

Optimal Domestic (and External) Sovereign Default*

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Abstract

Infrequent but turbulent episodes of outright sovereign default on domestic creditors are considered a “forgotten history” in macroeconomics. We propose a heterogeneous-agents model in which optimal debt and default on domestic and foreign creditors are driven by distributional incentives and endogenous default costs due to value of debt for self-insurance, liquidity, and risk-sharing. The government’s aim to redistribute resources across agents and through time in response to uninsurable shocks produces a rich dynamic feedback mechanism linking debt issuance, the distribution of government bond holdings, the default decision, and risk premia. Calibrated to Eurozone data, the model is consistent with key cyclical comovements and features of debt-crisis dynamics. Debt exhibits protracted fluctuations. Defaults have a low frequency of 1.2 percent, are preceded by surging debt and spreads, and occur with relatively low external debt. Default risk limits the sustainable debt, and yet spreads are zero most of the time.

Keywords: public debt, sovereign default, debt crisis, European crisis

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1 Introduction

The central finding of the seminal cross-country analysis of the history of public debt dating back to 1750 by Reinhart and Rogoff [42] is that governments defaulted outright on their *domestic* debt 68 times. Hall and Sargent [30] also document in detail a domestic sovereign default in the aftermath of the American Revolutionary War. These are de jure defaults in which governments reneged on the contractual terms of domestic debt via mechanisms such as forcible conversions, lower coupon rates, reductions of principal and suspension of payments, separate from de facto defaults due to inflation or currency devaluation. Domestic defaults are less frequent than external defaults, by a 1-to-3 ratio, but they are at least as important in terms of size and the macroeconomic instability that surrounds them. Also, all of them triggered external defaults, in several instances even at low external debt ratios.¹ Despite these striking facts, Reinhart and Rogoff found that domestic defaults represent a “forgotten history” in the macroeconomics literature.

Recent events raising the prospect of domestic defaults in advanced economies make this history much harder to forget. The European debt crisis and historically high public debt ratios in the U.S. and Japan suggest that the conventional wisdom treating domestic public debt as a risk-free asset is flawed and that there is a critical need to understand the riskiness of this debt and the dynamics of domestic defaults. The relevance of these issues is emphasized further by the sheer size of domestic public debt markets: The global market of local currency government bonds was worth about U.S.\$30 trillion in 2011, roughly half of the world’s GDP and six times larger than the market for investment-grade sovereign debt denominated in foreign currencies. Domestic debt also accounted for a large fraction of total public debt in most countries, almost two-thirds on average.²

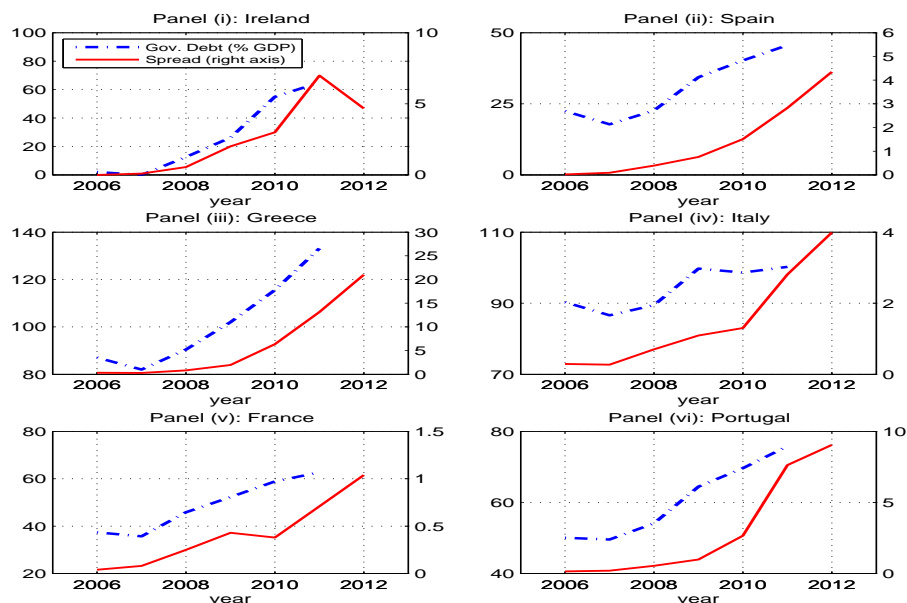
The European debt crisis is often, but in our view mistakenly, treated as a set of country-specific external sovereign debt crises. This view ignores three key features of the Eurozone that make a sovereign default by one member more akin to a domestic default than an external default: First, a large fraction of Eurozone public debt is held within Europe,

¹As Reinhart and Rogoff also highlighted, the decomposition of public debt into domestic and external is difficult. Several studies, including this paper, define domestic debt as that held by domestic residents, for which data are available for a limited number of countries in international databases (e.g., OECD Statistics). Other studies define domestic debt as debt issued under domestic, instead of foreign, jurisdiction. The two definitions are correlated, but not perfectly, and in some episodes have differed significantly (e.g. most of the bonds involved in the debt crises in Mexico, 1994 and Argentina, 2002 were issued domestically but with significant holdings abroad).

²Global bond market values and debt ratios are from *The Economist*, Feb. 11, 2012, and from the International Monetary Fund (IMF).

so default by one member can be viewed as a (partial) domestic default from the point of view of the Eurozone as a whole. Second, the Eurozone's common currency prevents individual countries from unilaterally reducing the real value of their debt through inflation (i.e., implementing country-specific de facto defaults). Lojsch, Rodriguez-Vives, and Slavik [34] report that about half of the public debt issued by Eurozone countries was held by Eurozone residents as of 2010, and 99.1 percent of this debt was denominated in euros.³ Third, and most important from the standpoint of the model proposed in this paper, policy discussions and strategies for dealing with the crisis emphasize the distributional implications of a default by one member country on all the Eurozone, and the high costs of damaging public debt markets. This is a critical difference relative to external defaults because it shows the concern of the parties pondering default decisions for the adverse effects of a default on the governments' creditors.⁴

Figure 1: Eurozone Debt Ratios and Spreads



During the European debt crisis, net public debt of countries at the epicenter of the crisis (Greece, Ireland, Italy, Portugal, and Spain) ranged from 45.6 percent to 133.1 percent of GDP, and their spreads versus Germany were large, ranging from 280 to 1,300 basis points (see Appendix A-1). Debt ratios in the large core countries, France and Germany, were also

³This 48 percent is only for Eurozone members. The fraction exceeds 85 percent if we add public debt holdings of European countries that are not in the Eurozone (particularly Denmark, Sweden, Switzerland, Norway, and the UK).

⁴Still, the analogy with a domestic default is imperfect, because the Eurozone lacks a fiscal authority with taxation powers across all its members, except for seigniorage collected by the European Central Bank.

relatively high at 62.7 percent and 51.5 percent, respectively. Figure 1 shows that both debt ratios and spreads were stable before 2008 but grew rapidly afterward (except in Italy, where the debt ratio was already high but spreads widened also after 2008). The fractions of each country’s debt held by residents of the same country ranged from 27 percent in Greece to 64 percent in Spain.

This paper proposes a model with heterogeneous agents and incomplete financial markets in which domestic default can be optimal for a government that uses debt and default to redistribute resources in response to idiosyncratic personal income shocks and aggregate government expenditure shocks. Default is optimal when the aggregation of individual utility gains from default across agents that are heterogeneous in bond holdings and income using a social welfare function with given weights is positive (i.e., when the social payoff of default exceeds that of repayment).

Default has endogenous costs that result from the role of public debt as a vehicle for self-insurance, liquidity-provision, and risk-sharing, and it also has an exogenous income cost. The first two endogenous costs result from roles that public debt typically serves in heterogeneous-agents models with incomplete markets: It provides agents with a vehicle for self-insurance against uninsurable shocks, and it provides liquidity (i.e., resources) to a fraction of agents who are endogenously credit-constrained. Default wipes out the public debt holdings of all agents, forcing them to restart the costly process of deferring consumption to rebuild their buffer stock of savings. Agents who have a stronger need to either draw from this buffer stock or to buy bonds to build them up incur a large utility cost if the government defaults. Moreover, the utility cost of default is also large for poor agents with low income and no bond holdings because they face binding borrowing limits and thus value the liquidity that public debt provides.

The risk-sharing role of public debt is due to the fact that, with debt, the government can redistribute resources across agents and through time. Current issuance of new debt causes “progressive redistribution” (i.e., in favor of agents with below-average bond holdings), while future repayment of that debt causes “regressive redistribution” in the opposite direction. Default can prevent the latter ex-post, but the ex-ante probability that this can happen lowers bond prices at which debt can be issued and thus hampers the government’s borrowing capacity and its ability to engage in progressive redistribution.

Since the distribution of bond holdings evolves endogenously over time and the government cannot discriminate among its creditors (in line with the *pari passu* clause typical of government debt), repayment and default affect the cross section of agents differently and these differences evolve over time.⁵ In each period, the social welfare gain of default

⁵The *pari passu* clause makes government bonds rank *pari passu* with each other *and* with other unsecured obligations of the government. Its meaning and enforceability had been subject of debate, but its enforcement

summarizes the trade-off between the government’s incentive to default to avoid regressive redistribution and the costs of default.

The government also levies a proportional income tax as an alternative vehicle for redistribution that operates in the usual way to improve risk-sharing of *idiosyncratic* income shocks. A 100 percent tax on individual income to finance a uniform lump-sum transfer provides perfect risk-sharing of these shocks but still does not provide insurance against the aggregate shocks. We study equilibria in which the income tax rate matches actual tax rate estimates, which are well below 100 percent.

The model includes the typical risk-neutral foreign creditors of the Eaton-Gersovitz [22] (EG hereafter) class of external default models, which yields the standard arbitrage condition linking default risk premia to default probabilities. This simplifies the determination of bond prices and enables us to study the distribution of debt across domestic versus foreign creditors. Default, debt, and risk premia dynamics, however, respond to very different forces from those at work in EG models because the government’s payoff function factors in the utility of all domestic agents, including its creditors.

Equilibrium dynamics in the model are governed by a rich dynamic feedback mechanism connecting the government’s debt issuance and default choices, the price of government bonds, the optimal plans of individual agents, and the dynamics of the distribution of bonds across agents (i.e., the wealth distribution). Wealth dynamics are driven by the agents’ optimal plans and determine the evolution of individual utility gains of default across the cross section of agents. In turn, a key determinant of the agents’ plans is the default risk premium reflected in the price of public debt, which is determined by the probability of default, which is itself determined by the governments aggregation of the individual default gains.

Public debt, spreads, and the social welfare gain of default evolve over time driven by this feedback mechanism as the exogenous shocks hit. With low debt, low realizations of government expenditure shocks, or both, repayment incentives are stronger producing “more negative” welfare gains of default, which in turn make repayment and increased debt issuance optimal. At higher debt, higher government expenditure shocks, or both, the balance changes, and as the dispersion of individual gains from default widens and the social welfare gain from default rises, debt can reach levels for which the latter becomes positive and default is optimal. Default wipes out the debt and sets the economy back to a state in which repayment incentives are strong because starting with zero debt the social value of debt is high. These dynamics also affect the allocation of public debt across domestic and external agents. After a default, the two grow at a similar pace as domestic demand grows gradually

in a 2000 case involving Peru’s debt and the recent case involving Argentina have significantly strengthened its legal standing (see Olivares-Caminal [38]).

because of the utility cost of postponing consumption to rebuild the buffer stock of savings, but over time, as self-insurance demand for debt continues to rise, domestic agents hold a larger share of public debt than foreign agents. As default approaches, the relative share of domestic agents falls, since debt at rising spreads is mainly sold to the risk-neutral foreign investors and sufficiently rich domestic agents.

The optimal debt moves across zones with one of three characteristics. First, a zone in which repayment incentives are strong (i.e., the social gain of default is “very negative”) and can sustain the optimal debt at zero default risk, and that debt is lower than the debt that maximizes the resources that can be gained by borrowing. Second, a zone in which the optimal debt is still offered default-risk-free but it is also the amount of debt that yields the most resources possible. Here, weaker repayment incentives result in bond prices that fall sharply if debt exceeds this amount, so debt is a risk-free asset but is still constrained by the government’s inability to commit to repay. Third, a zone in which repayment incentives are in between the first two cases, so that the optimal debt carries default risk but still generates more resources than risk-free debt and less than the maximum that could be gained with risky borrowing.

We study the model’s quantitative predictions by solving numerically the recursive Markov equilibrium without commitment using parameter values calibrated to data for the Eurozone.⁶ Most parameters are taken directly from empirical studies or our data estimates, while the discount factor, the welfare weights, and the exogenous default cost are set targeting the averages of the GDP ratios of total and domestic debt and spreads.

The model supports equilibria with debt and default, and the model’s dynamics both over the long run and around default events are in line with key features of the data. Comparing peak values for high-default-risk events excluding default (since most countries in the Eurozone did not default in the recent crisis), the model nearly matches the average total and domestic debt ratios and the ratio of domestic to total debt, while it produces spreads and external debt higher than in the data. In the long run, the model matches the qualitative ranking of the correlations of government expenditures with spreads, consumption, and net exports. Matching these correlations is important because government expenditure shocks (the model’s only aggregate shock) are central to the feedback mechanism we described because they weaken (strengthen) repayment incentives when they are high (low). The model also nearly matches the relative variability of consumption, net exports, and total public debt, and produces correlations with disposable income that have the same signs as in the data.

Defaults have a low long-run frequency of 1.2 percent, in line with Reinhart and Rogoff’s

⁶We use data from Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, and Spain.

[42] observation that domestic defaults are infrequent. As in the data, debt and spreads rise rapidly and suddenly in the periods close to a default, while in earlier periods, debt is stable and free of default risk. The ratio of external to domestic debt increases as a default approaches, but external debt is still only about 55 percent of total debt when default hits. Thus, to an observer of the model’s time series, a debt crisis looks like a sudden shock following a period of stability and with small variations in external debt. The debt buildup coincides with relatively low government expenditures, which strengthen repayment incentives and sharply reduce the social welfare gain of default to about -1.1 percent, while the default occurs with a modest increase in government purchases, which, at the higher debt, is enough to shift the distribution of individual default gains to yield a large increase in the social welfare gain of default to 0.9 percent.

