}_c(b(t)) \right] dt \\
s.t. \quad \dot{b}(t) = c(t) - y + [r^* + \sum_i \lambda^i \mathbb{1}_i(b)]b(t) & \\
& b(t) \in \bar{\Omega}, \\
& i \in \{s, c, p\}
\end{aligned}$$

The model admits multiple equilibria for $b > b^s$ because of strategic uncertainty amongst international investors in sovereign debt markets. The indicator functions $\mathbb{1}_s$, $\mathbb{1}_c$ and $\mathbb{1}_p$ take a value of one when the level of outstanding debt is in the solidarity, crisis or contagion region respectively.²⁰ In equilibrium, the government faces an interest rate schedule $r(b) = r^* + \sum_i \lambda^i \mathbb{1}_i(b)$, which reflects the probability that investors are not repaid. The government will receive \underline{V} in case of own default, $\hat{V}(b)$ in case of partner default and \hat{V} in case of joint default. In the symmetric model, $\lambda^c - \lambda^s$ is equal to the probability that exactly one government defaults. $\bar{\Omega} = [0, \bar{b}]$ is the set of all debt levels satisfying the participation condition for each government.

New strategies arise in equilibrium, as governments try to take advantage of bailout guarantees and avoid contagion. In the no crisis region, international investors demand $r(b) = r^*$ and the government optimally pursues a stationary debt strategy $\dot{b} = 0$, receiving $V(b) = \frac{u(y - r^*b)}{\rho}$. Self-fulfilling crises are possible for $b > b^s$. A credible bailout guarantee between countries will invalidate investor expectations of national default for debt below b^c . The government chooses to either maintain a constant level of debt, paying a risk premium λ^s and facing eventual default, or to run down debt in finite time to a level b^s , where $r = r^*$ and there can be no default in equilibrium. While saving, the govern-

²⁰The notation $\sum_{i \setminus c}$ refers to the summation for $i \in \{s, p\}$.

ment finds a constant consumption path at c^s optimal and consumes discretely when debt reaches b^s , $c(b^s) = y - r^*b^s$. As debt rises within the solidarity region, the consumption cost of running down debt in finite periods, $-u'(c)(b - b^s)$, is increasing and the government is indifferent between the two debt strategies at b^{s*} . A government faces the choice to maintain or run down its outstanding debt in \mathcal{C} and \mathcal{P} , while subject to a higher probability of self-fulfilling crisis and higher debt servicing costs. Since the level of debt is higher, the cost of running down debt to the sustainable level of debt, b^s , rises. However, the government may find it optimal to save the nearest discontinuity, $b^i \leq b$. Proposition 2 characterizes the optimal policy in equilibrium.

Proposition 2 (Optimal policy)

In the symmetric two country economy, the optimal government policy involves a constant or downward debt trajectory. Under a constant debt strategy, the government consumes $c = y - r(b)b$ and receives utility,

$$V(b) = \begin{cases} \frac{u(y - r^*b)}{\rho} & \text{if } b \in [0, b^s] \\ \frac{u(y - (r^* + \lambda^s)b)}{\rho + \lambda^s} + \frac{\lambda^s}{\rho + \lambda^s} \hat{V} & \text{if } b \in [b^{s*}, b^c] \\ \frac{u(y - (r^* + \lambda^c)b)}{\rho + \lambda^c} + \frac{\lambda^s}{\rho + \lambda^c} \hat{V} + \frac{\lambda^c - \lambda^s}{\rho + \lambda^c} (\hat{V}(b) + \underline{V}) & \text{if } b \in [b^{c*}, b^p] \\ \frac{u(y - (r^* + \lambda^p)b)}{\rho + \lambda^p} + \frac{\lambda^p}{\rho + \lambda^p} \hat{V} & \text{if } b \in [b^{p*}, \bar{b}] \end{cases}$$

where $b^{i*} = \frac{y - c^i}{r^* + \lambda^i}$ for $i \in \{s, c, p\}$ is the steady state debt level in each region above which a constant debt trajectory is optimal. If a downward trajectory is optimal, the government will save to a level $b^i \leq b$ while consuming c^i in each region i , defined implicitly by,

$$\begin{aligned} (\rho + \sum_i (\lambda^i \mathbb{1}_i(b^i))V(b^i) - \sum_{i \in \mathcal{C}} \lambda^i \mathbb{1}_i(b) \hat{V} - (\lambda^s \hat{V} + (\lambda^c - \lambda^s) \underline{V}) \mathbb{1}_c(b(t)) = \\ u(c^i) - u'(c^i) (c^i - y + r(b)b^i) + (\lambda^c - \lambda^s) \mathbb{1}_c(b) \hat{V}(b^i) \end{aligned}$$

The cost of running down debt to the nearest threshold $b^i < b$ is given by $-u'(c^i)(b - b^i)$.

Proof. Provided in the Appendix A5. ■

The value function will feature a concave kink at each b^i , $i \in \{s, c, p\}$. For $b > b^s$, investors

demand compensation for the possibility of self-fulfilling crises, resulting in a discontinuity in the interest rate and a kink in the value function.²¹ The kink at b^c is a novel result which reflects changing investor expectations for intervention by international agents. To the left of this discontinuity, investors expect an international agent to undertake a bailout in case of market panic, which reduces the probability of a self-fulfilling crisis relative to the one-country benchmark. Self-fulfilling crises in this region arise only due to uninsurable political risk (λ^s) and countries default jointly. To the right of the discontinuity, investors expect no international intervention in the case of market panic because the required bailout is too costly.

A change in investor expectations for the equilibrium actions of large international agents leads to sudden swings in sovereign risk premia. Consider the following instructive scenarios. Suppose $\Delta\hat{V}(b) \geq 0$ for the country undertaking the bailout. This could reflect a political push in favour of abandoning support for the partner country or a decrease in the spillover costs of default, due to the signing of new trade deals with other countries. If $\Delta\hat{V}(b)$ is large, then it may be the case that $V^{\text{sol}}(b) - \hat{V}(b) < 0$ and investors will no longer expect the country to undertake a bailout for its partner. The creditors of the partner country will require a higher risk premium, $\lambda^c > \lambda^s$, despite there being no change in domestic fundamentals.

Lemma 4 (Endogenous propagation of expectations)

Following an increase in the expectations of system-wide default or political risk, $\Delta\lambda^s > 0$, the risk premium on a country i 's sovereign debt can rise by as much as $\Delta\lambda^s + \lambda_i$.

Proof. Provided in the Appendix A6. ■

Expectations can change following statements from officials or other political events. Mario Draghi's seminal commitment to saving the Euro may have lead to $\Delta\lambda^s < 0$. An exit vote by Britain and growing anti-EU sentiment may have resulted in a $\Delta\lambda^s > 0$. Alternatively, a change in expectations of default can accompany a real shock that affects both countries. If expectations of joint default rise, in addition to the direct increase in borrowing costs, there exists a secondary effect through the disintegration of solidarity. The required bailouts become both larger and more expensive and the continuation equilibrium $V^{\text{hom}}(b)$ becomes less attractive. During the European debt crisis, markets assigned a non-zero probability to the dissolution of the European Monetary Union and sovereign debt yields spiked across

²¹This is a standard result in the sovereign debt literature originating in Cole and Kehoe (2000) and has also been identified in Aguiar et al. (2013).

the system. This can be partly attributed to the collapse of solidarity, as countries faced unfavourable borrowing conditions and political resistance when mobilizing multilateral rescue packages.

IV. Ex-ante borrowing incentives

In this section, I study the ex-ante effects of bailout guarantees on optimal borrowing policy. If default inflicts collateral damage on the partner economy $\hat{\alpha} > 0$, borrowing policies that lead to eventual default are privately optimal but socially inefficient outside of the contagion region. I present optimality conditions for a government pursuing a constant debt trajectory that leads to eventual self-fulfilling default (gambling). I then show that a bailout guarantee from an international agent with limited commitment (solidarity) can encourage governments to reduce their outstanding stock of debt.

Solidarity will have different effects on the optimal borrowing policy depending on the recipient government's level of debt. For levels of debt where investors expect a bailout off-equilibrium, the fall in the probability of a self-fulfilling default will lower debt servicing costs. This is the standard channel studied in the literature which raises moral hazard concerns. However, a limited bailout guarantee gives rise to a new equilibrium strategy, where governments target $b = b^c$ and default at rate λ^s . I show that the reduction in the distance to the nearest debt threshold, $b^i < b$, will make a deleveraging strategy more attractive for governments with sufficiently high levels of debt.

A country with sufficiently low debt benefits from solidarity, because investors anticipate a bailout by the partner country in the case of market panic. As a result, the country faces a lower probability of default and investors demand a lower interest rate on new debt issuances. Countries with debt $b \in (b^s, b^c]$, have an increased incentive to gamble under a bailout guarantee. However, if a country is sufficiently indebted $b > b^c$, investors expect an international agent not to intervene in the case of market panic because the necessary bailout is too costly. This provides an incentive to run down debt to b^c in order to manipulate investors' expectations for a bailout.

Proposition 3 (Solidarity and gambling)

For $b > b^c$, if λ^s is low, solidarity improves the return to running down debt to b^c and defaulting at rate $\lambda^s < \lambda^c$.

Proof. Provided in Appendix A7 . ■

Proposition 3 arises because I relax the assumption that an international agent can commit to a bailout for all $b \in \bar{\Omega}$. If solidarity was credible everywhere, there would be no incentive to run down debt.

Corollary 1 (Solidarity and gambling with perfect insurance)

When $\lambda^s = 0$, solidarity makes a constant debt strategy safe and optimal for all $b \leq b^c$. For $b > b^c$, the return to pursuing a safe strategy $b = b^c$ improves.

In section 2, I showed that perfect insurance increases the range of debt where solidarity is credible resulting in a higher sustainable level of debt. Corollary 1 states that under perfect insurance, solidarity provides stronger incentives for a borrowing country to run down its debt to take advantage of an extended region of solidarity. This further increases the set of states where a government avoids default following a bailout guarantee. An intervention by an international agent not subject to default expectations or political risk is more effective in preventing self-fulfilling crises and also provides stronger incentives for borrowing countries to reduce their outstanding stock of debt.

To conclude the section, I summarize how optimal borrowing policy differs in a model with solidarity and contagion. A limited commitment to off-equilibrium transfers, which arises from the optimizing behaviour of governments, provides an incentive for borrowing governments to reduce their outstanding level of debt such that its creditors expect a bailout in case of market panic. In addition, relative to a one-country model, a higher probability of default in the contagion zone, $\lambda^p \geq \lambda^c$, prompts the government to reduce its debt level to $b \leq \max\{b^c, b^p\}$. In models which feature a third party bailout agency that can commit to eliminate self-fulfilling crises, governments will tend to pursue debt levels near \bar{b} . This leaves them vulnerable to fundamental default. In a model where international agents cannot commit to a bailout, governments have an incentive to maintain debt levels at or below b^c .

V. Debt purchase programmes and enforcement of sanctions

This section investigates the scope for debt purchase programmes, by large international agents, to eliminate non-fundamental equilibria. In all models of sovereign debt, where expectations-driven default can arise, there is scope for intervention by a large, infinitely lived agent who will purchase the entire debt stock and act as an ultimate lender. In contrast to the market, a large agent is able to coordinate its actions and selects the *good* equilibrium in national debt markets. The borrowing government will not default for $b \in \bar{\Omega}$ if offered the positive competitive price for new debt. The international agent assumes the role of the lender of last resort, purchases new debt and requires the *restored equilibrium* interest rate detailed in $V^{hom}(b)$. The debt purchase programme can restore an equilibrium where investors believe governments will only default jointly, but fails to restore the fundamental equilibrium while $\lambda^s > 0$.

A large agent can resolve the multiplicity, however, it internalises the collateral damage it will incur from the borrower's default and therefore cannot credibly enforce sanctions in the case of no repayment. On the other hand, the market consisting of atomistic agents will always enforce sanctions. Anticipating this, borrowing governments will default on official lenders resulting in a deviation from *parri-passu*. Schlegl, Trebesch, and Wright (2016) present empirical evidence showing that official loans are defaulted upon almost twice as often as market held debt. In contrast to Tirole (2015) where market financing is always preferred due to enforceability concerns, in the presence of multiplicity, a large agent is needed to coordinate on the good equilibrium. The model presents a trade-off between enforceability and coordination. International investors, who constitute the market, will always credibly enforce sanctions if there is an ϵ benefit to doing so. The official lender can choose to withhold future lending, $b^I = 0$ in case of a missed payment. However, the lender knows that if this leads the borrower to default on market held debt, the market will impose default costs and it will itself incur collateral damage. The official lender can only credibly refuse debt purchases if the expected value of arrears outweighs that of collateral damage.

Lemma 5 (Unenforceable sanctions)

Sanctions for the non-repayment of official debt will not be enforced by an official lender for $b \leq b^c$ if $\delta > 0$ and $\hat{\alpha}y > \epsilon$.

Proof. Provided in Appendix A8. ■

Anticipating that sanctions will not be enforced, a borrowing sovereign will default on the bonds held by the official sector resulting in a large transfer.²² Lemma 5 is contingent on secondary markets not functioning. If the official lender can sell the debt to the market, the latter can commit to enforcing sanctions, extracting repayment from the borrower. This requires careful coordination between the official lender and the market, with the former coordinating on the good price schedule and the latter willing to buy the debt at face value. This relates to the argument in Broner, Martin, and Ventura (2010), that the existence of a repayment equilibrium in sovereign default models depends on the functioning of secondary markets. In their model, secondary markets maximize the ex-ante value of debt contracts. In the context of official lending, functioning secondary markets maximize the ex-ante value of a debt purchase programme for the official lender, for all $b \in \bar{\Omega}$, resulting in a wider solidarity zone.

The model suggests that official debt held by official lenders should be unwound and resold to the market. This will promote repayment from the borrowing government and will in turn encourage countries in good credit standing to undertake larger debt purchases. The ECB and the ESM hold a large share of GIIPS debt and face increasingly strong calls for debt relief. In addition, the outstanding debt constrains the ESM's future lending capacity, increasing the probability of future market panics. Schlegl, Trebesch, and Wright (2016) document a share up to 80% of Greek debt held by European official lenders and similar figures for other GIIPS.

VI. Information Asymmetry, Equilibrium Crises and Bailouts

The information asymmetries which plague international financial markets can, in the presence of roll over risk, help explain observed crises and bailouts. Sovereign default is

²²Note that b^c will necessarily be lower than \bar{b} as long as expected arrears are greater than zero. This can be the result of political risk, output risk or as discussed in the next section, asymmetric information.

a strategic choice therefore it is useful to interpret the outside option \underline{V} as the political willingness to default. Aguiar et al. (2015b) conduct an early study of equilibrium default that cannot be contracted away due to a stochastic, privately observed outside option. Agents have an incentive to pretend to have the highest outside option, in order to extract a large surplus. Perez (2015) also characterizes mispricing of debt in a pooling equilibrium and shows that long term debt becomes less attractive.

Suppose a government draws at $t = 0$ a privately observable default cost which incorporates its contingency plans and political and popular support for its choices. Let,

$$\underline{V} = \begin{cases} \underline{V}_l & \text{with probability } (1 - p) \\ \underline{V}_h & \text{with probability } p \end{cases}$$

where $\underline{V}_l < \underline{V}_h$.

I begin by investigating the implications in a one-country model if the market cannot observe the outside option of the government it is lending to. Each atomistic investor knows that there is a threshold \underline{b}_h below which a self-fulfilling crises is not an equilibrium outcome even if the issuing government has a high willingness to default,

$$\underline{b}_h = \max \left\{ b \leq \frac{y}{r^* + \chi} : V^{\text{off-eq}}(b) \geq \underline{V}_h \right\}. \quad (14)$$

A second threshold \underline{b}_l is derived by considering the willingness a lower willingness to default,

$$\underline{b}_l = \max \left\{ b \leq \frac{y}{r^* + \chi} : V^{\text{off-eq}}(b) \geq \underline{V}_l \right\}. \quad (15)$$

It follows that $\underline{b}_h < \underline{b}_l$. In the interval $(\underline{b}_h, \underline{b}_l]$, investors expect that an issuing government will default in case of market exclusion only if it has a high willingness to default, which occurs with probability p .

I assume that the market is indifferent between rolling over outstanding debt which will be repaid with certainty and repayment through an equilibrium restructuring process.²³ As before, following a self-fulfilling crisis where investors refuse to roll over debt, an issuing

²³I make this assumption to obtain sharp results at a small economic cost. The assumption requires a flow cost $\xi(t) = \chi b(0)e^{-\chi t}$ for investors during the restructuring. This could be motivated by legal or monitoring costs. The model can be generalised to allow for smaller $\xi(t)$ which would imply self-fulfilling crises only if p is sufficiently high.

government does not regain access to markets for a period δ . This can be thought of as the time until the next auction. Each individual investor will find it optimal not to purchase new debt from an issuing government if its debt level is in $(\underline{b}_h, \underline{b}_l]$, since there is a p probability of default. As a result, it follows from Lemma 2 that self-fulfilling crises arrive at rate λ . With probability $1 - p$, the government in exclusion faces \underline{V}^l and will choose to run down its debt, in equilibrium, at rate χ for a δ interval.²⁴ This can be viewed as a *pooling* equilibrium, wherein investors require a yield $r^* + \lambda p$ and debt is mispriced.²⁵

Lemma 6 (On-equilibrium self-fulfilling crises)

For $b \in (\underline{b}_h, \underline{b}_l]$, if the outside option for an issuing government is privately observed, a self-fulfilling crisis arrives on-equilibrium at $t = \tau$;

- i. If the country has a high willingness to default, it chooses to do so at $t = \tau$.*
- ii. If the country has a low willingness to default, it will run down its debt at rate χ for the duration of market exclusion, to avoid default.*

Proof: Provided in Appendix A9. ■

For $b \in (\underline{b}_l, \bar{b}_h]$, investors require compensation $r^* + \lambda$. The threshold \bar{b}_h is defined,

$$\bar{b}_h = \max \left\{ b \leq \frac{y}{r^* + \lambda} : V(b) \geq \underline{V}_h \right\} \quad (16)$$

The market knows that only governments with a low willingness to default will support levels of debt in $(\bar{b}_h, \bar{b}_l]$. This is a *separating* equilibrium where default choice acts as a signal.