The equilibrium recursive functions show significant dispersion in the effects of changes in debt and government expenditures on individual gains from default across agents with different bond holdings and income. This dispersion reflects differences in the agents’ valuation of the self-insurance, liquidity, and risk-sharing benefits of debt, and the effect of the exogenous income shock of default. As a result of these differences, the social distribution of default gains shifts markedly across states of debt and government purchases, producing large shifts in the social welfare gain of default in the dynamics near default events. The bond pricing function has a shape similar to that of EG external default models: starting at the risk-free price when debt is low and falling sharply as debt starts to carry default risk. The associated debt Laffer curves shift downward and to the left at higher realizations of government expenditures and display the three zones across which the optimal debt moves.

We conduct a sensitivity analysis to study the effects of changes in the social welfare weights, the parameters that drive self-insurance incentives, the income tax rate, and the exogenous cost of default. Some of the quantitative results hinge on how default incentives vary with each alternative scenario, but, overall in all the scenarios, the model sustains average debt ratios of similar magnitude as in the data at a low but positive default frequency. Spreads are negligible only when the exogenous default cost is removed completely, but in this scenario the amount of debt that is sustained is constrained by the government’s inability to commit. Debt is optimally chosen to be risk-free because otherwise bond prices drop too much so choosing risky debt generates few borrowed resources.

This paper is part of the growing research programs on optimal debt and taxation in incomplete markets models, both representative-agent and heterogeneous-agents models, and on external sovereign default. We make two main contributions: First, we propose a model in which optimal public debt issuance, default, and spreads are determined jointly with the dynamics of the distribution of debt holdings across a continuum of domestic heterogeneous-

agents and foreign investors. Second, we study the model’s quantitative predictions, including long- and short-run dynamics and contrast them with observed empirical regularities.

Well-known papers in the heterogeneous-agents literature explore the implications of public debt in models in which debt provides similar benefits as in our model (e.g., Aiyagari and McGrattan [7], Azzimonti, de Francisco, and Quadrini [11], Floden [23] and Heathcote [32]). Aiyagari and McGrattan [7] quantify the welfare effect of debt in a setup with capital and labor, distortionary taxes, and an exogenous supply of debt. Calibrating the model to U.S. data and solving it for a range of debt ratios, they found a maximum welfare gain of 0.1 percent. In contrast, a variant of our model without default risk predicts that the gain of avoiding an unanticipated, once-and-for-all default can reach 1.35 percent. Azzimonti et al. [11] link wealth inequality and financial integration with the demand and supply for public debt to explain growing debt ratios in the last decade. Heathcote [32] derives non-Ricardian implications from stochastic proportional tax changes because of borrowing constraints. Floden [23] shows that transfers rebating distortionary tax revenue dominate debt for risk-sharing of idiosyncratic risk. As in this paper, these papers embody a mechanism that hinges on the variation across agents in the benefits of public debt, but they differ from this paper in that they abstract from sovereign default.

Aiyagari, Marcet, Sargent, and Seppala [6] initiated a literature on optimal taxation and public debt dynamics with aggregate uncertainty and incomplete markets studying a representative agent environment without default. Bhandari, Evans, Golosov, and Sargent [12] study a model with heterogeneous agents in which fluctuations in transfers are socially costly because of redistributive effects, but also without default. Presno and Pouzo [41] added default and renegotiation, but in a representative agent setup. Corbae, D’Erasmus and Kuruscu [15] examined a heterogeneous-agents model. Their setup is similar to ours in that a dynamic feedback mechanism connects wealth dynamics and optimal policies but abstracting from debt and default.

The recent literature on external default models includes several papers that make theoretical and quantitative contributions to the classic EG model of external default, following the early studies by Aguiar and Gopinath [5] and Arellano [9].⁷ This literature has examined models with tax and expenditure policies, settings with foreign and domestic lenders, models with external debt denominated in domestic currency, and models of international coordination (e.g., Cuadra, Sanchez and Saprizza [16]), Dias, Richmond and Wright [18], Sosa Padilla [43] and Du and Schreger [21]). The key difference relative to our setup is that these studies assume a representative agent, and for the most part, they do not focus on default on domestic debt-holders.

⁷Panizza, Sturzenegger, and Zettelmeyer [39]; Aguiar and Amador [2]; and Aguiar, Chatterjee, Cole, and Stangebye [4] survey the literature in detail.

Other studies in the external default literature are also related to our work, in that they focus on the effects of default on domestic agents, optimal taxation, the role of secondary markets, discriminatory versus nondiscriminatory default and bailouts (e.g., Guembel and Sussman [26]; Broner, Martin and Ventura [13]; Gennaioli, Martin and Rossi [25]; Aguiar and Amador [1]; Mengus [37]; and Di Casola and Sichlimiris [19]).⁸ As in some of these studies, default in our setup is non discriminatory, but in general, these studies abstract from distributional default incentives and social benefits of debt for self-insurance, liquidity and risk-sharing.

There is also more recent literature on the intersection of heterogeneous-agents and external default models, which is more closely related to this paper. In particular, Dovis, Golosov, and Shourideh [20] study distributional incentives to default on domestic and external debt in a model with heterogeneous agents. Our work is similar in that both models produce debt dynamics characterized by periods of sustained increases followed by large reductions. The two differ in that they assume complete domestic asset markets, which alters the nature of the social benefits of public debt, and they study equilibria in which the sustainable debt is risk free. In addition, we conduct a quantitative analysis exploring the model’s ability to explain the observed dynamics of Spain’s debt and default spreads. Aguiar, Amador, Farhi and Gopinath [3] study a setup in which the heterogeneity is across country members of a monetary union, instead of across agents inside a country. They show how lack of commitment and fiscal policy coordination leads countries to overborrow due to a fiscal externality. They focus on public debt traded across countries by risk-neutral investors, instead of default on risk-averse domestic debt holders. Andreasen, Sandleris, and Van der Gucht [8] and Jeon and Kabukcuoglu [29] study models in which domestic income heterogeneity plays a role in the determination of external defaults.

The rest of this paper is organized as follows: Section 2 describes the model and defines the recursive Markov equilibrium we study. Section 3 examines two variants of the model simplified to highlight distributional default incentives (in a one-period setup without uncertainty) and the social value of public debt (as the welfare cost of a surprise once-and-for-all default). Section 4 discusses the calibration procedure and examines the models quantitative implications. Section 5 provides conclusions. An Appendix provides details on the data, solution method and additional features of the quantitative results.

⁸There is also a renaissance of the literature on debt crises driven by multiple equilibria motivated by the European crisis (e.g., Aguiar, Chatterjee, Cole, and Stangebye [4] and Lorenzoni and Werning [35]). Most of this literature studies representative agent settings.

2 A Bewley Model of Domestic Sovereign Default

Consider an economy inhabited by a continuum of private agents with aggregate unit measure and a benevolent government. There is also a pool of risk-neutral international investors that face an opportunity cost of funds equal to an exogenous, world-determined real interest rate. Domestic agents face two types of non insurable shocks: idiosyncratic income fluctuations and aggregate shocks in the form of fluctuations in government expenditures and the possibility of sovereign default. Asset markets are incomplete because the only available vehicle of savings are one-period, non-state-contingent government bonds, which both domestic agents and international investors can buy. The government also levies proportional income taxes, pays lump-sum transfers, and chooses whether to repay its debt or not (i.e., it cannot commit to repay). The government cannot discriminate among borrowers when it defaults.

2.1 Private Agents

Agents have a standard constant relative risk aversion (CRRA) utility function:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \quad u(c_t) = c_t^{1-\sigma} / (1-\sigma), \quad (1)$$

where $\beta \in (0, 1)$ is the discount factor, c_t is individual consumption, and σ is the coefficient of relative risk aversion.

Each period, an agent's idiosyncratic income realization is drawn from a bounded, non-negative set: $y_t \in \mathcal{Y}$. These shocks have zero mean across agents so that aggregate income is nonstochastic. Idiosyncratic income evolves as a discrete Markov process with realization set given by $\{\underline{y}, \dots, \bar{y}\}$ and a transition probability matrix defined as $\pi(y_{t+1}, y_t)$ with stationary distribution $\pi^*(y)$.

Agents can buy government bonds in the amounts denoted by $b_{t+1} \in \mathcal{B} \equiv [0, \infty)$. They are not allowed to take short positions, and hence, they face the no-borrowing constraint $b' \geq 0$. The distribution of agents over debt and income at a point in time is defined as $\Gamma_t(b, y)$, and we refer to it as the “wealth distribution” for simplicity.

If the government repays its outstanding debt, an individual agent's budget constraint at date t is:

$$c_t + q_t b_{t+1} = y_t(1 - \tau^y) + b_t + \tau_t. \quad (2)$$

The right-hand side of this expression determines the after-tax resources the agent has available for consumption and savings. The agent collects income from the payout on its individual debt holdings (b_t), its idiosyncratic income realization (y_t) net of a proportional income

tax levied at rate τ^y , and lump-sum transfers (τ_t). This total disposable income pays for consumption and purchases of new government bonds b_{t+1} at the price q_t .

Before writing the individual budget constraint in the states in which the government defaults, we need to note two important assumptions about default costs. First, we relax the standard assumption of EG external default models according to which one cost of default is that the government is excluded from credit markets either forever or for a stochastic number of periods. In this model, the bond market always reopens the following period after a default. Second, although the model can sustain debt without exogenous default costs (because of the endogenous costs due to the social value of debt), to calibrate the model and explore its quantitative predictions, we introduce an exogenous income cost akin to those widely used in the sovereign default literature. This cost is typically modeled as a function of the realization of a stochastic endowment and designed so that default costs are higher at higher income levels. Since aggregate income is constant in our setup, we model the cost instead as a function of the realization of g . Aggregate income in the period of default falls by the amount $\phi(g)$, which is a decreasing function of g , so that the default cost is higher when income is higher.

If the government defaults, an individual agent's budget constraint is:

$$c_t = y_t(1 - \tau^y) - \phi(g) + \tau_t. \quad (3)$$

Three important effects of government default on households are implicit in this constraint: (a) Bond holdings of all agents are written off (which hurts more agents with large bond holdings); (b) the public debt market freezes, so that agents drawing high- (low-) income realizations cannot buy (sell) bonds for self-insurance and credit-constrained agents cannot benefit from the liquidity benefit of public debt; and (c) everyone's income falls by the amount $\phi(g)$.

2.2 Government

Each period, the government collects $\tau^y Y$ in income taxes, pays for g_t , and, if it repays existing debt, it chooses the amount of new bonds to sell B_{t+1} from the non-negative set $B_{t+1} \in \mathcal{B} \equiv [0, \infty)$. The income tax rate τ^y is exogenous, time- and state-invariant, and strictly positive. Government expenditures evolve according to a discrete Markov process with realizations defined over the set $\mathcal{G} \equiv \{\underline{g}, \dots, \bar{g}\}$ and associated transition probability

matrix $F(g_{t+1}, g_t)$.⁹ The processes for y and g are assumed to be independent for simplicity.¹⁰ Lump-sum transfers are determined endogenously as explained below, and their sign is not restricted, so $\tau_t < 0$ represents lump-sum taxes. Notice also that since both τ^y and Y are constant at the aggregate level, aggregate income tax revenue $\tau^y Y$ is constant (whereas *individual* income tax bills fluctuate with y). As discussed in Heathcote et al. [33], affine tax functions, like the one used in here, approximate actual tax and transfer programs pretty well.

The government has the option to default on the outstanding debt B_t at each date t . The default choice is denoted by the binary variable d_t (with $d_t = 1$ indicating default). As in Bhandari, et. al. [12] we assume that the government's preferences over stochastic process for consumption are ordered by

$$E_0 \sum_{t=0}^{\infty} \beta^t \int_{\mathcal{B} \times \mathcal{Y}} \sum_{y_i \leq y} u(c_t(b_t, y_t)) d\omega(b_t, y_t), \quad (4)$$

that is, the government is a benevolent planner who maximizes a standard utilitarian social welfare function, which aggregates the utility of individual agents identified by a pair (b, y) using a joint cumulative distribution function of welfare weights $\omega(b, y)$. We assume that $\omega(b, y)$ takes the following form:

$$\omega(b, y) = \sum_{y_i \leq y} \pi^*(y_i) \left(1 - e^{-\frac{b}{\bar{\omega}}}\right). \quad (5)$$

In the y dimension, the distribution $\omega(b, y)$ matches the long-run distribution of individual income $\pi^*(y)$. In the b dimension, the distribution is given by an exponential function with scale parameter $\bar{\omega}$, which we label “creditor bias” (with a higher $\bar{\omega}$ the government weights more the utility of agents who hold larger bond positions).

B_{t+1} and τ_t are determined after the default decision. Lump-sum transfers are set as needed to satisfy the government budget constraint. If the government repays, once the debt is chosen, the government budget constraint implies:

$$\tau_t^{d=0} = \tau^y Y - g_t - B_t + q_t B_{t+1}. \quad (6)$$

If the government defaults, the current repayment is not made and new bonds cannot be

⁹Note that in principle nothing rules out that consumption of some agents could be nonpositive in default states (i.e., $c_t = y_t(1 - \tau^y) - \phi(g) - g_t + \tau^y Y < 0$), but this does not happen in our baseline calibration. Otherwise we would need an additional restriction on the y and g processes: $g + \tau^y Y < (1 - \tau^y)\underline{y} - \phi(g)$, which implies that consumption is positive for the lowest value of individual income \underline{y} and all values of g .

¹⁰The independence assumed here is between *individual* income and *aggregate* government expenditures.

issued. Thus, default entails a one-period freeze of the public debt market. The government budget constraint implies then:

$$\tau_t^{d=1} = \tau^y Y - g_t. \quad (7)$$

The above treatment of transfers is analogous to that of the EG models of external default. In EG models, the resources the government generates by borrowing (plus the primary surplus if any) are transferred to a representative agent, whereas here the resources are transferred to a continuum of heterogeneous agents. In the calibration, these transfers will approximate a data average on welfare and entitlement payments to individuals net of capital tax revenues, which are not modeled.

2.3 International Investors

International investors are risk-neutral agents with “deep pockets” with an opportunity cost of funds equal to the world real interest rate \bar{r} . Their holdings of domestic government debt are denoted \hat{B}_{t+1} , which is also the economy’s net foreign asset position.

The investors’ expected profits from bond purchases are $\Omega_t = -q_t \hat{B}_{t+1} + \frac{(1-p_t)}{(1+\bar{r})} \hat{B}_{t+1}$. In this expression, p_t is the probability of default at $t+1$ perceived as of date t , $-q_t \hat{B}_{t+1}$ represents the value of bond purchases in real terms (i.e., the real resources lent out to the government at date t), and $\frac{(1-p_t)}{(1+\bar{r})} \hat{B}_{t+1}$ is the expected present value of the payout on government debt at $t+1$, which occurs with probability $(1-p_t)$. Arbitrage implies that $\Omega_t = 0$, which yields the standard arbitrage condition:

$$q_t = \frac{(1-p_t)}{(1+\bar{r})}. \quad (8)$$

2.4 Timing of transactions

The timing of decisions and market participation in the model is as follows:

1. Exogenous shocks y and g are realized.
2. Individual states $\{b, y\}$, wealth distribution $\Gamma_t(b, y)$, and aggregate states $\{B, g\}$ are known.
3. Agents pay income taxes. The government makes its debt and default decisions:
 - If it chooses to repay, $d_t = 0$, B_t is paid, the market of government bonds opens, new debt B_{t+1} is issued, lump-sum transfers are set according to equation (6), private agents choose b_{t+1} , and q_t is determined.

- If the government defaults, $d_t = 1$, B_t and all domestic and foreign holdings of government bonds are written off, the debt market closes, and lump-sum transfers are set according to equation (7).

4. Agents consume, and date t ends.

2.5 Recursive Markov Equilibrium

We study a Recursive Markov Equilibrium (RME) in which the government chooses debt and default optimally from a set of Conditional Recursive Markov Equilibria (CRME) that represent optimal allocations and prices for given debt and default choices. To characterize both RME and CRME, we first rewrite the optimization problem of domestic agents and the arbitrage condition of foreign investors in recursive form.

The aggregate state variables are B and g .¹¹ The optimal debt issuance and default decision rules are characterized by the recursive functions $B'(B, g)$ and $d(B, g) \in \{0, 1\}$ respectively.¹² The probability of default at $t + 1$ evaluated as of t , denoted $p(B', g)$, can then be defined as follows:

$$p(B', g) = \sum_{g'} d(B', g') F(g', g). \quad (9)$$

For any B' , the default probability is formed by adding up the transitional probabilities from g to g' for which, at the corresponding values of g' and B' , the government would choose to default. Hence, the default probability is the cumulative probability of $F(g', g)$ across the realizations of g' for which $d(B', g') = 1$.

The state variables for an individual agent's optimization problem are the agent's bond holdings and income (b, y) and the aggregate states (B, g) . Agents take as given $d(B', g)$, $B'(B, g)$, $\tau^{d=0}(B, g)$, and $\tau^{d=1}(g)$, a recursive bond pricing function $q(B', g)$, and the Markov processes of y and g . This set of recursive functions allows agents to project the evolution of aggregate states and bond prices, so that an agent's continuation value if the government *has chosen* to repay ($d(B, g) = 0$) and issued $B'(B, g)$ bonds can be represented as the solution to the following problem:

$$V^{d=0}(b, y, B, g) = \max_{\{c \geq 0, b' \geq 0\}} \{u(c) + \beta E_{(y', g')|(y, g)}[V(b', y', B', g')]\} \quad (10)$$

¹¹ $\Gamma_t(b, y)$ does not enter as a state variable, despite the presence of aggregate risk, because the wealth distribution does not affect bond prices directly, which in turn is the case because q_t satisfies the foreign investors' risk neutral arbitrage condition, and the weights of the social welfare function are set by $\omega(b, y)$.

¹²In the recursive notation, variables x_t and x_{t+1} are denoted as x and x' respectively.

s.t.

$$c + q(B'(B, g), g)b' = b + y(1 - \tau^y) + \tau^{d=0}(B, g), \quad (11)$$

where $V(b', y', B', g')$ (without superscript) is the next period's continuation value for the agent *before* the default decision has been made that period.

Similarly, the continuation value if the government *has chosen* to default is:

$$V^{d=1}(y, g) = u(y(1 - \tau^y) - \phi(g) + \tau^{d=1}(g)) + \beta E_{(y', g')|(y, g)}[V^{d=0}(0, y', 0, g')]. \quad (12)$$

Finally, the continuation value at date t and evaluated *before* the default decision has been made is given by:

$$V(b, y, B, g) = (1 - d(B, g))V^{d=0}(b, y, B, g) + d(B, g)V^{d=1}(y, g). \quad (13)$$

The solution to this problem yields the individual decision rule $b' = h(b, y, B, g)$ and the associated value functions $V(b, y, B, g)$, $V^{d=0}(b, y, B, g)$ and $V^{d=1}(y, g)$. By combining the agents' bond decision rule, the exogenous Markov transition matrices of y and g , and the government's default decision, we can obtain expressions that characterize the evolution of the wealth distribution in the repayment and default states. The wealth distribution at the beginning of $t + 1$ is denoted $\Gamma' = H^{d' \in \{0,1\}}(\Gamma, B, g, g')$. If $d(B', g') = 0$, for $\mathcal{B}_0 \subset \mathcal{B}$, $\mathcal{Y}_0 \subset \mathcal{Y}$, Γ' is:

$$\Gamma'(\mathcal{B}_0, \mathcal{Y}_0) = \int_{\mathcal{Y}_0, \mathcal{B}_0} \left\{ \int_{\mathcal{Y}, \mathcal{B}} I_{\{b' = h(b, y, B, g)\} \in \mathcal{B}_0} \pi(y', y) d\Gamma(b, y) \right\} db' dy', \quad (14)$$

where $I_{\{\cdot\}}$ is an indicator function that equals 1 if $b' = h(b, y, B, g)$ and zero otherwise. Note that g' is an argument of $H^{d' \in \{0,1\}}$ because Γ' is formed after d' is known, and d' depends on g' . If $d(B', g') = 1$, for $\mathcal{Y}_0 \subset \mathcal{Y}$, Γ' is given by:

$$\Gamma'(\{0\}, \mathcal{Y}_0) = \int_{\mathcal{Y}_0} \left\{ \int_{\mathcal{Y}, \mathcal{B}} \pi(y', y) d\Gamma(b, y) \right\} db' dy', \quad (15)$$

and zero otherwise. This is because at default all households' bond positions are set to zero, and hence, Γ' is determined only by the evolution of the income process (i.e., if the government defaults, $\Gamma'(b, y) = \pi^*(y)$ for $b = 0$ and zero for any other value of b).

The foreign investors' arbitrage condition in recursive form is:

$$q(B', g) = \frac{(1 - p(B', g))}{(1 + \bar{r})}. \quad (16)$$

This arbitrage condition is functionally identical to the one typical of EG models of ex-

ternal default: Risk-neutral arbitrage against the opportunity cost of funds requires a wedge between the price at which foreign investors are willing to buy government debt ($q(\cdot)$) and the price of international bonds ($1/(1 + \bar{r})$) that compensates them for the risk of default measured by the default probability. At equilibrium, bond prices and risk premia are formed by a combination of exogenous factors (the Markov process of g) and the endogenous government decision rules $B'(B, g)$ and $d(B, g)$. Note, however, that the arbitrage condition in this model embodies a very different mechanism determining default probabilities from that driving EG models. In EG models, these probabilities follow from the values of continuation versus default of a representative agent, while here they are determined by comparing those values for the social welfare function. In turn, these social valuations depend on the dispersion of individual payoffs of default versus repayment (and on the welfare weights). Hence, inequality affects default probabilities via changes in the dispersion of individual payoffs of default versus repayment. Later in this section, we characterize further some features of these payoffs and in Section 4, we examine their properties quantitatively.

We now define the CRME for given debt and default decision rules. The definition includes the following three aggregate variables. First, aggregate consumption is given by:

$$C = \int_{\mathcal{Y} \times \mathcal{B}} c(b, y, B, g) d\Gamma(b, y), \quad (17)$$

where $c(b, y, B, g)$ corresponds to individual consumption by each agent identified by a (b, y) pair when the aggregate states are (B, g) . Second, aggregate (nonstochastic) income is:

$$Y = \int_{\mathcal{Y} \times \mathcal{B}} y d\Gamma(b, y). \quad (18)$$

Third, aggregate domestic demand for newly issued bonds is:

$$B^{d'} = \int_{\mathcal{Y} \times \mathcal{B}} h(b, y, B, g) d\Gamma(b, y). \quad (19)$$

The ratio of domestic debt to total public debt is defined as $\min\{B^{d'}/B', 1\}$.

Definition: Given an initial wealth distribution $\Gamma_0(b, y)$, a default decision rule $d(B, g)$, a government debt decision rule $B'(B, g)$, an income tax rate τ^y , and lump-sum transfers $\tau^{d \in \{0,1\}}$ defined by (6) and (7), a **CRME** is defined by a value function $V(b, y, B, g)$ with associated household decision rule $b' = h(b, y, B, g)$, a transition function for the wealth distribution $H^{d' \in \{0,1\}}(B, g, g')$, a default probability function $p(B', g)$, and a bond pricing function $q(B', g)$ such that:

1. Given the bond pricing function and government policies, $V(b, y, B, g)$ and $h(b, y, B, g)$

solve the individual agents' optimization problem.

2. The foreign investors' arbitrage condition (equation (16)) holds.
3. The transition function of the wealth distribution satisfies conditions (14) and (15) in states with repayment and default, respectively.
4. The government budget constraints (6) and (7) hold.
5. The market of government bonds clears:¹³

$$\hat{B}' + B^{d'} = B'. \quad (20)$$

6. The aggregate resource constraint of the economy is satisfied. If the government repays:

$$C + g = Y + \hat{B} - q(B', g)\hat{B}', \quad (21)$$

and if the government defaults:

$$C + g = Y - \phi(g). \quad (22)$$

We now formulate the model's RME as a CRME in which $B'(B, g)$ and $d(B, g)$ are optimal government choices. If $B > 0$ at the beginning of period t , the government sets its optimal $d(B, g)$ as the solution to the following problem:

$$\max_{d \in \{0,1\}} \{W^{d=0}(B, g), W^{d=1}(g)\}, \quad (23)$$

where the social value of continuation is:

$$W^{d=0}(B, g) = \int_{\mathcal{Y} \times \mathcal{B}} V^{d=0}(b, y, B, g) d\omega(b, y),$$

and the social value of default is:

$$W^{d=1}(g) = \int_{\mathcal{Y} \times \mathcal{B}} V^{d=1}(y, g) d\omega(b, y).$$

$W^{d=0}(B, g)$ and $W^{d=1}(g)$ are social welfare functions with weights given by $\omega(b, y)$.

If the government chooses to repay, it also chooses an optimal amount of new debt to issue. To characterize this choice, assume that the government first considers an intermediate

¹³When $\hat{B}' \geq 0$ the country is a net external borrower because the bonds issued by the government are less than the domestic demand for them, and when $\hat{B}' < 0$, the country is a net external saver.

step in which it evaluates how any arbitrary debt level (denoted \tilde{B}') affects each agent. The corresponding value for each agent is the solution to the following problem:

$$\begin{aligned} \tilde{V}(b, y, B, g, \tilde{B}') &= \max_{\{c \geq 0, b' \geq 0\}} u(c) + \beta E_{(y', g')|(y, g)}[V(b', y', \tilde{B}', g')] \\ \text{s.t. } &\begin{cases} c + q(\tilde{B}', g)b' = y(1 - \tau^y) + b + \tau \\ \tau = \tau^y Y - g - B + q(\tilde{B}', g)\tilde{B}'. \end{cases} \end{aligned} \quad (24)$$

Note that $V(\cdot)$ in the right-hand side of this problem is given by the solution to the household problem (10), which implies that the government is assessing the value of deviating from the optimal policy only in the current period.

The optimal debt issuance decision rule can then be characterized as the solution to this problem:

$$\max_{\tilde{B}'} \int_{\mathcal{Y} \times \mathcal{B}} \tilde{V}(b, y, B, g, \tilde{B}') d\omega(b, y). \quad (25)$$

Now we can define the model's RME:

Definition: A **RME** is a CRME in which the default decision rule $d(B, g)$ solves problem (23) and the debt decision rule $B'(B, g)$ solves problem (25).

2.6 Feedback Mechanism

Here we discuss some important key features of the model's optimality conditions that together form the feedback mechanism linking default incentives, default risk, the wealth distribution, and the dispersion of individual gains from a government default. This material will also be used for the analysis of the quantitative results of Section 4.

(a) *Default risk and demand for government bonds.*

Assuming the agents' value functions are differentiable, the first-order condition for b' in a state in which the government has repaid (i.e., in the optimization problem that defines $V^{d=0}(b, y, B, g)$) is:

$$-u'(c)q(B', g) + \beta E_{(y', g')|(y, g)}[V_1(b', y', B', g')] \leq 0, = 0 \text{ if } b' > 0, \quad (26)$$

where $V_1(\cdot)$ denotes the derivative of the value function with respect to its first argument. Using the envelope theorem, this condition can be rewritten as:

$$u'(c) \leq \beta E_{(y', g')|(y, g)} \left[(1 - d(B', g')) \frac{u'(c')}{q(B', g)} \right], \quad (27)$$

which holds with equality if $b' > 0$. The right-hand side of this expression shows that,

in assessing the marginal benefit of buying an extra unit of b' , agents take into account the possibility of a future default. In states in which a default is expected, $d(B', g') = 1$ and agents assign zero marginal benefit to buying bonds.¹⁴ In states in which repayment is expected, the marginal benefit of buying bonds is $\frac{u'(c')}{q(B', g')}$, which includes the default risk premium embedded in the price paid for newly issued bonds.

These results imply that, conditional on B' , a larger default set (i.e., a larger set of values of g' for which the government defaults) reduces the expected marginal benefit of an extra unit of savings. In turn, this implies that, everything else equal, a higher default probability reduces individual domestic demand for government bonds unless an agent has high enough (b, y) to be willing to take the risk of demanding more bonds at higher risk premia (lower bond prices) and expect future adjustments in τ . This has important distributional implications because, as we explain below, the government internalizes when making the default decision how it affects the probability of default and bond prices. Notice also that future default risk at any date later than t , not just $t + 1$ influences the agents' demand for b_{t+1} because of the time-recursive structure of the above Euler equation (27). Hence, even if debt is offered at the risk-free price at t , bond demand still responds negatively to default risk if default has positive probability beyond $t + 1$ (i.e., agents factor in the risk of a future default wiping out their wealth as they build their individual stock of savings).