Information asymmetry between the issuing government and the market explains a wider set of observed debt dynamics than previously considered in the literature. Preceding the Greek referendum in 2015, which asked the population to decide on whether or not to accept a bailout from the Troika (required to avoid default), there were fluctuations in sovereign bond yields in line with predictions and polls for the referendum results.

²⁴Since governments facing \underline{V}_h will default at $t = 0$, the markets will recognise that all deleveraging governments have a low willingness to default. Allowing for a $\delta' < \delta$ in case of equilibrium deleveraging would give the issuing government an incentive to pretend to have a low willingness to default.

²⁵This complicates ex-ante borrowing incentives since governments with high willingness to default will find it optimal to accumulate debt at the prevailing interest rate whereas government with a low willingness will choose to deleverage. Borrowing strategies in turn can act as a signal to the market although this will not lead to a separating equilibrium in the presence of debt shocks or imperfect monitoring.

Information asymmetries also impede solidarity. In the benchmark model, a country in good credit standing is certain of the transfer necessary to exactly compensate a country on the brink of default for its outside option. For simplicity, I assume that the bailout contract is written at $t = \tau$, before the revelation of \underline{V} and is not revisable. The country in good credit standing will choose b^I from a menu of contracts $\{b^{I,l}, b^{I,h}\}$ where $b^{I,l} < b^{I,h}$ for every t . If the country in good credit standing undertakes $b^{I,l}$ then the bailout will fail with probability p as a recipient government with a high willingness to default it will reject the bailout and default at $t = \tau$.

The bailout contracts are defined as follows,

$$b^{I,h}(b, t) = \min\{\tilde{b} : V^{\text{off-eq}}(b, \tilde{b}, t) > \underline{V}^h\}$$

$$b^{I,l}(b, t) = \min\{\tilde{b} : V^{\text{off-eq}}(b, \tilde{b}, t) > \underline{V}^l\}$$

If $b^{I,h}$ is offered, then the bailout is certain to be successful if undertaken. The bailout agent will undertake $b^{I,h}$ for $b \leq b^{c,h}$ where,

$$b^{c,h} = \max\left\{b \leq \frac{y - b^{I,h}(b)}{r^* + \lambda^c} : V^{\text{sol}}(b, b^{I,l}) \geq \hat{V}(b)\right\} \quad (17)$$

In contrast, if $b^{I,l}$ is offered, the country in good credit standing anticipates that the bailout may fail to compensate the government experiencing a market panic for its outside option. The recipient government will choose to default with probability p . Suppose the country in good credit standing would pursue consumption $\bar{c}(b^{I,l})$ while undertaking a bailout in the case $p = 0$. Let $\bar{c}^d(b^{I,l})$ denote the consumption of the government in good credit standing while undertaking a bailout which is doomed to fail. It follows that $\bar{c} \geq \bar{c}^d$ because the latter factors in collateral damage costs and the absence of risk sharing following a partner default at $t = \delta$. Bailouts $b^{I,l}$ will be undertaken for for $b \leq b^{c,l}$ where,

$$b^{c,l} = \max\left\{b \leq \frac{y - b^{I,l}(b)}{r^* + \lambda^c} : \frac{u((1-p)\bar{c}(b^{I,l}) + p\bar{c}^d(b^{I,l}))}{r^* + \lambda^c} + \frac{\lambda^c}{r^* + \lambda^c} \hat{V} \geq \hat{V}(b)\right\} \quad (18)$$

The solidarity zone \mathcal{S} can be divided into two regions, one where all bailouts are successful (*pooling bailout equilibrium*) and one where only bailouts of governments with outside option \underline{V}^l are successful (*separating bailout equilibrium*).

Proposition 4 (Country solidarity and information asymmetry)

Bailouts are observed on-equilibrium for levels of debt $b \in (b^{c,h}, b^{c,l}]$ and the bailout fails to prevent default with probability p .

Proof: Provided in Appendix A10. ■

In the region where equilibrium bailouts are possible, investors anticipate default at rate $\lambda^c - p\lambda_i > \lambda^s$.

VII. Conclusion

Self-interested bailouts (solidarity) between governments can restore confidence in sovereign debt markets. In a model of self-fulfilling crises à la Cole and Kehoe (2000) I show that sovereign risk premia will critically depend on investors' expectations for the equilibrium and off-equilibrium actions of international agents. Solidarity can invalidate investor expectations of default in national debt markets but generally fails to restore a fundamental equilibrium because international agents cannot commit to the intervention. The risk of system-wide default, which can resemble political risk in the Eurozone, means that a bailout may never materialize and this endogenously increases its cost. Furthermore, I show that solidarity with limited credibility can encourage a heavily indebted government to reduce its outstanding debt such that its creditors anticipate a bailout in case of a self-fulfilling crisis.

In the model there is a conflict between enforceability and coordination. I show that large international agents such as governments or the ESM, in contrast to the market, can coordinate their actions and select a good equilibrium. However, the market has a comparative advantage in the enforcement of sanctions therefore, in line with empirical evidence, market held debt is repaid more often than officially held debt. Finally, I show that the introduction of asymmetric information can help explain a rich set of equilibrium dynamics. Market panics need not lead to default and bailouts can be observed on the equilibrium path.

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VIII. Appendix I

A. Proofs of propositions and lemmata

A.1. Proof of Lemma 1

I derive the off-equilibrium optimal policy which defines $V^{\text{off-eq}}(b)$ and \underline{b} . I guess that optimal consumption is as follows,

$$\tilde{c}(t) = \begin{cases} y - (\chi + r^*)b(t) & \text{for } t \in [0, \delta) \\ y - r^*b(\delta) & \text{for } t \in [\delta, \infty) \end{cases}$$

with the optimal debt trajectory satisfying,

$$\dot{\tilde{b}}(t) = \begin{cases} -\chi b(t) & \text{for } t \in [0, \delta) \\ 0 & \text{for } t \in [\delta, \infty) \end{cases}$$

As a result,

$$V^{\text{off-eq}}(b; \tilde{c}) = \int_0^\delta e^{-r^*t} u(y - (\chi + r^*)b(0)e^{-\chi t}) dt + e^{-r^*\delta} \frac{u(y - r^*b(0)e^{-\chi\delta})}{r^*}$$

Time t now enters as a state variable, since the outstanding debt stock is $b(0)e^{-\chi t}$. I consider the effects of a tighter deleveraging constraint or equivalently shorter maturity debt. At the optimal policy, shortening the repayment schedule should never improve welfare since a government always has the option of early repayment.

$$\begin{aligned} \frac{d}{d\chi} \left[V^{\text{off-eq}}(b; \tilde{c}) \right] &= \int_0^\delta e^{-r^*t} u'(y - (\chi + r^*)b(0)e^{-\chi t}) b(0)e^{-\chi t} \left(t(\chi + r^*) - 1 \right) dt \\ &\quad + e^{-r^*\delta} u'(y - r^*b(0)e^{-\chi\delta}) \delta b(0)e^{-\chi\delta} < 0 \end{aligned}$$

To see why the derivative is negative, I can derive an upper bound for the derivative and show that this is still negative. Since $u' > 0$ and $u'' < 0$, then for $\delta > 1/(r^* + \chi)$,

$$\begin{aligned} \int_0^\delta e^{-r^*t} u'(y - (\chi + r^*)b(0)e^{-\chi t}) b(0)e^{-\chi t} (t(\chi + r^*) - 1) dt &\leq \\ \min_{t \in [0, 1/(r^* + \chi)]} [u'(y - (r^* + \chi)be^{-\chi t})] \left(-\frac{1}{(r^* + \chi)e} \right) &+ \\ \max_{t \in (1/(r^* + \chi), \delta]} [u'(y - (r^* + \chi)be^{-\chi t})] \frac{1}{r^* + \chi} \left[\frac{1}{e} - \delta e^{-(r^* + \chi)\delta} (r^* + \chi) \right] & \\ = -\delta e^{-(r^* + \chi)\delta} u' \left(y - (r^* + \chi)be^{-\frac{\chi}{r^* + \chi}} \right) & \end{aligned}$$

Note that for $\delta > 1/(r^* + \chi)$,

$$\int_{1/r^* + \chi}^\delta e^{-(r^* + \chi)t} ((r^* + \chi)t - 1) dt = \frac{1}{r^* + \chi} \left[\frac{1}{e} - \delta e^{-(r^* + \chi)\delta} (r^* + \chi) \right] > 0$$

Then since $u'' < 0$,

$$\begin{aligned} e^{-(r^* + \chi)\delta} \delta u' \left(y - (r^* + \chi)be^{-\frac{\chi}{r^* + \chi}} \right) &> \\ e^{-(r^* + \chi)\delta} \delta u' \left(y - r^*be^{-\chi\delta} \right) & \end{aligned}$$

The case of $\delta < 1/(r^* + \chi)$ follows since the integrand is strictly negative for these values.