(b) Public debt, self-insurance, liquidity, and risk-sharing

The role of public debt as a vehicle for self-insurance, liquidity, and risk-sharing can be illustrated by combining the agents' budget constraint with the government budget constraint and adopting the variable transformation $\tilde{b} = (b - B)$ to obtain:

$$c = y + \tilde{b} - q(B', g)\tilde{b}' - \tau^y(y - Y) - g \quad (28)$$

$$\tilde{b}' \geq -B' \quad (29)$$

These expressions make it evident that public debt issuance (B') relaxes the borrowing constraint for agents who are hitting it. That is, it provides them with liquidity in the form of extra resources for consumption.

There are two additional key effects of debt that also result from the incompleteness of financial markets. First, debt issuance provides a valuable asset used for self-insurance. Agents with sufficiently high income, regardless of their existing holdings of b , would want to buy more debt, and agents with sufficiently low income would want to use their accumulated precautionary savings. Second, debt redistributes resources across agents, enabling

¹⁴The model can be extended to allow for partial defaults (e.g., reductions in the real value of the debt via inflation). With a partial default, bond positions would be reduced uniformly across agents by the fraction of the debt that represents the partial default, and as a result the marginal benefit of buying bonds in the default state would be positive, instead of zero.

the government to improve risk-sharing. In each period, repayment of B results in regressive redistribution in favor of the relatively wealthy in the beginning-of-period wealth distribution (i.e., agents with $\tilde{b} > 0$, or “above average” holdings relative to B). In contrast, new debt B' causes progressive redistribution in favor of the relatively poor in the end-of-period wealth distribution (i.e., agents with $\tilde{b}' < 0$, or below average holdings relative to B'). The magnitude and cross-sectional dispersion of these effects changes over time as the endogenous wealth distribution evolves.

These two forms of redistribution are connected intertemporally. Assuming repayment, more progressive redistribution at t implies more regressive redistribution in the future. Because of the government’s inability to commit to repay, however, the extent to which progressive redistribution can be implemented at t is inversely related to the expectation that in the future the planner will be tempted to avoid regressive redistribution by defaulting. This is because the price at which new debt is sold at t depends negatively on the probability of a default at $t + 1$. This reduces the government’s ability to produce progressive redistribution, because $q(B', g)$ falls as B' rises, since the default probability is nondecreasing in B' . Hence, the resources generated by debt, $q(B', g)B'$, follow a Laffer curve similar to the familiar one from EG models of external default. In EG models, there is a debt Laffer curve also because bond prices fall and default probabilities rise as debt rises, but the resources generated by debt are transferred to a representative agent. In contrast, in this model the resources generated by debt are transferred to heterogeneous agents, and although τ is uniform across agents, the heterogeneity in bond holdings effectively makes the transfers generated by debt vary across agents (inversely with the value of \tilde{b}').

The role of income taxation as an alternative means to improve risk-sharing of idiosyncratic income shocks is also evident in condition (28): The term $-\tau^y(y - Y)$ implies that agents with below (above) average income effectively receive (pay) a subsidy (tax). If income is taxed 100 percent, full social insurance against these shocks is provided, and all agents after-tax income equals Y . But this still would not remove the need for precautionary savings, because aggregate shocks to government expenditures as well as government defaults cannot be insured away. In the absence of aggregate shocks, however, the 100 percent income tax would provide full insurance.¹⁵

(c) Feedback mechanism

The dynamic feedback mechanism driving the model’s dynamics follows from the features of the model highlighted in (a) and (b). In particular, it is critical to note that the extent that the probability of default and the price of debt at t depend on the dispersion of payoffs of default versus repayment across agents at $t + 1$, because the government’s social welfare

¹⁵There is also no tax-smoothing role for debt because the income tax is nondistortionary since individual income is exogenous and aggregate income is constant. Hence, income tax revenue is constant over time.

function aggregates these payoffs to make the default decision. This is a feedback mechanism because the debt issued at t becomes the initial debt outstanding at $t+1$ and this matters for the dispersion of the agents' payoffs, affecting agents with different (b, y) differently, as we illustrate quantitatively in Section 4. Thus, the debt issued at t affects the default decision at $t+1$, which affects default probabilities and bond prices at t , which in turn affects the agents' date- t demand for bonds and the government's debt choice. The links of this chain are connected via the distributional effects of debt issuance and the dispersion of payoffs of default versus repayment across agents.

The feedback mechanism cannot be fully characterized analytically in closed form, but we can gain further intuition about it as follows. Define $\Delta c \equiv c^{d=0} - c^{d=1}$ as the difference in consumption across repayment and default in a given period for an agent who has a particular \tilde{b} when the aggregate states are (B, g) . Δc can be expressed as:

$$\Delta c = \tilde{b} - q(B', g)\tilde{b}' + \phi(g) \quad (30)$$

The right-hand side of this expression includes the distributional effects noted in (b). If inequality in the initial wealth distribution is high, so that a larger fraction of agents have $\tilde{b} < 0$, and strong default incentives make default risk high, so that $q(B', g)$ is low, a larger fraction of agents have $\Delta c < 0$ and are more likely to be better off with a default, which in turn justifies the distributional incentives to default. The opposite is true if initial inequality and default risk are low. Moreover, given initial inequality and bond prices, higher inequality in the end-of-period wealth distribution (i.e., a larger fraction of agents with $\tilde{b}' < 0$) reduces the fraction of agents with $\Delta c < 0$. Hence, changes in wealth inequality, default incentives, and default risk interact in determining the dispersion of $\Delta c < 0$ across agents. The interaction does not follow a monotonic pattern, however, because Δc can be negative also for agents with sufficiently high (b, y) who buy more risky debt attracted by the higher risk premia. Thus, as we look across agents with different wealth, $\frac{db'}{dB}$ changes sign and, for some wealthy individuals, it can even be the case that Δc decreases with B .

It is also important to note that Δc alone does not determine individual payoffs of default or repayment. These depend on both date- t differences in consumption (or utility) and differences in the continuation values $V^{d=0}(b', y', B', g')$ and $V^{d=0}(0, y', 0, g')$. Still, the interaction between the wealth distribution, consumption differentials across default and repayment states, and default risk discussed previously is illustrative of the feedback mechanism driving the model. Moreover, we can also establish that, since $V^{d=0}$ is increasing in b as in standard heterogeneous-agents models, there is a threshold value of bond holdings $\hat{b}(y, B, g)$, for given (y, B, g) , such that agents with $b \geq \hat{b}$ prefer repayment (since $V^{d=0}(b, y, B, g) \geq V^{d=1}(y, g)$),

and those with $b < \hat{b}(y, B, g)$ prefer default. That is,

$$\hat{b}(y, B, g) = \{b \in \mathcal{B} : V^{d=0}(b, y, B, g) = V^{d=1}(y, g)\}. \quad (31)$$

We can conjecture that $\hat{b}(y, B, g)$ is increasing in B because the difference in τ under repayment versus default widens at higher levels of public debt: Higher debt reduces transfer payments both because of the higher repayment on B even without default risk and because higher risk premia reduces the price at which B_{t+1} is sold, causing a debt-overhang effect (i.e., additional borrowing is used to service debt). As a result, agents need to have higher individual wealth in order to prefer repayment as B rises. This conjecture stating that $\hat{b}(y, B, g)$ is increasing in B was verified numerically (see Figure 14 in the Appendix).

3 Distributional Incentives & Social Value of Debt

This section examines two simplified variants of the model. First, a one-period variant with a predetermined wealth distribution, designed to isolate the distributional default incentives and highlight the roles of consumption dispersion, the distribution of bond holdings and the welfare weights in the default choice. By construction, this setup abstracts from the social benefits of debt for self-insurance, liquidity, and risk-sharing. The second variant is a version of the model without default risk, designed to isolate these social benefits by conducting a quantitative analysis of the welfare cost of a once-and-for-all default. There is no default risk because the government is committed to repay after the once-and-for-all default, and the default itself is unanticipated and exogenous. The quantitative analysis of the full model presented in the next section combines the elements isolated in these two exercises.

3.1 Distributional default incentives

Consider a one-period variant of the model without uncertainty and a predetermined distribution of debt ownership. There are two types of agents: A fraction γ is L -type agents with low bond holdings denoted b^L , and the complement $(1 - \gamma)$ is H -type agents with high bond holdings b^H . The government has an exogenous stock of debt B , which is deciding whether to repay or not, and default may entail an exogenous cost that reduces income by a fraction $\phi \geq 0$.¹⁶ The budget constraints of the government and households under repayment are $\tau^{d=0} = B - g$ and $c^i = y + \tau^{d=0} + b^i$ (for $i = L, H$), respectively, and under default are $\tau^{d=1} = -g$ and $c^i = (1 - \phi)y + \tau^{d=1}$ (for $i = L, H$) respectively. The utility function can be

¹⁶We include this cost because, as we show here, distributional incentives alone cannot sustain debt in this simple model unless the social welfare function weights L types by less than γ . This cost can proxy for the endogenous default costs driven by the social value of debt in the full model.

as in Section 2, but what is necessary for the results derived here is that it be increasing and strictly concave.

In this one-period setup, the agents choices of b^L and b^H (or equivalently their consumption allocations) are predetermined. For a given exogenous “decentralized” distribution of debt holdings characterized by a parameter ϵ , the bond holdings of L-type agents are $b^L = B - \epsilon$. Market clearing in the bond market then requires $b^H = B + \frac{\gamma}{1-\gamma}\epsilon$. Since we are still assuming agents cannot borrow, it must be that $\epsilon \leq B$, and since by definition $b^H \geq b^L$, it must be that $\epsilon \geq 0$. Using the budget constraints, the decentralized consumption allocations under repayment are $c^L(\epsilon) = y - g - \epsilon$ and $c^H(\gamma, \epsilon) = y - g + \frac{\gamma}{1-\gamma}\epsilon$, and under default, they are $c^L = c^H = y(1 - \phi) - g$. Notice that under repayment, ϵ determines also the dispersion of consumption across agents, which increases with ϵ , and under default there is zero consumption dispersion.

The main question to understand distributional incentives to default is: How does an arbitrary distribution of bond holdings (i.e., dispersion of consumption) differ from the one that is optimal for a government with the option to default? To answer this question, we solve the optimization problem of the social planner with the default option. The planner’s welfare weight on L -type agents is ω . The optimal default decision solves:

$$\max_{d \in \{0,1\}} \{W_1^{d=0}(\epsilon), W_1^{d=1}(\phi)\}, \quad (32)$$

where social welfare under repayment is:

$$W^{d=0}(\epsilon) = \omega u(y - g + \epsilon) + (1 - \omega)u\left(y - g + \frac{\gamma}{1 - \gamma}\epsilon\right) \quad (33)$$

and under default is:

$$W_1^{d=1}(\phi) = u(y(1 - \phi) - g). \quad (34)$$

We characterize the solution to the above problem as a choice of the socially optimal consumption dispersion ϵ^{SP} , which is the value of ϵ that maximizes $W^{d=0}(\epsilon)$. Since default is the only instrument available to the government to improve consumption dispersion relative to what decentralized allocations for some ϵ support, the planner repays only if doing so allows it to either attain ϵ^{SP} or get closer to it than by defaulting.

The optimality condition for the choice of ϵ^{SP} reduces to:

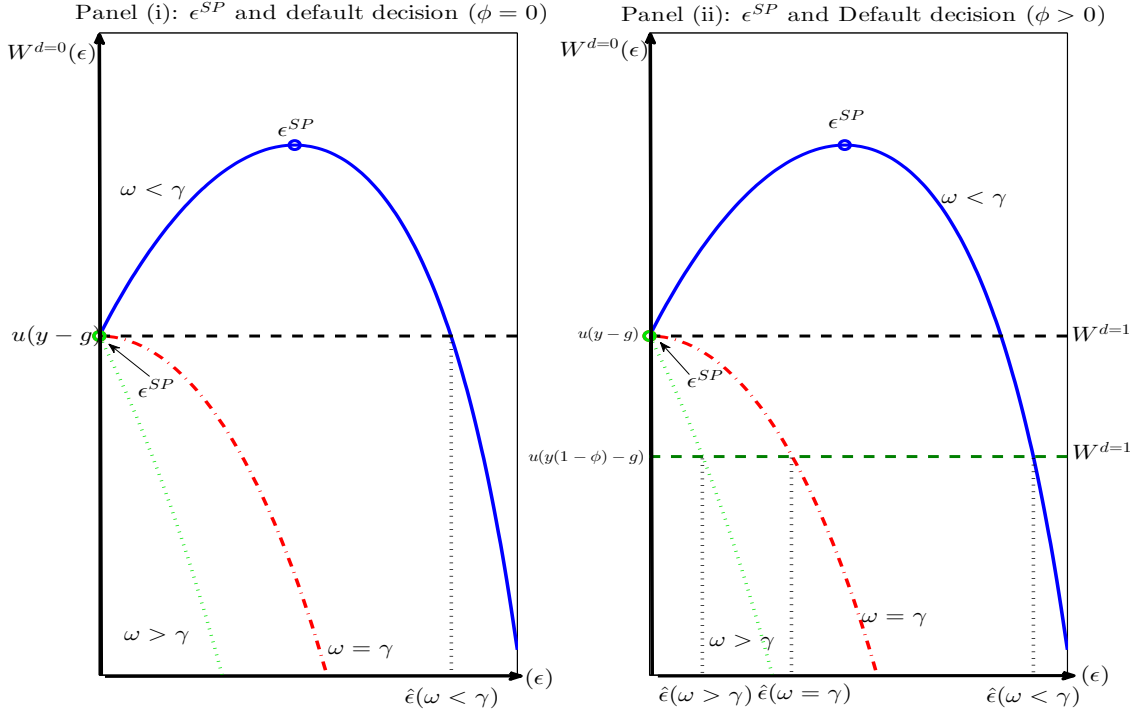
$$\frac{u'(c^H)}{u'(c^L)} = \frac{u'\left(y - g + \frac{\gamma}{1-\gamma}\epsilon^{SP}\right)}{u'(y - g - \epsilon^{SP})} = \left(\frac{\omega}{\gamma}\right) \left(\frac{1 - \gamma}{1 - \omega}\right). \quad (35)$$

This condition implies that the socially optimal ratio of c^L to c^H increases as ω/γ rises (i.e.,

as the ratio of the planner's weight on L types to the actual existing mass of L types rises). If $\omega/\gamma = 1$, the planner desires zero consumption dispersion; for $\omega/\gamma > 1$, the planner likes consumption dispersion to favor L types; and the opposite holds for $\omega/\gamma < 1$. As we show, if $\phi = 0$, debt cannot be sustained for $\omega/\gamma \geq 1$ because default is optimal. This is the case because, for any $\epsilon > 0$, the consumption allocations feature $c^H > c^L$ while the socially efficient consumption dispersion requires $c^H \leq c^L$. Hence, there is no way to implement ϵ^{SP} (since the only instrument is the default choice), and default is therefore a second-best policy that brings the planner the closest it can get to ϵ^{SP} .

The choice of ϵ^{SP} and the default decision in the absence of default costs (i.e., $\phi = 0$) are illustrated in Panel (i) of Figure 2. This figure plots the functions $W^{d=0}(\epsilon)$ for $\omega \gtrless \gamma$. The value of social welfare at default and the values of ϵ^{SP} for $\omega \gtrless \gamma$ are also identified in the plot. Notice that the vertical intercept of $W^{d=0}(\epsilon)$ is always $W^{d=1}$ for any values of ω and γ because, when $\epsilon = 0$, there is zero consumption dispersion and that is also the outcome under default. In addition, the bell-shaped form of $W^{d=0}(\epsilon)$ follows from $u'(\cdot) > 0, u''(\cdot) < 0$.¹⁷

Figure 2: Default Decision with and without Default Costs



¹⁷Note in particular that $\frac{\partial W^{d=0}(\epsilon)}{\partial \epsilon} \gtrless 0 \iff \frac{u'(c^H(\epsilon))}{u'(c^L(\epsilon))} \gtrless \left(\frac{\omega}{\gamma}\right)\left(\frac{1-\gamma}{1-\omega}\right)$. Hence, social welfare is increasing (decreasing) at values of ϵ that support sufficiently low (high) consumption dispersion so that $\frac{u'(c^H(\epsilon))}{u'(c^L(\epsilon))}$ is above (below) $\left(\frac{\omega}{\gamma}\right)\left(\frac{1-\gamma}{1-\omega}\right)$.

Assume first that $\omega > \gamma$. In this case, ϵ^{SP} would be negative because condition (35) implies that the planner's optimal choice features $c^L > c^H$. However, these consumption allocations are not feasible (since they imply $\epsilon < 0$), and by choosing default the government attains $W^{d=1}$, which is the highest feasible social welfare for $\epsilon \geq 0$. Assuming instead $\omega = \gamma$, it follows that $\epsilon^{SP} = 0$ and default attains exactly the same level of welfare, so default is chosen and it also delivers the efficient level of consumption dispersion. In short, if $\omega \geq \gamma$, the government always defaults for any $\epsilon > 0$, and thus equilibria with debt cannot be supported.

Equilibria with debt can be supported when $\omega < \gamma$. In this case, the intersection of the downward-sloping segment of $W^{d=0}(\epsilon)$ with $W^{d=1}$ determines a threshold value $\hat{\epsilon}$ such that default is optimal only for $\epsilon \geq \hat{\epsilon}$. Default is still a second-best policy because with it the planner cannot attain $W^{d=0}(\epsilon^{SP})$, it just gets the closest it can get. As Figure 2 shows, for $\epsilon < \hat{\epsilon}$, repayment is preferable because $W^{d=0}(\epsilon) > W^{d=1}$. Thus, in this simple setup, when default is costless, equilibria with repayment require two conditions: (a) that the government weights H types by more than their share of the government bond holdings *and* (b) that the debt holdings of private agents do not produce consumption dispersion in excess of $\hat{\epsilon}$.

Now we introduce the exogenous cost of default. The solutions are shown in Panel (ii) of Figure 2. The key difference is that now it is possible to support repayment equilibria even when $\omega \geq \gamma$. Now there is a threshold value of consumption dispersion, $\hat{\epsilon}$, separating repayment from default decisions for all values of ω and γ . The government chooses to repay whenever ϵ exceeds $\hat{\epsilon}$ for the corresponding values of ω and γ . It is also evident that the range of values of ϵ for which repayment is chosen widens as γ rises relative to ω . Thus, when default is costly, equilibria with repayment require only that the debt holdings of private agents implicit in ϵ do not produce consumption dispersion in excess of the value of $\hat{\epsilon}$ associated with given values of ω and γ . Intuitively, the consumption of H type agents must not exceed that of L type agents by more than what $\hat{\epsilon}$ allows. If it does, default is optimal.

D'Erasmus and Mendoza [17] extend this analysis to a two-period model with shocks to government expenditures, optimal bond demand choices by private agents, and optimal bond supply and default choices by the government. The results for the distributional default incentives derived above still apply. In addition, we show that the optimal debt and default choices of the government are characterized by a socially optimal deviation from the equalization of marginal utilities across agents, which calls for higher debt the higher the liquidity benefit of debt in the first period (i.e., the tighter the credit constraint on L -types) and the higher the marginal distributional benefit of a default in the second period. We also show that the model still sustains debt with default risk if we introduce a consumption tax as a second tool for redistribution, an alternative asset for savings, and foreign creditors.

3.2 Social Value of Debt

We now study the variant of the model that isolates the endogenous costs of default captured by the social value of debt. In particular, we compute the social cost of a once-and-for-all, unanticipated default, which captures the costs of wiping the buffer stock of savings of private agents, preventing debt issuance from providing liquidity to credit-constrained agents, and precluding private agents from purchasing government bonds for self-insurance. The goal is to show that default in the model of Section 2, in which the government is excluded from credit markets only in the period in which it defaults, can entail significant endogenous costs.

We compare social welfare across two economies. As in the full model, in both economies, there is a continuum of heterogeneous agents facing idiosyncratic (income) and aggregate (government expenditure) shocks. In the first economy, the government is fully committed to repay, while in the second there is an exogenous once-and-for-all, unanticipated default in the first period (i.e., a “surprise” default). After that, the government is committed to repay. We perform the experiment across different initial levels of government debt. Since there is no default risk, bond prices are always equal to $1/(1+\bar{r})$ and the domestic aggregate demand for bonds is the same for the different values of B (what changes is the amount traded abroad).

This experiment is related to the one conducted by Aiyagari and McGrattan [7], but with some important differences. First, we are computing the social cost of a surprise default relative to an economy with full commitment, whereas they calculate the welfare cost of changing the debt ratio always under full commitment. Second, their model features production and capital accumulation with distortionary taxes, which we abstract from, but considers only idiosyncratic shocks, while we incorporate aggregate shocks. Third, in our setup, the equilibrium interest rate is always $1/(1+\bar{r})$, whereas they study a closed-economy model with an endogenous interest rate.

We quantify the social value of public debt as the welfare cost of a surprise default computed as follows: Define $\alpha(b, y, B, g)$ as the individual welfare effect of the surprise default. This corresponds to a compensating variation in consumption such that, at a given aggregate state (B, g) , an individual agent defined by a (b, y) pair is indifferent between living in the economy in which the government always repays and the one with the surprise default.¹⁸ Formally, $\alpha(b, y, B, g)$ is given by:

$$\alpha(b, y, B, g) = \left[\frac{V^{d=1}(y, g)}{V^c(b, y, B, g)} \right]^{\frac{1}{1-\sigma}} - 1,$$

¹⁸We measure welfare relative to this scenario, instead of permanent financial autarky, because it is in line with the one-period debt-market freeze when default occurs in our model. The costs relative to full financial autarky would be larger but less representative of the model’s endogenous default costs.

where $V^{d=1}(y, g)$ represents the value of the surprise default, and $V^c(b, y, B, g)$ is the value under full commitment. For a given (B, g) , there is a distribution of these individual welfare measures across all the agents defined by all (b, y) pairs in the state space. The social value of public debt is then computed by aggregating these individual welfare measures using the social welfare function defined in Section 2:

$$\bar{\alpha}(B, g) = \int \alpha(b, y, B, g) d\omega(b, y). \quad (36)$$

Table 1 shows results for four scenarios corresponding to surprise defaults with debt ratios ranging from 5 to 20 percent of GDP.¹⁹ For each scenario, the table shows GDP ratios of total public debt, B/GDP , domestic debt B^d/GDP , transfers τ (evaluated at average $g = \mu_g$ and the corresponding level of B), as well as $\bar{\alpha}(B, g)$ for different values of g (average μ_g , minimum, \underline{g} , and maximum, \bar{g}). We also report the fraction of agents with $\alpha(b, y, B, \mu_g) > 0$ (i.e., the fraction of agents benefiting from a default). All figures come from solutions of the household and government problems described in Section 2. Since computing B^d also requires in addition the wealth distribution $\Gamma(b, y)$, we report B^d for a “panel average,” calculated by first averaging over the cross-section of (b, y) pairs within each period, and then averaging across a long time series simulation.

Table 1: Social Value of Public Debt

B/GDP	B^d/GDP	$\tau(B, \mu_g)/GDP$	$\bar{\alpha}(B, \mu_g)\%$	$\bar{\alpha}(B, g)$	$\bar{\alpha}(B, \bar{g})$	hh's $\alpha(b, y, B, \mu_g) > 0$
5.0	4.25	25.96	-1.87	-4.66	-1.13	0.90
10.0	4.25	23.87	-0.90	-3.76	-0.12	29.1
15.0	4.25	20.83	0.04	-2.88	0.89	66.0
20.0	4.25	17.29	1.00	-1.99	1.90	83.9

Note: Values are reported in percentage. B^d/GDP corresponds to the average of 10,000-period simulations with the first 2,000 periods truncated. Positive values of $\bar{\alpha}(B, g)$ denote that social welfare is higher in the once-and-for-all default scenario than under full repayment commitment. “hh’s” denotes households.

The results show that the social value of debt (i.e., the welfare cost of a surprise default) is large and monotonically decreases as debt rises. For $g = \mu_g$, the results range from a social cost of -1.87 percent for defaulting on a 5 percent debt ratio to a gain of 1.00 for defaulting on a 20 percent debt ratio (i.e. the social value of debt ranges from 1.87 to -1.00 percent). Surprise defaults are very costly for debt ratios of 10 percent or less, while they yield welfare gains at debt ratios of 15 percent or higher. For the low value of g , default

¹⁹The parameter values used here are the same as those of the calibration described in the following section and listed in Table 2.

remains significantly costly even at a 20 percent debt ratio. Interestingly, at the high value of g the welfare costs are smaller and the gains larger than for average g , and they change from costs to gains at a debt ratio between 10 and 15 percent. These estimates of the social value of public debt are significantly larger than those obtained by Aiyagari and McGrattan [7]. The maximum social value of debt in their results is roughly 0.1 percent, while we obtain 1.87 percent (for $g = \mu_g$).

The smaller social value of debt (higher social value of default) at higher debt ratios follows from the fact that higher debt reduces transfers (τ decreases monotonically) and thus the extent to which the government can redistribute resources across domestic agents by repaying, while the benefits of debt for self-insurance, liquidity, and risk-sharing fall. Accordingly, the fraction of agents that favor a default on average increases monotonically with the debt ratio. At relatively low debt (below 10 percent of GDP) only up to 30 percent of the population favors a default. These are agents with relatively low wealth who benefit from a smaller cut in transfers after a government default. The larger cut in transfers due to higher debt service when debt increases beyond 10 percent of GDP induces even agents with sizable wealth to favor default. For instance, with a 20 percent debt ratio, the average fraction of agents in favor of default is roughly 83.9 percent.

In summary, this experiment shows that, in the absence of default risk, the social value of public debt under incomplete markets is significant but falls monotonically as debt rises. At sufficiently high debt, the debt service costs grow large enough to overtake the social benefits of public debt, making default socially beneficial.

4 Quantitative Analysis

In this section, we study the quantitative predictions of the model using a set of parameter values calibrated to an average Eurozone country. We focus on the Eurozone since we view the European crisis to be much closer to a crisis with domestic holders of sovereign debt in which European institutions internalize default tradeoffs across the entire region. Moreover, during this period, there was a significant increase in default risk for several countries in the region (including a sovereign default by one of its members, Greece). The moments we use for the calibration are computed as GDP-weighted averages of country specific moments.²⁰

The section begins with the model's calibration, followed by an analysis of time series properties and properties of the equilibrium recursive functions, closing with a sensitivity analysis. The solution algorithm tracks closely the layout of the model in Section 2, solving for the RME using a backward-recursive solution strategy over a finite horizon of arbitrary

²⁰We use data from Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Spain. See Appendix A-2 for a description of the data, sources, and the country specific moments.

length until the value functions, decision rules, and bond pricing function converge (see Appendix A-3 for details).

4.1 Calibration

The Markov processes of y and g are constructed as numerical approximations to log-AR(1) time-series processes:

$$\log(y_{t+1}) = (1 - \rho_y) \log(\mu_y) + \rho_y \log(y_t) + u_t, \quad (37)$$

$$\log(g_{t+1}) = (1 - \rho_g) \log(\mu_g) + \rho_g \log(g_t) + e_t, \quad (38)$$

where $|\rho_y| < 1$, $|\rho_g| < 1$ and u_t and e_t are i.i.d. over time and normally distributed with zero means and standard deviations σ_u and σ_e respectively. These moments are calibrated to data following the procedure we describe below. The Markov processes are constructed using Tauchen's [44] method, set to produce grids with five evenly spaced nodes for y and 25 for g , centered at the means, and with the lowest and highest nodes set at plus and minus 2.5 standard deviations from the mean in logs. The variances of the Markov processes are within 1 percent of their AR(1) counterparts.

The model is calibrated at an annual frequency. The parameter values that need to be assigned are the subjective discount factor, β ; the coefficient of relative risk aversion, σ ; the moments of the AR(1) processes of individual income $(\mu_y, \rho_y, \sigma_u)$ and government expenditures $(\mu_g, \rho_g, \sigma_e)$, the income tax rate, τ^y ; the opportunity cost of funds of foreign investors, \bar{r} , the parameters that define the default cost function $\phi(g)$; and the scale parameter of the welfare weights (which is also the mean), $\bar{\omega}$.

The parameter values are assigned in two steps. First, the values of all parameters except β , $\bar{\omega}$ and the function $\phi(g)$ are set to values commonly used in the literature or to estimates obtained from the European data. Second, β , $\bar{\omega}$ and $\phi(g)$ are calibrated using the Simulated Method of Moments (SMM) to minimize the distance between target moments taken from the data and their model counterparts. Thus, these parameters are set by solving the model repeatedly until the SMM converges, conditional on the parameter values set in the first step. We use data from several sources. The sample period for most variables is 1981–2015. Appendix A-2 provides a detailed description of the data and related transformations.