The off-equilibrium problem satisfies all condition required for sufficiency as detailed in Kamien and Schwartz (1971). A government faced with market exclusion will repay its outstanding level of debt at the minimum rate.

A.2. Proof of Lemma 2

If the government chooses to repay debt coming due in $t \in [0, \delta]$ while excluded from markets, it earns $V^{\text{off-eq}}(b)$. Since $V^{\text{off-eq}}(b)$ is a decreasing function of b , the level of debt below which a government will never choose to default is \underline{b} given by (6). For $b > \underline{b}$, the government chooses to default if offered a zero price for new debt issuances for a δ

period.

For $b \in (\underline{b}, \bar{b}]$, there exist multiple equilibria and the government will repay only if investors offer the *no default expectations* competitive price. For levels of debt below \underline{b} , each individual investor will find it optimal to deviate lend to the government who will repay. In a symmetric equilibrium, all investors will lend and there is a unique equilibrium.

$\lim_{\delta \rightarrow 0} V^{\text{off-eq}}(b) = V(b)$ because the stock of debt that needs to be retired during the period of market exclusion approaches zero. Moreover, $\lim_{\chi \rightarrow 0} V^{\text{off-eq}}(b) = V(b)$ because bonds held by investors are redeemable only by option of the government, who will choose not to exercise it.

A.3. Proof of Lemma 3

Investors form distinct expectations of sovereign default in each debt market. In the one country model, a self-fulfilling crisis is possible for $b \in (\underline{b}, \bar{b}]$, for each sovereign. In the two country model, the formation of default expectations is no longer independent across national debt markets.

Suppose one country, denoted by j , is offered a zero price on new debt issuances while its partner, country i , is offered a positive market price. If $V^{\text{sol}}(b) \geq \hat{V}(b)$, then by the definition of $V^{\text{sol}}(b)$ (7-8) and $b^I(b)$ (9), it is optimal for both countries to avoid default. Each investor holding country j debt has an incentive to deviate and purchase new debt since j will repay following a bailout from i . Investors anticipate the bailout and lend in response therefore a self-fulfilling crisis is avoided with no bailout on the equilibrium path. Investors expect the countries to default only jointly.

For $b > b^c$, $V^{\text{sol}}(b) < \hat{V}(b)$, country i does not make the off-equilibrium transfer and country j finds default optimal. Investors anticipate that i will not give the bailout and in equilibrium investors refuse to purchase debt from country j .

Suppose both countries i and j are offered a zero price on new debt issuances. The rate of arrival of joint default is $\lambda^s = \lambda_{ij}$. At $b = b^s$ each sovereign is indifferent between repayment and default and consequently unwilling to bail out its partner. Anticipating there will be no bailout, investors will refuse to purchase new debt from either country.

A.4. Proof of Proposition 1

The break-even condition for investors requires equating the expected return from holding a bond of duration T , to be equal to the opportunity cost,

$$\underbrace{e^{r(b)T}}_{\text{return from holding a bond}} \times \underbrace{e^{-\sum_i \lambda^i \mathbb{1}_i T}}_{\mathbb{P}(T > \tau)} = \underbrace{e^{\rho T}}_{\text{return from risk free asset}}$$

where τ is the arrival time of a self-fulfilling default. Taking logs and dividing by T yields the equilibrium interest rate,

$$r(b) = \rho + \sum_i \lambda^i \mathbb{1}_i(b)$$

(i) follows from Lemma 2.

(ii) follows from Lemma 3. I prove that the solidarity region is non-empty for $\hat{\alpha} > 0$, conditional on a non-empty region of multiplicity $b^s < \bar{b}$. For $b > b^s$, each country has an incentive to undertake a bailout for $b^I \leq \hat{\alpha}y$ even as $\delta \rightarrow \infty$. Notice that as $\delta \rightarrow 0$, $V^{\text{sol}} \rightarrow V(b)$ but $\hat{V}(b) \leq V(b)$ if $\hat{\alpha} \geq 0$. The solidarity region will be non empty if $V^{\text{sol}}(b) \in (\hat{V}(b), V(b)]$, which is true for $\hat{\alpha} > 0$.

(iv) *Contagion*: Suppose one country, denoted by j , is offered a zero price on its new debt issuances and i is offered the *no default expectations* price. If $V^{\text{sol}} < \hat{V}$, i finds it optimal not to undertake the bailout and j experiences self-fulfilling default inflicting spillovers $\hat{\alpha}y$ on i . Country i then finds default optimal because $\hat{V}(b) \leq \hat{V}$, by the definition of b^p . Anticipating this, investors will not lend to either country. Both sovereigns find default optimal, conditional on one receiving the *default expectations* price. Default arrives at rate $\lambda^p = \lambda_{ij} + \lambda_i + \lambda_j$.

If $b^c > b > b^p$ and $V^{\text{sol}} \geq \hat{V}$, then country i would undertake a bailout. By Proposition 2, this invalidates expectations of country j default. Country i investors anticipate that country j investors will lend, therefore spillover costs $\hat{\alpha}y$ never materialize and country i finds repayment optimal $V(b) \geq \hat{V}$ for all $b \leq \bar{b}$. Default is avoided in both countries conditional on one country receiving the *no default expectations* price.

Conditional on a non-empty multiplicity region, the contagion zone is non-empty. When $\max\{b^c, b^p\} = b^p$, non-emptiness is ensured when $\hat{\alpha} > 0$, since $\hat{V}(b) \leq V(b)$. When $\max\{b^c, b^p\} = b^c$, the contagion zone is non-empty because $V^{\text{sol}} < V$ $\delta > 0$ implies $b^c <$

\bar{b} .

(v) For $b > \bar{b}$, a government always prefers default to repayment.

Finally, to show that the multiplicity region is non-empty, I must show $\underline{b} \leq \bar{b}$. \underline{b} is defined in (6) and,

$$\bar{b} = \max \left\{ b : V(b) \geq \frac{u((1 - \alpha - \hat{\alpha})y)}{\rho} \right\}$$

In the $\hat{\alpha} = 0$ benchmark, this requires that $V(b) > V^{\text{off-eq}}(b)$. Since $V^{\text{off-eq}}(b) \leq V(b; r^*)$ for $\delta \geq 0$ and $V(b) \leq V(b; r^*)$ for $\lambda \geq 0$, $\underline{b} \leq \bar{b}$ requires $\delta > 0$ and λ small. Notice that \bar{b} is increasing in the cost of default αy . Allowing for $\hat{\alpha} > 0$, multiplicity can be sustained for higher λ . (v) holds because $V(b)$ is a decreasing function. For $b > \bar{b}$, investors anticipate the government will default even when offered the positive competitive price and do not lend.

Conditional on a non-empty multiplicity zone, there exist non-empty solidarity and contagion zones. The crisis zone is empty if $b^c \geq b^p$ and is defined as the set of b which satisfy $V^{\text{sol}}(b) > \hat{V}(b)$ and $\hat{V}(b) \leq \underline{\hat{V}}(b)$ where there is no bailout and and no contagion, resembling the one-country benchmark.

A.5. Proof of Proposition 2

The HJB for the stochastic optimal control problem is given by,

$$\rho V(b(t)) = \sup_{c(t)} \left\{ u(c(t)) + \frac{1}{dt} \mathbb{E} dV(b(t)) \right\} \quad (19)$$

By Ito's lemma,

$$\begin{aligned} dV(b(t)) = & \left[\frac{dV(b(t))}{dt} + \frac{dV(b(t))}{db} (c(t) - y + r(b(t))b(t)) \right] dt + \\ & \left([\hat{V}(b(t)) - V(b(t))] dx_1(t) + [\underline{V} - V(b(t))] dx_2(t) + [\hat{\underline{V}} - V(b(t))] dx_2(t) \right) \\ & + \sum_{l \in \{4,5\}} \left[\hat{\underline{V}} - V(b(t)) \right] dx_l(t) \end{aligned}$$

I use five independent Poisson processes to characterize the arrival of default expectations. These are expectations of joint default in each of the regions and individual default of either country in the crisis region. Moreover passing through the expectations operator,

$$\mathbb{E}dx_1(t) = \mathbb{E}dx_2(t) = \mathbb{1}_c(b(t))(\lambda^c - \lambda^s) \quad (20)$$

$$\mathbb{E}dx_3(t) = \mathbb{1}_c(b(t))\lambda^s \quad (21)$$

$$\mathbb{E}dx_4(t) = \mathbb{1}_s(b(t))\lambda^s \quad (22)$$

$$\mathbb{E}dx_5(t) = \mathbb{1}_p(b(t))\lambda^p \quad (23)$$

Substituting (20)-(23) in (19), the Hamilton-Jacobi-Bellman equation is given by,

$$\begin{aligned} & (\rho + \sum_i (\lambda^i \mathbb{1}_i(b(t)))V(b) - \sum_{i \setminus c} \lambda^i \mathbb{1}_i(b(t))\hat{V} - \left(\lambda^s \hat{V} + (\lambda^c - \lambda^s)V \right) \mathbb{1}_c(b(t)) = \\ & \max_{c \in C} \left\{ u(c(t)) + \frac{dV(b)}{db} (c(t) - y + r(b(t))b(t)) + (\lambda^c - \lambda^s) \mathbb{1}_c(b(t)) \hat{V}(b(t)) \right\} \end{aligned} \quad (24)$$

A technical background to the above derivation is given in Sennewald (2007). The jump processes contribute additional terms to the HJB which show the fall in flow utility in case of arrival of a default event.