The first step of the calibration proceeds as follows: We set $\sigma = 1$ (i.e. log utility), which is in the range commonly used in macro models. The risk-free interest rate is set to $\bar{r} = 0.013$, which is the average annual real return on German EMU-convergence criterion government bonds in the European Commission's Eurostat database for the period 2002–2015 (these are secondary market returns, gross of tax, with around 10 years' residual maturity). We

start in 2002, the year the euro was introduced, to isolate spreads from currency risk.

Data on earnings across countries is relatively scarce and difficult to find. For this reason, to calibrate the individual income process, we set $\rho_y = 0.85$, which is a standard value in the heterogeneous-agents literature (e.g., Guvenen [27]). Then, we target $\text{Var}(\log(y)) = 0.30$ that corresponds to the middle range of the residual cross-sectional variance of log-earnings observed in Italy, Germany, and Spain (see Fuchs-Schundeln, Krueger and Sommer [24] for Germany, Japelli and Pistaferri [28] for Italy and Pijoan-Mas and Sanchez Marcos [40] for Spain). Using this estimate of the variance of income and the fact that in a stationary process $\sigma_u^2 = \text{Var}(\log(y))(1 - \rho_y^2)$, we set $\sigma_u = 0.3116$.²¹ Average income is calibrated such that the aggregate resource constraint is consistent with national accounts data with GDP normalized to one. This implies that Y in the model must equal GDP net of fixed investment because the latter is not explicitly modeled. The GDP-weighted average of investment averaged 22.26 percent of GDP during the period 1981-2015, which implies that $Y = \mu_y = 0.7774$.²²

The g process is calibrated using data on government final consumption expenditures from National Accounts for the period 1981–2015 from the World Bank’s World Development Indicators, and fitting an AR(1) process to the logged government expenditures-GDP ratio. The GDP-weighted average of the country specific estimated parameters yield: $\rho_g = 0.8604$, $\sigma_e = 0.024$ and $\mu_g = 0.1998$.

The value of τ^y is set to 38.59 percent to match the average level of revenue from effective labor taxes levied on individuals, accruing to both individual labor income and consumption taxes, and excluding all forms of capital income taxation (that equals 30.01% of GDP). Consumption tax revenues and the split of labor and capital components of individual income taxes are obtained using the effective tax rates constructed by Mendoza, Tesar, and Zang [36]).

The default cost function is decreasing in g above a threshold level set at μ_g (so that the default cost is akin to those used in EG models in which it rises with income after a threshold). The cost of default function is:

$$\phi(g) = \phi_1 \max\{0, (\mu_g - g)^{1/2}\}. \quad (39)$$

This functional form implies that aggregate consumption in the default state is given by $C = Y - g - \phi_1 \max\{0, (\mu_g - g)^{1/2}\}$.

In the second calibration step, we use the SMM algorithm to set the values of β , \bar{w} , and ϕ_1 targeting these three data moments: the 1981–2015 average ratio of domestic pub-

²¹The panel dimension available for these countries is limited (or not available), so estimates of the persistence of the transitory component of residual earnings is readily available.

²²Appendix A-2 provides tables with individual country estimates.

lic debt holdings to total public debt (55.53 percent), the 2002-2015 average bond spread relative to German bonds (0.92 percent), and the 1981-2015 average, maturity-adjusted public debt-GDP ratio (7.45 percent).²³ The maturity adjustment is necessary because the model considers only one-period debt while total debt statistics include multiple maturities. To make the adjustment, we follow the approach of the studies on external default with long-term debt by Hatchondo and Martinez [31] and Chatterjee and Eiyigunor [14], which capture the maturity structure of debt by expressing the observed debt as a consol issued in year t that pays one unit of consumption goods in $t + 1$ and $(1 - \delta)^{s-1}$ units in year $t + s$ for $s > 1$. Under this formulation, an observed outstanding debt, \bar{B} , with a given mean duration, D , has an equivalent one-period representation (i.e., the maturity-adjusted debt) given by $B = \frac{\bar{B}}{D}$, where D is the Macaulay duration rate of the consol (see Appendix A-2 for details). The GDP-weighted average for the period between 1981 and 2015 average debt-GDP ratio was 0.4823 and the average maturity is 6.35 years, that yield a maturity-adjusted debt ratio of 7.45 percent.²⁴

Table 2: Model Parameters and Targets

<i>Calibrated from data or values in the literature</i>			
Risk-Free Rate (%)	\bar{r}	0.013	Real Return German Bonds
Risk Aversion	σ	1.00	Standard Value
Autocorrel. Income	ρ_y	0.85	Güvenen (2009)
Std. Dev. Error	σ_u	0.31	Variance Residual Log-Earnings
Avg. Income	μ_y	0.78	GDP Net of Fixed Capital Investment
Autocorrel. G	ρ_g	0.86	Autocorrel. Government Consumption
Std Dev Error	σ_e	0.02	Std. Dev. Government Consumption
Avg. Gov. Consumption	μ_g	0.20	Avg. G/Y
Proportional Income Tax	τ^y	0.39	Marginal Labor Income Tax
<i>Estimated using SMM to match target moments</i>			
Discount Factor	β	0.871	Avg. Ratio Domestic Debt
Welfare Weights	ω	0.065	Avg Spread (vs Germany)
Default Cost	ϕ_1	0.793	Avg. Debt to GDP (maturity adjusted)

²³Total public debt refers to total general government net financial liabilities as a fraction of GDP. The ratio of domestic to total debt corresponds to the fraction of general government gross debt held by domestic investors from Arslanalp and Tsuda [10], extended with the ratio of marketable debt held by residents to total marketable central government debt from Organization for Economic Co-operation and Development Statistics. See Appendix A-2 for further details.

²⁴The value of duration is calibrated to the value of average maturity for total central government debt and it is available only until 2010 (source: OECD Statistics).

The SMM algorithm minimizes this loss function:

$$J(\Theta) = [M^d - M^m(\Theta)]' [M^d - M^m(\Theta)], \quad (40)$$

where $M^m(\Theta)$ and M^d are 3×1 vectors with model- and data-target moments, respectively.²⁵ The model moments are averages obtained from 160 repetitions of 10,000 period simulations, with the first 2,000 periods truncated to avoid dependency on initial conditions, and excluding default periods because most Eurozone countries did not default in the data sample period.

Table 2 shows the calibrated parameter values. Table 3 shows the target data moments and the model's corresponding moments in the SMM calibration.

Table 3: Results of SMM Calibration

Moments (%)	Data	Model
Avg. Ratio Domestic Debt	55.53	55.47
Avg. Spread Eurozone	0.92	1.22
Avg. Debt to GDP (maturity adjusted)	7.45	7.87

4.2 Equilibrium Time Series Properties

The quantitative analysis aims to answer two main questions. First, from the perspective of the theory, does the calibrated model support an equilibrium in which debt exposed to default risk can be sustained and default occurs along the equilibrium path? Second, from an empirical standpoint, to what extent are the model's time series properties in line with those observed in the data?

To answer these questions, we study the model's dynamics using a time series simulation for 10,000 periods, truncating the first 2,000 to generate a sample of 8,000 years, large enough to capture the long-run properties of the model. This sample yields 121 default events, which implies an unconditional default probability of 1.21 percent. Thus, the model produces optimal domestic (and external, since the government cannot discriminate debtors) sovereign defaults as a low-probability equilibrium outcome. In contrast with typical results from external default models, these defaults do not require costs of default in terms of exclusion from credit markets, permanently or for a random number of periods, and rely

²⁵The model moments depend on all parameter values, but we argue that β , $\bar{\omega}$, and ϕ_1 are well-identified using the chosen moments because, everything else equal, β affects the domestic demand for assets, $\bar{\omega}$ affects the social welfare function and thus the optimal debt choice, and ϕ_1 affects the default frequency, which is informative about debt prices and spreads.

in part on endogenous default costs that reflect the social value of debt for self-insurance, liquidity, and risk-sharing.

Table 4 compares moments from the model’s simulation with data counterparts. Since not all countries in the Eurozone have defaulted during our sample period but default risk spiked during the European debt crisis, we show model averages excluding default years to compare with data averages, and averages for the years before defaults occur (“prior default”) to compare with the crisis peaks in the data (the “peak crisis” column, which shows the highest values observed during the 2008-2012 period). Table 4 shows that the model does well at matching several key features of the data. The averages of total debt, the ratio of domestic to total debt, government expenditures, tax revenue and spreads were calibration targets, so these moments in the model are close to the data by construction. The rest of the model averages (domestic and external debt and transfers) approximate well the data averages.

Table 4: Long-run and Pre-Crisis Moments: Data versus Model

Moment (%)	Data		Model	
	Avg.	Peak Crisis	Average	Prior Default
Gov. Debt B	7.45*	10.94	7.87	10.82
Domestic Debt B^d	4.14	5.92	4.37	4.87
Foreign Debt \hat{B}	3.31	5.02	3.50	5.95
Ratio B^d/B	55.53*	54.15	55.47	44.97
Tax Revenues $\tau^y Y$	30.01*	29.20	30.01	30.01
Gov. Expenditure g	19.98*	21.34	19.99	19.15
Transfers τ	8.15	16.78	9.90	10.35
Spread (%)	0.92*	3.34	1.22	9.53

Note: * identifies moments used as calibration targets. See Appendix A-2 for details on sources, definitions, and sample periods for data moments. Since GDP was normalized to 1, all variables in levels are also GDP ratios.

The model is within a 20-percent margin at matching the crisis peaks of total debt, domestic debt, and the ratio of domestic to total debt. The model overestimates external debt at the crisis peak by one-fifth, and has its largest misses in that, while g increases, its crisis peak is 4 percent smaller than in the data and spreads are nearly 300 basis points higher than the peak observed in the data. On the other hand, the large spreads can be viewed as a positive result, because external default models with risk-neutral lenders typically find it very difficult to produce large spreads at reasonable debt ratios.

Table 5 compares an additional set of model and data moments, including standard deviations (relative to the standard deviation of income), income correlations, and correlations with government expenditures. We use disposable income instead of GDP or national income

because both of these are constant in the model, and we report correlations with government expenditures because g is the model’s exogenous aggregate shock. Given the parsimonious structure of the model, it is noteworthy that it can approximate well several key moments of the data, including most co-movements. The model does a good job at approximating the standard deviation of disposable income, as well as the relative standard deviations of consumption, the trade balance, and total debt. On the other hand, the model overestimates the variability of spreads and underestimates that of domestic debt.

Table 5: Cyclical Moments: Data versus Model

Variable x	Standard Deviation		Correl($x, hhdi$)		Correl($x, g/GDP$)	
	Data	Model	Data	Model	Data	Model
Consumption	0.89	0.84	0.60	0.98	-0.31	-0.71
Trade Balance/GDP	0.68	0.60	-0.21	-0.86	0.03	0.10
Spreads	0.35	1.79	-0.33	-0.02	-0.07	-0.26
Gov. Debt / GDP	2.82	1.31	-0.26	-0.08	0.40	-0.62
Dom. Debt / GDP	2.15	0.24	-0.27	-0.31	0.32	-0.30

Note: $hhdi$ denotes household disposable income. In the model, $hhdi = (1 - \tau^y)Y + \tau$ and $TB = Y - C - g$. $hhdi$ and C are logged and HP filtered with the smoothing parameter set to 6.25 (annual data). GDP ratios are also HP filtered with the same smoothing parameter. Standard deviations are ratios to the standard deviations of $hhdi$. Since the data sample for spreads is short (2002-2015) and for a period characterized by a sustained rise in spreads since 2008, we generate comparable model data by isolating events spanning 10 years before spikes in spreads, defining spikes as observations in the 95 percentile. The standard deviation of spreads is demeaned to provide a comparable variability ratio. See Appendix A-2 for details on data sources.

The correlations with disposable income produced by the model line up very well with those found in the data. As in the data, the model generates a negative correlation between income and total debt and income and domestic debt. These correlations are driven by the correlation between government transfers and total and domestic debt. Transfers decline with government expenditures but the government smooths those changes via government debt. The model also yields uncorrelated spreads consistent with the data. The correlation of consumption with disposable income is close to 1 in the model v. 0.60 in the data, and the model yields uncorrelated spreads and disposable income while in the data the correlation is -0.44.

Most correlations with government expenditures produced by the model qualitatively match those found in the data. The correlations with debt, domestic debt and spreads are of particular importance for the mechanism driving the model. As we document later in this section, the model predicts that periods with relatively low g weaken default incentives and thus enhance the government’s borrowing capacity. Accordingly, the model yields a negative correlation of government expenditures with spreads (-0.26 versus -0.07 in the data).

However, this mechanism drives the negative correlations of debt and domestic debt with government expenditures in the model v. positive correlations observed in the data. We note that the correlation of government expenditures with domestic debt is relatively noisy in the data with the GDP-weighted average driven mostly by the high positive value of Germany (equal to 0.71) while there are a few important countries with negative correlations (e.g., Spain and Netherlands where correlations are -0.19 and -0.03, respectively). The model is also very close to matching the correlation between the trade balance and spreads (0.03 in the data versus 0.10 in the model, respectively), which is driven by the same forces, since trade deficits are financed with the share of the public debt sold abroad.

We study next dynamics around default events. Figure 3 shows a set of event analysis charts based on the simulated data set with its 121 defaults. The plots show 13-year event windows centered on the year of default at $t = 0$ starting from the median debt level of all default events at $t = -6$.²⁶ Panel (i) shows total public debt (B) and domestic and foreign debt holdings (B^d and \hat{B} , respectively). Panel (ii) shows g and τ . Panel (iii) shows bond spreads. Panel (iv) shows the social welfare gain of default denoted $\bar{\alpha}$.