Where the value function is differentiable, the first order condition is given by $u'(c) = -V'(b)$. A marginal unit of consumption today yields $u'(c)$ utility units today at the expense of an increased debt burden tomorrow. For levels of debt $b \geq b^s$, I deal with the discontinuities as follows. Equilibria are monotone (Definition 1) therefore r is weakly increasing in $b(t)$. In the neighbourhood to the right of any discontinuous point b^i in $r(b)$, the government can achieve discretely higher utility by saving, $\dot{b}(t) < 0$ therefore $c < y - r(b)$. I solve the HJB in the viscosity sense following Aguiar et al. (2015a), as defined in Bressan and Hong (2007) and I provide an explanation for the solution concept in the Appendix B3.

I now proceed to show that $V(b)$ satisfies the HJB at points of differentiability, suppressing the dependence on t for ease of exposition. $V(b)$ can be expressed as follows,

$$V(b) = \frac{u(y - r(b)b)}{r(b)} + \sum_{i \setminus c} \frac{\lambda^i}{r(b)} \mathbb{1}_i(b) \hat{V} + \left(\frac{\lambda^s}{r(b)} \hat{V} + \frac{(\lambda^c - \lambda^s)}{r(b)} (V + \hat{V}(b)) \right) \mathbb{1}_c(b) \quad (25)$$

Substituting (25) in (24) yields,

$$\max_{c \in \bar{C}} \left\{ u(c) + \frac{dV(b)}{db} (c - y + r(b)b) + (\lambda^c - \lambda^s) \mathbb{1}_c(b) \hat{V}(b) \right\}$$

For stationary debt trajectories $c = y - r(b)b$ therefore (24) is satisfied for all points where $V(b)$ is differentiable in $\bar{\Omega}$.

Finally I characterise tangent trajectories, which arise for levels of debt where the government finds saving optimal, $c < y - r(b)b$. The envelope condition to the HJB (24) is given by,

$$V''(b) \left[c - y + r(b)b \right] = 0$$

Should consumption be lower than the steady state level due to the sovereign saving, then $V''(b) = 0$, implying the slope of the value function is linear in this region. From the first order condition, we know that this requires a constant consumption policy $c(t) = c^i$, given by the HJB evaluated as $b \downarrow b^i$, while the sovereign is saving. At b^{i*} , $c^i = y - r(b)b^{i*}$. The cost of running down debt is therefore given by $-u'(c^i)(b - b^i)$ where b^i is the target level of debt.

A.6. Proof of Lemma 4

Lemma 3 shows that the cost of a bailout is increasing in λ^s , therefore $V^{\text{sol}}(b; \lambda^s)$ decreasing in both arguments. Define $\Delta\lambda^s = \lambda^{s'} - \lambda^s$. If $V^{\text{sol}}(b; \lambda^{s'}) < \hat{V}(b) \leq V^{\text{sol}}(b; \lambda^s)$, then the increase in expectations of joint default is sufficient to change investor expectations of a bailout. Under the new level $\lambda^{s'}$, investors no longer deem the bailout guarantee credible, and country-specific expectations of default λ_i are relevant.

A.7. Proof of Proposition 3

For $b > b^c$, a bailout guarantee is not credible and there is no change in the interest rate. A government can choose to run down its debt to $b^c \geq b^s$ in order the benefit from the

bailout guarantee and earns utility,

$$\frac{u(y - (r^* + \lambda^s)b^c)}{\rho + \lambda^s} + \frac{\lambda^s}{\rho + \lambda^s}\hat{V} - u'(c^c)(b - b^c)$$

The cost of running down debt $-u'(c^s)(b - b^c)$ is lower than the cost of reducing debt to \underline{b} in the no cooperation benchmark. As $\lambda^s \rightarrow 0$, the return to a debt reduction strategy improves and in the limit, the return is strictly higher relative to the analogous strategy in the no intervention benchmark which earns utility,

$$\frac{u(y - r^*\underline{b})}{\rho} - u'(c^s)(b - \underline{b})$$

because $b^c > \underline{b}$.

A.8. Proof of Lemma 5

For $b > b^s$, if the official lender enforces market exclusion $\overline{b^I} = 0$, the borrower will default on market held debt and incur sanction αy . The official lender will incur collateral damage $\hat{\alpha}y$ as well. Assuming no repayment on official debt, the official lender needs to make a transfer of b^I , given by (9). The lender's utility, if it makes the bailout, is $V^{\text{sol}}(b)$ given by (7-8). The lender will find it optimal to undertake the bailout if $V^{\text{sol}}(b) \geq \hat{V}(b)$ which holds for $b \leq b^c$. A threat of $b^I = 0$ is not credible and the borrower has no incentive to repay official debt.

In contrast to the market, if the official lender can recoup ϵ by enforcing sanctions, it will still choose not to do so for $\epsilon < \hat{\alpha}y$.

A.9. Proof of Lemma 5

I assumed that investors pay $\xi(t) = b(0)e^{-\lambda t}$ during restructuring making them indifferent between repayment with or without restructuring. In the region $(\underline{b}_h, \underline{b}_l]$, each individual investor faces a $p > 0$ probability of no repayment and refuses to lend resulting in a self-fulfilling crisis at τ .

(i) Countries with outside option \underline{V}_h choose to default at τ since (14) implies $V^{\text{off-eq}}(b^h) = \underline{V}_h$ and $V^{\text{off-eq}}(b)$ is a decreasing function. (ii) Countries with outside option \underline{V}_l choose to

continue repaying, deleveraging in $[\tau, \tau + \delta]$ since (15) implies $V^{\text{off-eq}}(b^l) = \underline{V}_h$.

A.10. Proof of Proposition 4

By Lemma 6, self-fulfilling crises are possible only for $b > \underline{b}_h$. For $b \leq b^{c,h}$, it follows from Proposition 1 (ii) that investors expect joint default only, since countries will undertake large bailouts $b^{l,h}$ and therefore bailouts are off-equilibrium only. In the region $(b^{c,h}, b^{c,l}]$, governments will undertake small bailouts $b^{l,l}$, which will only be successful if the recipient has outside option \underline{V}^l . Since in this region, there is a p probability of default, investors zero-profit condition is not satisfied and they refuse to lend. A bailout will be observed in equilibrium since $V^{\text{sol}}(b) \geq \hat{V}$ for the country in good credit standing by (18). For $b > b^{c,l}$, no bailouts are made. The region will be non-empty for sufficiently low p since as $p \rightarrow 0$, $b^{c,l} > b^{c,h}$.

B. Solution to Optimal Control Problems

B.1. Optimal strategy for bailout agent

I characterize the optimal consumption policy for the country in good credit standing. The Hamiltonian to the optimal control (7-8) problem is given by,

$$H(b, c, t, p) := \max_c \left\{ e^{-(r^* + \lambda^c)t} \left[u(c(t)) + \lambda^c \hat{V} \right] + p(t) \left(c(t) - y + r(b(t))b(t) + b^l(t) \right) \right\}$$

I consider the optimal policy for the government at δ and work backwards. $V^{\text{hom}}(b)$, detailed in Appendix C1 is discontinuous in b . For sufficiently low debt, $b(\delta) \in (b^s, b^{s*})$, the government will find it optimal to reduce its debt to a sustainable level b^s in finite time. Over the region of savings, the value function is linear with gradient $-u'(c^s)$ (Proposition 2). I consider the interest rate to be fixed in $[0, \delta]$ while investors assess the situation.

By Pontryagin's maximum principle there exists a function $p^*(t)$ which satisfies the terminal condition,

$$p^*(\delta) = e^{-(\rho + \lambda^c)\delta} \frac{dV(b(\delta))}{db}$$

and the adjoint equation,

$$-\dot{p}^*(t) = \frac{H(b^*, p^*, c^*, t)}{db} = p^*(t) (\rho + \lambda^c)$$

This defines the law of motion for p^* . Moreover, the condition for a maximum requires,

$$0 = \frac{dH(b^*, p^*, c^*, t)}{dc} = e^{-(\rho + \lambda^c)t} u(c(t)) + p^*(t)$$

Using the law of motion for $p^*(t)$, the terminal condition and the maximum condition I arrive at,

$$u'(c) = u'(c(t)) \quad \forall t \in [0, \delta] \quad (26)$$

therefore a constant consumption strategy is optimal if the endogenous credit ceiling \bar{b} is not binding. For sufficiently low levels of debt, the government consumes $c^s \forall t$ and runs down debt to b^s in finite periods.