In order to compute $\bar{\alpha}$, we proceed as in Section 3 and calculate first compensating variations in consumption for each agent that equate expected lifetime utility across default and repayment. Hence, $\alpha(b, y, B, g)$ denotes a permanent percent change in consumption that renders an agent identified by a (b, y) pair indifferent between the payoffs $V^{d=0}(b, y, B, g)$ and $V^{d=1}(y, g)$ at the aggregate states (B, g) :

$$\alpha(b, y, B, g) = \exp \left((V^{d=1}(y, g) - V^{d=0}(b, y, B, g))(1 - \beta) \right) - 1.$$

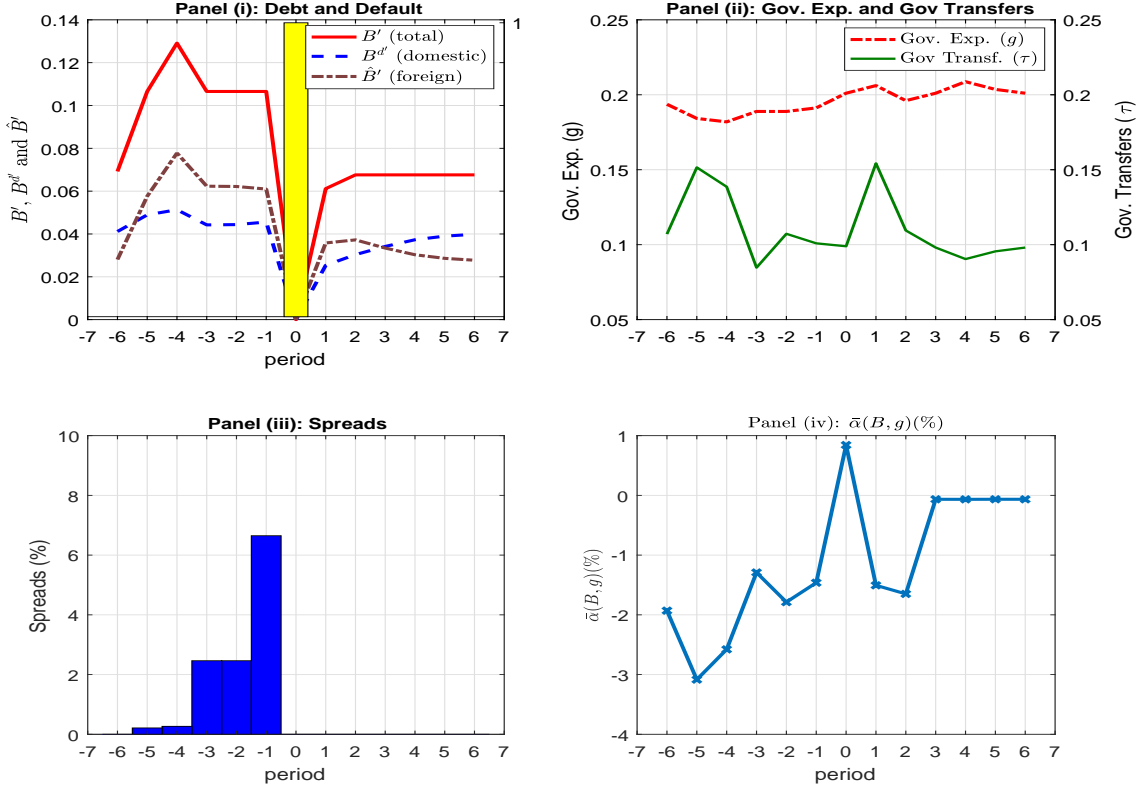
$\alpha(b, y, B, g) < 0$ implies that agents with (b, y) prefer repayment. The social welfare gain of default is then obtained by aggregating these individual gains using the social welfare function:

$$\bar{\alpha}(B, g) = \int_{\mathcal{B} \times \mathcal{Y}} \alpha(b, y, B, g) d\omega(b, y).$$

Note that, since the functions involved are nonlinear, this aggregation does not yield the same result as the compensating consumption variation that equates $W^{d=0}(B, g)$ and $W^{d=1}(g)$. The differences between the two calculations, however, turned out to be negligible, and, in particular, both are positive only when the government defaults. We chose $\bar{\alpha}(B, g)$ to make it easier to relate social and individual welfare gains.

²⁶Appendices A-4 and A-5 present results of two alternative approaches to study these dynamics. Appendix A-4 examines event windows similar to Figure 3 but starting from the lowest and highest debts at $t = -6$ across all 121 default events. Appendix A-5 examines two default events separated by a nondefault phase that matches the mode duration of the nondefault state in the full simulation. These approaches yield similar qualitative findings as those reported in the text.

Figure 3: Default Event Analysis



The event analysis plots show that debt increases in the initial periods of the sample. This change is driven by the reduction in government expenditures. Intuitively, a reduction in the default probability results in better credit terms and induces the government to take on more debt (mostly foreign). This initial increase in debt levels finances higher government transfers enabling the government to redistribute more resources and provide more liquidity to credit-constrained agents. The sharp drop in $\bar{\alpha}$ shows that using the newly gained borrowing capacity in this way is indeed socially optimal. Consistent with this drop in $\bar{\alpha}$, spreads are zero or close to zero during this period.

There is, however, a sudden change during the three years prior to the default even though the government is decreasing its debt position in $t = -3$. Spreads rise very sharply to values above 200 basis points in periods $t = -3$ and $t = -2$ and above 600 basis points in period $t = -1$. This increase in spreads derives from the increase in government expenditures observed during the same period that leads to the rise in the social value of default $\bar{\alpha}$ (particularly large at $t = -1$). As spreads increase, they attract domestic agents with sufficiently high (b, y) to buy more debt and domestic holdings of sovereign debt also rise. Government transfers follow a path similar to that of total government debt. At $t = 0$, g

risks slightly again but, at the high debt, this is enough to cause a large change in $\bar{\alpha}$ by about 200 basis points from -1.1 to 0.9 percent, causing a “sudden” default on a debt ratio practically unchanged from two years prior. The surge in spreads at $t = -1$ and the default that followed, both occurring with an unchanged debt, could be viewed as suggesting that equilibrium multiplicity or self-fulfilling expectations were the culprit, but in this simulation this is not the case. In addition, default occurs with relatively low external debt, which is roughly 55 percent of total debt.

In the early years after a default, g hardly changes but, since the agents’ precautionary savings were wiped out, domestic debt holdings rise steadily from 0 to 3 percent of GDP by $t = 6$. This reflects the optimal (gradual) buildup of precautionary savings by agents that draw relatively high income realizations. Total debt and transfers rise sharply in the first year, as the social value of debt starting from zero debt is very high and debt that is not sold at home is sold abroad at zero spread, because repayment incentives are strong ($\bar{\alpha}$ is around -1 percent). Foreign holdings of debt fall steadily after the initial increase, as domestic agents gradually demand more debt for self-insurance and the supply of debt remains constant. Total debt cannot rise more because repayment incentives are weak as government expenditures remain relatively high (the social welfare gain of default rises to become only slightly negative). By $t = 6$, debt and its foreign and domestic component are approaching the levels they had at $t = -6$. Repayment incentives are weak but still enough to issue debt at zero spread. We show in the analysis of the decision rules below that in this situation (i.e., when domestic agents desire to increase bond holdings but high g realizations weaken repayment incentives), the government optimally chooses to place as much debt as it can at virtually zero default risk.

It is important to recall that the social valuations in Panel (iv) aggregate individual payoffs of default v. repayment derived from the agents’ value functions, and as such reflect expected lifetime utility valuations, not just comparisons of contemporaneous utility effects. Thus, in both choosing to repay and issue risky debt at $t - 1$ and choosing to default at t , the government considers the dynamic equilibrium effects of both decisions, particularly the tradeoffs between progressive redistribution by defaulting and the costs of default.

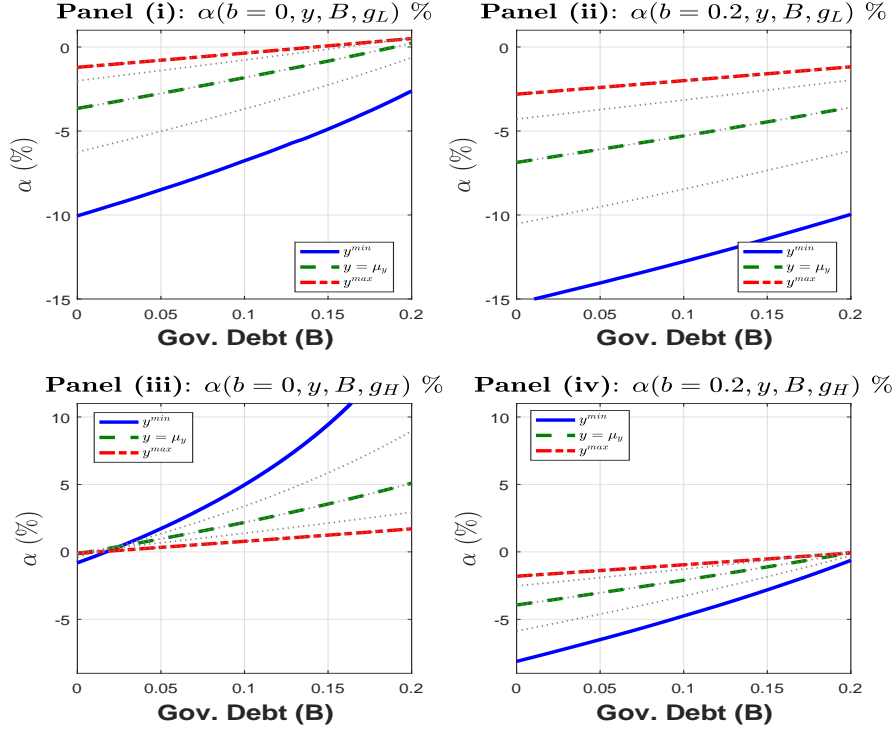
4.3 Recursive Equilibrium Functions

We analyze next the quantitative features of the equilibrium recursive functions. This analysis illustrates the feedback mechanism that drives the model and clarifies further the intuition behind the time series results.

First we study how individual welfare gains of default $\alpha(b, y, B, g)$ respond to changes in

the aggregate states B, g across the cross section of agents defined by (b, y) pairs.²⁷ Start with the response to variations in B . Figure 4 shows four graphs that plot the gains as a function of B for a range of realizations of y . Each plot is for a different combination of b and g . Panels (i) and (ii) are for $b = 0$ and $b = 0.2$, respectively, both with $g = g_L$. Panels (iii) and (iv) are also for $b = 0$ and $b = 0.2$, respectively, but now for $g = g_H$.

Figure 4: Dispersion in Individual Gains from Default as a Function of B



Figures 4 and 5 illustrate three key features of the way in which changes in public debt affect the dispersion of individual default gains:

(1) *The gains differ sharply across debt and nondebt holders.* They are mostly positive in the domain of B across income realizations for agents that do not hold debt when g is high (Panel (iii) in Figure 4), as these agents pay the same tax rate as debt holders, do not suffer wealth losses from a default, and, unless they draw high enough y , do not use the bond market to save. For agents with low income in Panel (iii), however, the gains are negative when B is very low because these agents value highly the liquidity and risk-sharing benefits of public debt, and hence prefer repayment even when incentives to repay are weak. In contrast, default gains are almost always negative in the domain of B for agents with either

²⁷In the charts that follow, B_H and B_L denote 50 percent above and below the long-run average of debt $B_M = 0.079$; y^{max} and y^{min} denote plus and minus 2 standard deviations of mean income $\mu_y = 0.78$; and g_H and g_L denote plus and minus 2 standard deviations of mean government expenditures $\mu_g = 0.20$.

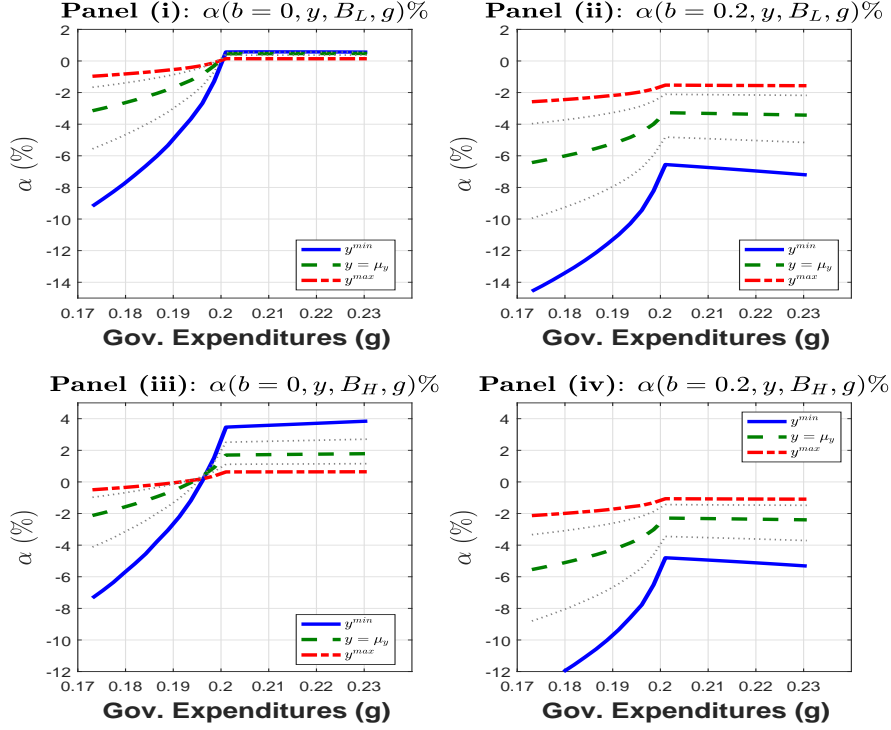
low or high b when g is low, and for agents with high b when g is high (Panels (i), (ii) and (iv)). The exception are agents that do not hold debt and draw sufficiently high income when g is low and B is large (see Panel (i)). These agents value much less the benefits of public debt. For agents with $b = 0.2$ (Panels (ii) and (iv)), the gains are always negative and large in absolute value because the loss of wealth becomes the dominant factor and makes default very costly for them.

(2) *The gains are nonmonotonic in y .* With $b = 0$ and $g = g_h$ (Panel (iii)), the gains are higher for agents with lower y (except when B is very low for the reasons explained in (1) above) because low-wealth, high-income agents value more having access to the bond market as a vehicle for self-insurance and transfers are smaller when g is high. In contrast, with all the other combinations of b and g (Panels (i), (ii) and (iv)), the gains are smaller (or default costs larger) for agents with lower income. Low-income agents with high b value more the loss of their assets due to a default precisely when they would like to use their buffer stock of savings for self-insurance (recall that defaults occur in periods of high g , which together with the debt freeze reduce τ sharply).