The government in good credit standing makes the minimum transfer needed to prevent partner default. In particular, the bailout agent assumes responsibility for all future debt servicing costs of a portion of period 0 debt of the borrower. It is useful to define the level of debt b^{oe} above which a country facing arrival rate of default λ^s would choose to default while deleveraging at rate χ ,

$$b^{oe} = \max \left\{ b(0) : \int_0^\delta e^{-(\rho + \lambda^s)t} \left[u(y - (\chi + r^* + \lambda^s)b(0)e^{-\chi t}) + \lambda^s \hat{V} \right] dt + e^{-(\rho + \lambda^s)t} V^{hom}(b(0)e^{\chi \delta}) > \hat{V} \right\}$$

The country in good credit standing will make a transfer b^I to cover the shortfall such that international investors break even. The bailout schedule is given by,

$$b^I(t) = (\chi + r^* + \lambda^s)(b(0) - b^{oe})e^{-\chi t}$$

In $[0, \delta]$, for sufficiently low debt, I guess that the debt policy for the country in good credit standing is given by,

$$\dot{b}(t) = (\chi + r^* + \lambda^s)(b(0) - b^{oe})e^{-\chi t} + (r^* + \lambda^c)(b(t) - b(0)) - (y - (r^* + \lambda^c)b(0) - c^s) \quad (27)$$

I verify by substituting the constraint faced by the the country in good standing,

$$c(t) - y + (r^* + \lambda^c)b(t) + e^{-\chi t} ((\chi + r^* + \lambda^s)(b(0) - b^{oe})) = \left[(\chi + r^* + \lambda^s)(b(0) - b^{oe})e^{-\chi t} + (r^* + \lambda^c)(b(t) - b(0)) - (y - (r^* + \lambda^s)b(0) - c^s) \right]$$

Indeed $c(t) = c^s$ in $[0, \delta)$ satisfying (27). Considering the above ordinary differential equation (ODE) in the form $\dot{b}(t) - (r^* + \lambda^c)b(t) = \alpha(t)$, where $\alpha(t) = (\chi + r^* + \lambda^s)(b(0) - b^{oe})e^{-\chi t} - (r^* + \lambda^c)b(0) - (y - (r^* + \lambda^c)b(0) - c^s)$. Using the factor $e^{\int -(r^* + \lambda^c)dt}$ for a given $b(0)$, for $t \leq \delta$,

$$b^l(t) = \left(e^{(r^* + \lambda^c)t} - e^{-\chi t} \right) \frac{\chi + r^* + \lambda^s}{\chi + r^* + \lambda^c} (b^b(0) - b^{oe}) + b^l(0) - \left(e^{(r^* + \lambda^c)t} - 1 \right) \frac{c(0) - c^s}{r + \lambda^c} \quad (28)$$

where superscripts b and l are used to distinguish the debt of the borrower and the lender.

For sufficiently high levels of debt, $b(\delta) \in [b^{s*}, \bar{b}]$ the sovereign will pursue a strategy whereby from δ onwards, $\dot{b} = 0$ is optimal. This is also the relevant case in the asymmetric scenario where the lender is not vulnerable to rollover crises. The optimal level of constant consumption \bar{c} must satisfy $\dot{b}(\delta) = \bar{b}$ and $\bar{c} = y - r(b)\bar{b}$. The level of consumption pursued if the government, while in the crisis zone, has no intention to run down debt at δ is given by,

$$\bar{c} = e^{-(r^* + \lambda^c)\delta} \left\{ y - (r^* + \lambda^c) \left[\left(e^{(r^* + \lambda^c)\delta} - e^{-\chi\delta} \right) \frac{\chi + r^* + \lambda^s}{\chi + r^* + \lambda^c} (b^b(0) - b^{oe}) + b^l(0) - \left(e^{(r^* + \lambda^c)\delta} - 1 \right) \frac{c(0)}{r + \lambda^c} \right] \right\}$$

I next search for an initial level of debt of debt $b_{-\delta}^{s*}$, below which $c(t) = c^s$ is indeed the optimal control for the solidarity problem. Assume $c(t) = c^s \leq \bar{c}$, then the law of motion for debt is given by (27) and the particular solution is given by (28). Using $b(\delta) = b^{s*}$,

$$b_{-\delta}^{s*} = e^{-(r^* + \lambda^c)\delta} b^{s*} - \left(e^{-(r + \lambda^c)\delta} - 1 \right) \frac{c(0) - c^s}{r + \lambda^c} + \left(1 - e^{-(r^* + \lambda^c)\delta} \right) b^l(0) + \left(e^{-(\chi + r + \lambda^c)\delta} - 1 \right) (b^b(0) - b^{oe})$$

The optimal consumption policy is,

$$c(t) = \begin{cases} c^s & \text{if } b(0) \in [b^s, b_{-\delta}^{s*}) \\ \bar{c} & \text{if } b(0) \in [b_{-\delta}^{s*}, \bar{b}] \end{cases}$$

where c^s is a constant level of consumption pursued while the government saves and \bar{c} is the level of consumption pursued if the government does not intend to run down debt at δ .

The utility for the government in good credit standing if $b \leq b_{-\delta}^{s*}$ is given by,

$$V^{\text{sol}}(b) = \frac{1 - e^{-(r^* + \lambda^c)\delta}}{r^* + \lambda^c} [u(c^s) + \lambda^c \hat{V}] + e^{-(\rho + \lambda^c)\delta} \left(\frac{u(y - r^* b^s)}{\rho} - u'(c^s)(b(\delta) - b^s) \right)$$

where $b(\delta)$ is given by (27) evaluated at $t = \delta$ and the utility for the government in good credit standing, if $b > b_{-\delta}^{s*}$, in the crisis zone, is given by,

$$V^{\text{sol}}(b) = \frac{u(\bar{c})}{r^* + \lambda^c} + \frac{\lambda^c}{r^* + \lambda^c} \hat{V}.$$

Analogous results can be derived for the solidarity and contagion zones using Proposition 2.

The optimal control problem (7-8) satisfies all but one conditions of Kamien and Schwartz (1971) who prove sufficiency using a basic calculus of variations argument. The maximand in (7) does not satisfy the condition that $V^{\text{hom}}(b(\delta))$ must be everywhere differentiable and it remains to show that the continuation value attains a maximum at the optimal control. This is satisfied because $V^{\text{hom}}(b(\delta))$ is itself the value function for the homogeneous expectations optimal control problem,

$$V^{\text{hom}}(b(t)) = \sup_{c \in C} P_{b,t}[c(t)]$$

where,

$$P_{b,t}[c(t)] = \int_0^\infty e^{-\rho t - \lambda^s \int_0^t \mathbb{1}_s(b(s)) ds} \left[u(c(t)) + \lambda^s \hat{V} \mathbb{1}_s(b(t)) \right] dt$$

and denoting the pay off of the continuation value to the solidarity problem as $P_{b,t}^{\text{sol}}[c(t)]$,

$$\operatorname{argmax}_{c \in C^{\text{sol}}} P_{b,t}^{\text{sol}}[c(t)] = \operatorname{argmax}_{c \in C} P_{b,t}[c(t)]$$

therefore $P_{b,t}[c^*(t)] - P_{b,t}[c(t)] \geq 0$ as required.

IX. Appendix II

.1. Optimal control on stratified domains

This appendix summarises and applies Proof 1 in Aguiar et al. (2013). I present the technical conditions required to prove that $V(b)$ is the unique, Lipschitz continuous viscosity solution to the HJB using the results in Bressan and Hong (2007) henceforth BH.

Conditions on R : The conditions summarised below allow me to apply the results in BH. They are analogous to those presented in Definition 1 of Aguiar et al. (2015a). R is a set of functions on $\bar{\Omega}$, $r : \bar{\Omega} \rightarrow \mathbb{R}_+$. The function r is bounded and lower semi continuous. Furthermore, r must be such that it is always feasible to have positive consumption with $\dot{b} = 0$ on $\bar{\Omega}$. Finally r contains a finite number of discontinuities $N - 1$ and is Lipschitz continuous on sets Ω_n for all $n \in N$ as described by the decomposition above. This satisfies condition (H1) in BH.

I use the definition of a viscosity solution relative to the stratification of the domain in BH. The analysis considers a Hamilton-Jacobi-Bellman equation of the form,

$$\rho V(b) = H(b, V'(b)) = \max_{(h, w \in G)} \{w + V'(b)h\}$$

I decompose $\bar{\Omega} \in \mathbb{R}$ into the disjoint union of N intervals starting from $b = 0$ and using the debt thresholds derived in the paper to characterize open sets and points of discontinuity, $(0, b_1), \dots, (b_{N-2}, b_{N-1})$. The debt trajectories and the running utility payoff will be sufficiently regular within each manifold but may change discontinuously across.

All admissible control trajectories are tangent trajectories within the open sets and steady state trajectories at the boundaries satisfying condition (H2) in BH. Having also imposed an upper bound on u , I can use Theorem 1 in BH to prove existence of an optimal control for the HJB. For every point of non differentiability b_n , $V(b^+) = V(b^-)$, where $V(b^+)$ is the limit as b approaches b_n from above and $V'(b^+) \neq V'(b^-)$ with $V'(b^+)$ and $V'(b^-)$ exist and are bounded. It follows that the value function is Lipschitz continuous satisfying (H3).

Finally, the running utility payoff is Lipschitz continuous with respect to b within each interval satisfying equation (46) in BH albeit in a broader sense than in Aguiar et al.

(2015a). Corollary 1 of BH states that under assumptions (H1)-(H4), if equation (46) is satisfied then $V(b)$ is the unique, Lipschitz continuous viscosity solution to the HJB.

A. Auxiliary Problems

A.1. No heterogeneity in expectations

The homogeneous expectations case is relevant when investors believe that countries will only ever default jointly. This is also the limit of $V(b)$ as $b^I(t)$, the cost of intervention approaches 0 as would be the case in the limit $\chi, \delta \rightarrow 0$. The lower multiplicity threshold is unchanged in the symmetric model, since it corresponds to the maximum amount of debt that a country can sustain when excluded from markets absent an intervention from its partner. Each government faces the following problem,

$$V^{hom}(b) = \max_c \int_0^\infty e^{-\rho t - \lambda^s \int_0^t \mathbb{1}_s(b(s)) ds} \left[u(c(t)) + \lambda^s \mathbb{1}_s(b(t)) \hat{V} \right] dt$$

$$s.t \quad \dot{b}(t) = c(t) - y + r(b(t))b(t),$$
(29)

The participation condition is satisfied for a higher level of debt than that of the heterogeneous expectations case since sovereigns will face a lower debt servicing costs and a lower probability of default. The associated-Hamilton Jacobi Bellman equation is,

$$(\rho + \lambda^s \mathbb{1}_s(b(t)))V^{hom}(b) - \lambda^s \mathbb{1}_s(b(t))\hat{V} = \max_{c \in C} \left\{ u(c(t)) + V'(b)(c(t) - y + r(b(t))b(t)) \right\}$$

The value function for each government is given by $V(b) : \Omega \rightarrow \mathbb{R}$,

$$V^{hom}(b) = \begin{cases} \frac{u(y - r^*b)}{\rho} & \text{if } b \in [0, b^s] \\ V(b^s) - u'(c^s)(b - b^s) & \text{if } b \in (b^s, b^{s*}) \\ \frac{u(y - (r^* + \lambda^s)b)}{\rho + \lambda^s} + \frac{\lambda^s}{\rho + \lambda^s} \hat{V} & \text{if } b \in [b^{s*}, \bar{b}] \end{cases}$$

where c^s is given by the HJB taking the limit $b \downarrow b^i$ and the interest rate schedule is given by $r(b(t)) = r^* + \lambda^s \mathbb{1}_s(b(t))$.

A.2. Equilibrium when partner has defaulted

The problem faced by a government when its partner has defaulted is the outside option for a country in good standing choosing to undertake a bailout. It is also required to derive the necessary condition for the threshold level of debt above which contagion can arise. A country in good credit standing faces the following problem when its partner is in default,

$$\hat{V}(b) = \max_c \int_0^\infty e^{-\rho t - \lambda^c \int_0^t \mathbb{1}_c(b(s)) ds} \left[u(c(t)) + \lambda^c \mathbb{1}_c(b(t)) \hat{V} \right] dt$$

$$s.t. \quad \dot{b}(t) = c(t) - (1 - \hat{\alpha})y + r(b(t))b(t)$$

and faces a default utility of \hat{V} . The country in default will still have an incentive to intervene as it wishes to avoid the collateral damage, but I assume it is unable to do so effectively since it is itself excluded from markets. The debt thresholds are given as follows,

$$\hat{b}^s = \max \left\{ b \leq \frac{(1 - \hat{\alpha})y}{\chi + r^*} : \hat{V}^{\text{off-eq}}(b) \geq \hat{V} \right\}$$

$$b^p = \max \left\{ b \leq \frac{(1 - \hat{\alpha})y}{r^* + \lambda^c} : \hat{V}(b) \geq \hat{V} \right\}$$

Notice that for $b > b^p$, $\hat{V}(b) = \hat{V}$. Construction of the equilibrium is analogous to Proposition 2.

A.3. Comparative statics for \underline{b}

I investigate how the sustainable level of debt \underline{b} varies with the repayment schedule χ and the duration of market exclusion δ . As $\chi \rightarrow 0$, as in the case of a consol, self-fulfilling crises will not arise in equilibrium because the government will not need to deleverage in case of market exclusion. As χ rises, the deleveraging constraint tightens and the government is less able to smooth its consumption. This makes default more attractive.

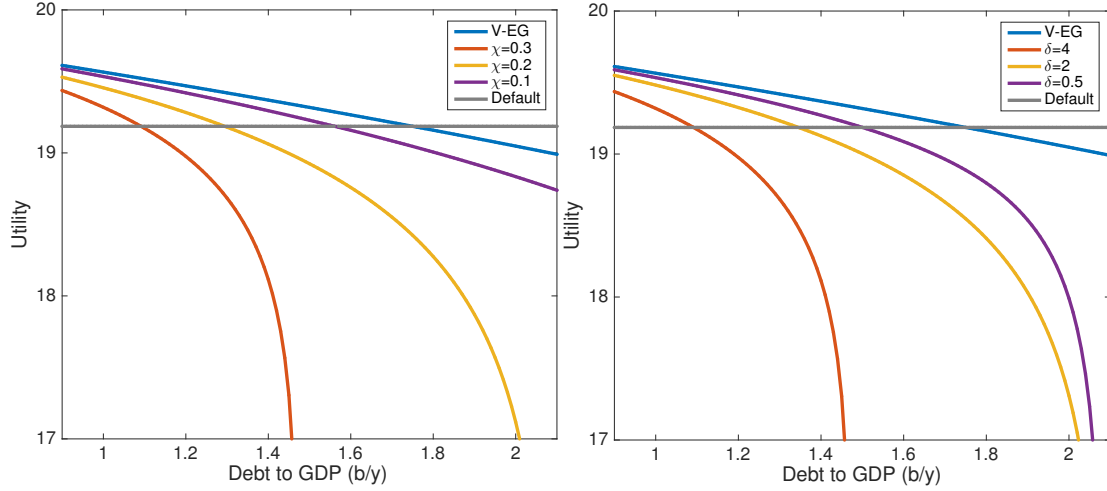


Figure 1. Numerical simulation and comparative statics for the utility of repayment in case of market exclusion. The LHS presents the effects of varying the repayment schedule χ when the period of exclusion $\delta = 4$ and the RHS presents the effects of varying δ when $\chi = 0.2$.

A.4. Conditions for optimal gambling strategy

I derive the payoff for a government with $b(0) \in \mathcal{C}$, who finds it optimal to save to b^s . This is optimal if $\nexists b^{s*} \in \mathcal{S}$. The government first faces the choice of gambling in \mathcal{C} or running down debt to b^c where there exists a discontinuity in $r(b)$ below which the interest rate is lower in equilibrium due to credible solidarity (Lemma 2). At b^c , the government faces the choice of maintaining a constant level of debt b^c or running debt down to b^s , where no crises are possible and the interest rate is r^* . Starting at b^c , running down debt is optimal if,

$$\frac{u(y - (r^* + \lambda^s)b^c)}{\rho + \lambda^s} + \frac{\lambda^s}{\rho + \lambda^s} \hat{V} < \frac{u(y - r^*b^s)}{\rho} - u'(c^s)(b^c - b^s) \quad (30)$$

The condition for gambling in the crisis zone is given by,

$$\frac{u(y - (r^* + \lambda^c)b)}{\rho + \lambda^c} + \frac{(\lambda^c - \lambda^s)}{\rho + \lambda^c} (\hat{V}(b) + \underline{V}) + \frac{\lambda^s}{\rho + \lambda^c} \hat{V} \geq \frac{u(y - r^*\tilde{b})}{\rho} - u'(\tilde{c}^c)(b - \tilde{b}) \quad (31)$$

. If (30) holds and (31) does not, then a government with $b \in (b^c, b^{c^*})$ will save to b^s as in the dotted dynamics in figure 2. The pay-off to a safe strategy is given by,

$$V(b) = V(b^c) - u'(c^c)(b - b^c), \quad (32)$$

where $V(b^c)$ is given by the RHS of (30) and c^s, c^c are presented in Proposition 2.

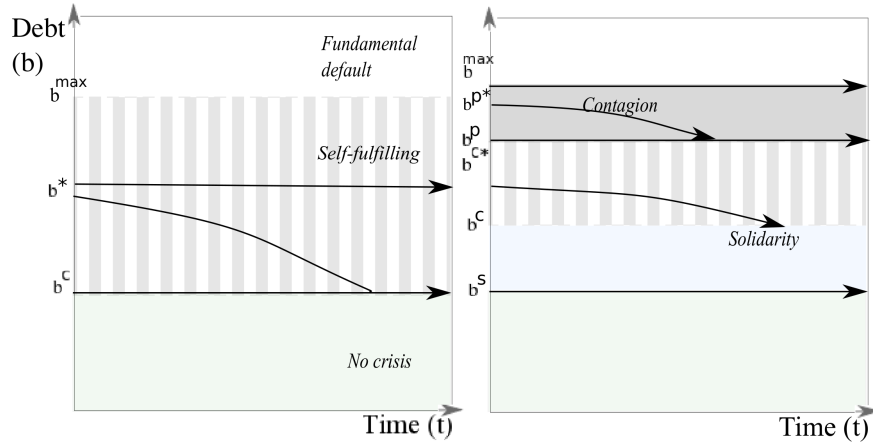
A country may choose to pursue a constant debt trajectory at any level of debt below b^s and in each of the regions $\mathcal{S}, \mathcal{C}, \mathcal{P}$. The pay offs to each strategy are presented in Proposition 2. Should the government be given a choice of starting at any of these gambling strategies, the government will always prefer to gamble at a lower level of debt facing lower debt servicing costs and a lower probability of default. Within any debt region above b^s , a gambling strategy is more attractive as debt increases. A gambling equilibrium is more likely to exist for a country i if the jump in the interest rate at the discontinuity λ^i is small.

B. Extensions

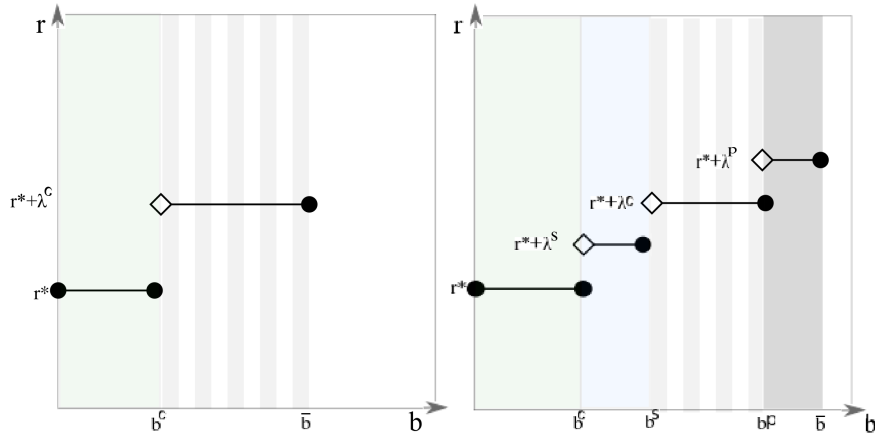
B.1. Joint liability

In this section, I analyse the implications of imposing joint liability contracts on market financing costs and optimal policy. Part of the problem in Europe lies in the architecture of sovereign debt. Under joint liability, each country is fully responsible for repaying bonds issued by either country. The model no longer admits outcomes of national default since countries will only default jointly, therefore, $\underline{V} = \hat{V}(b) = \frac{(1 - (\hat{\alpha} + \alpha))y}{r^*}$. In section 3, I analyse a model where countries optimally choose to share the debt burden out of self-interest. However, under joint liability, debt contracts are such that partner default will imply joint default. If expectations of default are not symmetric between countries, the benefits of joint liability are unevenly distributed.

Tirole (2015) studies whether joint liability will arise endogenously. In a stylized two-period, two country model, he finds that joint liability is enforceable if spillover costs of default are sufficiently high and if countries face symmetric fundamental risk. Brunnermeier et al. (2016) discuss the issuance of European Safe Bonds (ESBies), a senior tranche backed by a diversified portfolio of Euro-area bonds. They advocate that these are held by banks



(a) Equilibrium debt trajectories conditional on no default.



(b) Equilibrium interest rate demanded by international investors.

Figure 2. Debt dynamics and prevailing interest rates in sovereign risk models with self-fulfilling crises. LHS reflects standard results from the one country benchmark without international cooperation whereas RHS reflects the optimal policy from Proposition 2.

to insulate them from domestic sovereign risk and thus break the *diabolic loop*.

Under joint liability, if a government is excluded from the market, its partner assumes full responsibility for the outstanding stock of debt. In a symmetric equilibrium, the lower multiplicity threshold will be higher than under individual liability because sovereigns face an outside option of joint default, $\hat{V} \leq \underline{V}$. The no crisis region will exist as an equilibrium

interval for $[0, b_{jl}^s]$, where the threshold is defined by,

$$b_{jl}^s = \max \left\{ b \leq \frac{y}{\chi + r^*} : V^{\text{off-eq}}(b) \geq \hat{V} \right\}$$

As a result, $b_{jl}^s > b^s$ and the size of the no crisis zone is increasing in $\hat{\alpha}$. Moreover, a country in good credit standing will be willing to undertake a bailout for a larger range of debt because it now faces the outside option of joint default. In a symmetric equilibrium, the threshold for debt below which a bailout is undertaken is given by,

$$b_{jl}^c = \max \left\{ b \leq \frac{y - b^I(b)}{r^* + \lambda^c} : V^{\text{sol}}(b) \geq \hat{V} \right\}$$

In the new solidarity zone, $b \in (b_{jl}^s, b_{jl}^c]$, sovereigns will default jointly with rate of arrival λ^s when a system wide sunspot arrives. The fundamental default threshold will be unchanged and for $b \in (b_{jl}^c, \bar{b}]$ expectations of national default result in joint default. In this region, investors demand an interest rate of $r^* + \lambda^p$. National defaults are reduced only at the expense of increased system wide defaults in the region $(b_{jl}^c, b^p]$.²⁶ Intuitively, under joint liability, the country in good credit standing is forced to assume liability for its partner's debt for levels at which it would have preferred to let its partner default.

Joint liability has implications on the optimal borrowing policy. Since $b_{jl}^s > b^s$, there is an increased incentive for governments, with any level of outstanding debt, to pursue a safe strategy. For $b \in (b^c, b_{jl}^c]$, lower borrowing costs provide an incentive to pursue a constant debt strategy where default arrives at λ^s . In this region, joint liability will increase the government's incentive to save to b_{jl}^s only if λ^s is high.

In the case of asymmetric expectations of default, $\lambda_i \neq \lambda_j$, a country i facing a higher probability of default in the no cooperation benchmark than j will benefit more from joint liability. Both countries will experience the same increase in the sustainable level of debt b_{jl}^s . The increase in the solidarity region, due to the introduction of joint liability will be larger for country i , since j can afford to undertake a larger bailout.

²⁶To understand when this region would be non-empty notice that $\lim_{\hat{\alpha} \rightarrow 0} \hat{V} = V$. However, if $\delta > 0$ then $V^{\text{sol}} < V$ even if $\hat{\alpha}=0$ which implies $b_{jl}^c < b^p < \bar{b}$ is possible.

B.2. International integration

I assess the implications of introducing a third country k to the symmetric model, with economic ties to the existing ones. The analysis can address concerns about the enlargement of the European Union. An additional country will widen the insurance pool by introducing an additional layer of off-equilibrium transfers. However, the system becomes vulnerable to a wider set of default expectations. The additional layer of collateral damage can increase the severity and likelihood of system-wide default.

I outline the implications of introducing a third country to the model above, while holding the cost of default and \hat{V} fixed. Wider international integration is unlikely to increase exposure to foreign defaults but will rather spread costs across economic partners. If default expectations for country k arrive at rate $\lambda_k^c = \gamma_z + \gamma_k$, the probability of default in the solidarity region is unchanged, $\lambda^s = \gamma_z$. However, the bailout threshold b^c will rise because the two countries in good credit standing will share the burden of the bailout and benefit from solidarity while undertaking the bailout. If default expectations for country k arrive at rate $\lambda_k^c = \gamma_k$ then $p_{kz} = 0$ and k is not vulnerable to the arrival of γ_z , the system wide sunspot. In this case, an additional and distinct region of solidarity will arise for low levels of debt above b^s . In the new region, k will credibly undertake a bailout for countries i and j and default arrives at rate γ_k . This region is wider when agent k is large and γ_k is small. This is arguably the relevant case when modelling international institutions.

Conversely, for levels of debt near \bar{b} , the system will be vulnerable to a wider set of market sentiments. A new contagion zone arises for sufficiently high debt below \bar{b} and default arrives at $\lambda^p = \gamma_i + \gamma_j + \gamma_k + \gamma_z$.

The analysis is complicated by changes in existing thresholds and the need to derive additional ones. There will also be implications on the optimal borrowing policy. In the symmetric equilibrium the lower multiplicity threshold will remain unchanged, unless $\lambda_k = 0$. If $\lambda_k = 0$ but $\lambda_{ij} > 0$, then the introduction of the third country provides perfect insurance and enlarges the no crisis zone. The threshold for fundamental default, \bar{b} , will generally fall due to a higher interest rate in the contagion zone.

The analysis reveals that in a symmetric model, for low levels of debt, international integration is beneficial due to a wider resource pool for bailouts. This is true even if the new countries are subject to the same global sunspot, γ_z . For high levels of debt, the intro-

duction of new countries exposes debt markets to a wider set of market sentiments which can increase the probability of self-fulfilling default. This implies that the expansion of the Eurozone is likely to have reduced vulnerability to self-fulfilling crises in the early 2000s, when fundamentals were relatively strong, but may have contributed to the probability of contagion in recent years when fundamentals were weak. Since the risk of self-fulfilling crises is more pronounced when fundamentals are weak, the expansion of the European Union may have contributed to the vulnerability of debt markets to self-fulfilling crises and contagion.

In addition, the analysis shows that a large agent k , not subject to the global sunspot γ_z can lead to better outcomes. The ESM is considered safe from expectations-driven defaults, because of a system of sophisticated guarantees and securitization. The establishment of the ESM has largely addressed the issue of political risk in the Eurozone and has provided a credible backstop to sovereign debt shielding member states from self-fulfilling crises. The explicit conditionality which accompanies debt purchases has also succeeded in preventing over borrowing by governments.