(3) The gains are increasing, convex functions of B for all income levels. This is most evident for agents with $b = 0$ in Panel (iii), as they value increasingly more the redistribution of resources in their favor when a larger B is defaulted on. For low B , default risk is not an issue, and hence gains from default are linearly increasing, simply because of the cut in transfers triggered by a default. As B rises, however, default risk starts to affect bond prices and demand for bonds, hampering the ability of using bonds for self-insurance and liquidity-provision, and requiring increasingly larger cuts in transfers under repayment (as more resources are devoted to debt service because of the debt-overhang effect). This happens when default is a positive probability event at $t + 1$ from the perspective of date t , which is the case for $B > 0.05$ and it is particularly evident for households with low income that rely heavily on government transfers to finance consumption expenditures.

Figure 5 shows how $\alpha(b, y, B, g)$ responds to variations in g across various income realizations. The figure is divided in four plots as the previous figure, but now for different combinations of b and B . Panels (i) and (ii) are again for $b = 0$ and $b = 0.2$, respectively, both now for a low supply of debt B_L . Panels (iii) and (iv) again are also for $b = 0$ and $b = 0.2$, respectively, but now both for a high supply of debt B_H .

Figure 5: Dispersion in Individual Gains from Default as a Function of g



As was the case for changes in B , Figure 5 shows a large dispersion in the responses of individual default gains to changes in g across agents with different b and y and for high and low b . In addition, it highlights the effect of the exogenous income cost of default making default costlier in “better” states of nature (recall disposable income is lower if default occurs when g is relatively low—below the mean in our calibration). In all four panels, the individual default gains are increasing and convex in g for $g < \mu_g$. This is due to two forces at work in this interval. First, the exogenous default cost falls as g rises. Second, default risk increases with g and this lowers bond prices and affects demand for bonds, resulting in lower transfers which reduce the value of repayment. The response of default gains to increases in g is weaker for high-income agents (i.e. α curves are flatter for higher y), because the variations in transfers and the exogenous default cost represent a smaller share of their disposable income. For $g > \mu_g$, the gains from default become nearly independent of g , and this is because, without the exogenous default cost, the effects of higher g on repayment and default payoffs nearly balance each other out.

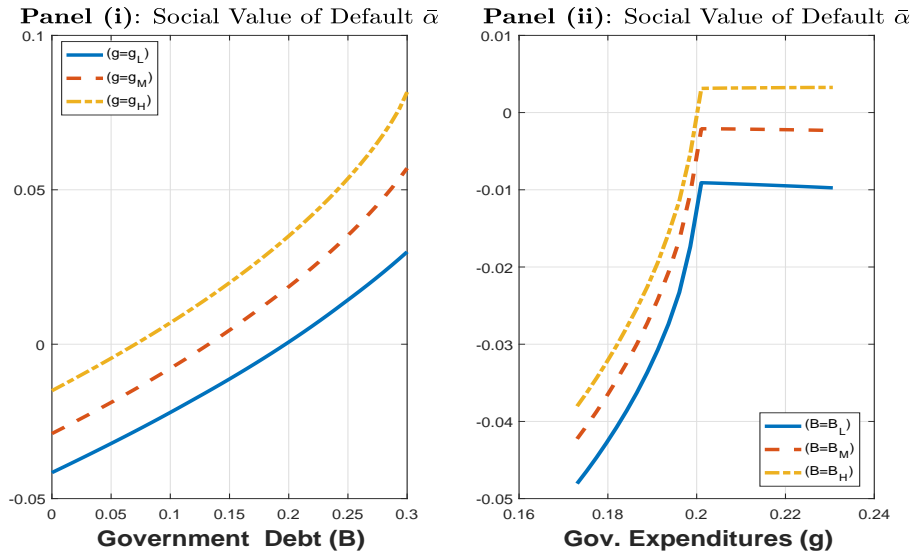
Comparing agents with $b = 0.2$ versus $b = 0$, default gains at a given value of y are uniformly higher for the latter in all the domain of g , just like it was the case for all values of B in Figure 4. This is because transfers under repayment are lower and default risk is higher for higher g . For $g \geq \mu_g$, the gains are lower (higher) at lower y for agents with

(without) bonds. For $g < \mu_g$, however, the gains are for the most part lower for agents with lower y regardless of whether they hold bonds or not because, in this range of g , disposable individual income falls by both the lower y and the exogenous income cost of default, which is uniform across agents.

As a result of the heterogeneity in the responses to g shocks across agents, we find that, while for negative g shocks almost all agents favor repayment, for positive g shocks agents without bond holdings favor default and favor it more the lower their income, while agents with $b = 0.2$ favor repayment and favor it more the lower their income. This reaffirms the result from the event analysis indicating that below-average realizations of g feature stronger repayment incentives for the government and thus sustain more debt, since all individual default valuations move in the same direction and all favor repayment, while above-average realizations of g strengthen default incentives because non-bond holders prefer default (with those with low income preferring it the most) while bond holders do not (with those with low income disliking it the most).

Next we study how the large dispersion in individual default gains we documented affects the social welfare gains of default and the default decision rule. Figure 6 shows plots of the social welfare gains as functions of B (Panel (i)) and g (Panel (ii)).

Figure 6: Social Value of Default

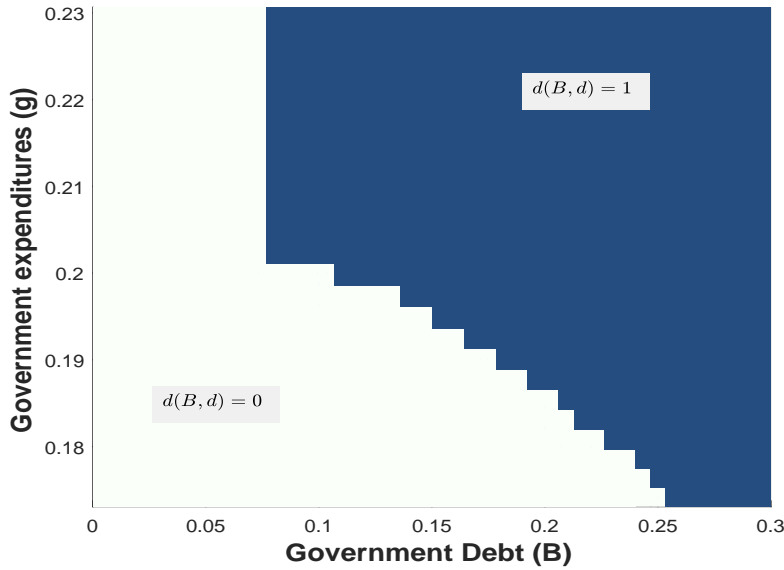


The two plots inherit the properties observed in the individual default gains, but aggregated across agents using the welfare weights: The social value of default is increasing and convex in B and in the range of $g \leq \mu_g$, while for $g > \mu_g$ the social gain of default is nearly independent of g (with the kinks at μ_g again deriving from the kink in the exogenous default cost).

Social gains yield much smaller numbers in absolute value than individual gains because they reflect the government's aggregation of winners and losers from default across the cross section of agents with different bond holdings and income. The points at which they change sign identify thresholds above which default is socially preferable to repayment. In Panel (i) ((ii)), the threshold moves to a lower B (g) for higher g (B) because repayment requires larger transfer cuts. It follows from this result that, if the economy is at an aggregate (B, g) below the corresponding default thresholds, the government would always repay and debt would be issued risk-free. For instance, in Panel (ii), for sufficiently low B the social gain of default is always negative for any g .

Figure 7 shows the default decision rule $d(B, g)$. The default and repayment sets are identified by the (B, g) pairs for which default or repayment is chosen, respectively.

Figure 7: Equilibrium Default Decision Rule $d(B, g)$



Note: The dark blue area represents $d(B, g) = 1$ and light grey area represents $d(B, g) = 0$.

In line with the previous finding that for sufficiently low B the social gain of default is negative for all values of g , for $B < 0.07$, the government chooses to repay regardless of the value of g (as Figure 6 shows, $\bar{\alpha}(B, g)$ is negative for all g when $B < 0.07$). If the optimal debt choice were to fall in this region, the government would be optimally choosing to issue risk-free debt. For $B \geq 0.07$, there is always a high enough threshold value of g such that above it the government defaults and below it repays, and the threshold is lower at higher B (i.e., the default set expands as g and B increase). This is again consistent with the shifts in the thresholds of the social welfare gains from default noted above.

Notice that the default decision rule is not symmetric because of the asymmetry in the

exogenous cost of default, which lowers disposable income only if default occurs with below-average g . Default is never optimal for $B < 0.07$; then for $0.07 \leq B \leq 0.11$; default is still not optimal for below average g (because in this region default carries the exogenous cost) but it is optimal for above-average g , then as B increases more default is optimal even for below-average g . This is again consistent with the properties of $\bar{\alpha}(B, g)$ we described.

An important drawback in looking at both the social and individual default gains is that, on one hand, by aggregating the individual gains, $\bar{\alpha}$ hides the dispersion of those individual gains, while, on the other hand looking at the individual α s is uninformative about the default choice because it hinges on social valuations. To illustrate how the dispersion of default gains affects both the social gain of default and the default decision, Figure 8 shows the social distributions of default gains for particular (B, g) pairs. These are distributions of the α s induced by the welfare weights $\omega(b, y)$ for four pairs of (B, g) formed by combining B_L, B_H and g_L, g_H .²⁸ The averages of these distributions correspond to the points in the plots of the $\bar{\alpha}$ curves shown in Figure 6 for the corresponding combination of $g = \{g_L, g_H\}$ and $B = \{B_L, B_H\}$.

Figure 8: Social Distributions of Default Gains α (for Different B and g)

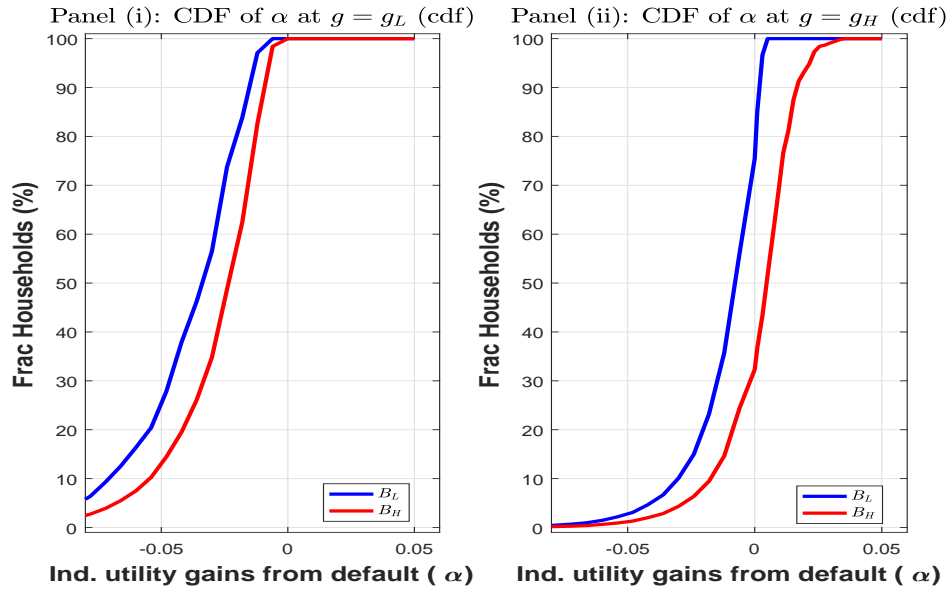


Figure 8 illustrates a key feature of the model: The social distribution of gains from default across agents varies endogenously with the aggregate states (B, g) , even though the welfare weights $\omega(b, y)$ are exogenous. The social distribution of default gains is not the same

²⁸These plots show cumulative distribution functions (CDF) of $\alpha(b, y, B, g)$ for given (B, g) across all (b, y) pairs. Given a (B, g) pair, each (b, y) maps into a value of $\alpha(b, y, B, g)$ and the government assigns to agents with that wealth and income a weight $\omega(b, y)$ in the social distribution of default gains. The CDFs are constructed by sorting the $\alpha(b, y, B, g)$ values from low to high and integrating over (b, y) using $\omega(b, y)$.

as $\omega(b, y)$, because the nonlinear, nonmonotonic responses of the individual α s to changes in B and g discussed earlier imply that the α s move in different directions across (b, y) pairs when (B, g) changes.²⁹

In line with Figure 6, the social distribution of default gains shifts to the right as B rises, indicating that the planner assesses a larger fraction of agents as benefiting from a default when the outstanding debt is higher. In Panel (i) of Figure 8, we see that consistent with the observation from Figure 7 that for $g = g_L < 0.177$ default is never chosen, the social distributions of welfare gains of default for either B_L or B_H have most of their mass in the negative domain, which represents agents that are better off with repayment. In contrast, the distribution in Panel (ii) for the case with $g = g_H$ and $B = B_H$ has enough mass in the positive domain to yield a positive mean, which makes default socially optimal. Even in this case, however, about 32 percent of agents are better off under repayment in the planner's valuation (this is the cumulative social weight of the agents with negative α s for the aggregate state (B_H, g_H)). Note also that the asymmetric effects of above- versus below-average g shocks on the individual α s are reflected in these distributions, because the distributions in Panel (i) for g_L are skewed to the left compared with those for g_H in Panel (ii), even though the g shocks are symmetric, the two panels use the same two values of B , and the welfare weights are the same.

How do the welfare weights differ from the actual wealth distribution? A comparison between $\omega(b, y)$ and the average $\Gamma(b, y)$ in the model simulation shows that while in $\Gamma(b, y)$ 81.0 percent of agents end up with $b \leq 0.01$, the government assigns them a cumulative welfare weight of 14.5 percent.³⁰ The corresponding values for $b \leq 0.10$ are 87.9 and 77.7 percent, respectively. Both distributions display 90 percent of agents below $b = 0.15$ and very close to 100 percent at $b = 0.50$.

Relative to Eurozone's distribution of wealth, it is worth noting that the welfare weights function $\omega(b, y)$, which was calibrated via SMM, is much closer to the distribution of wealth than the model's average wealth distribution $\Gamma(b, y)$. The European Central Bank (ECB) in its most recent report Household Finance and Consumption Survey reports that the share of net wealth held by the top 10 and 5 percent of agents is 51.2 percent and 37.8 percent, respectively.³¹ In the model, the corresponding weights implied by ω are 33.1 percent and 20.3 percent, respectively, while those implied by $\bar{\Gamma}$ are 86.8 percent and 62.7

²⁹This is also evident in the intensity plots of $\alpha(b, y, B, g)$ in the (b, y) space included in Appendix A-6, which display regions with similar colors (i.e., similar α s) for different (b, y) pairs.

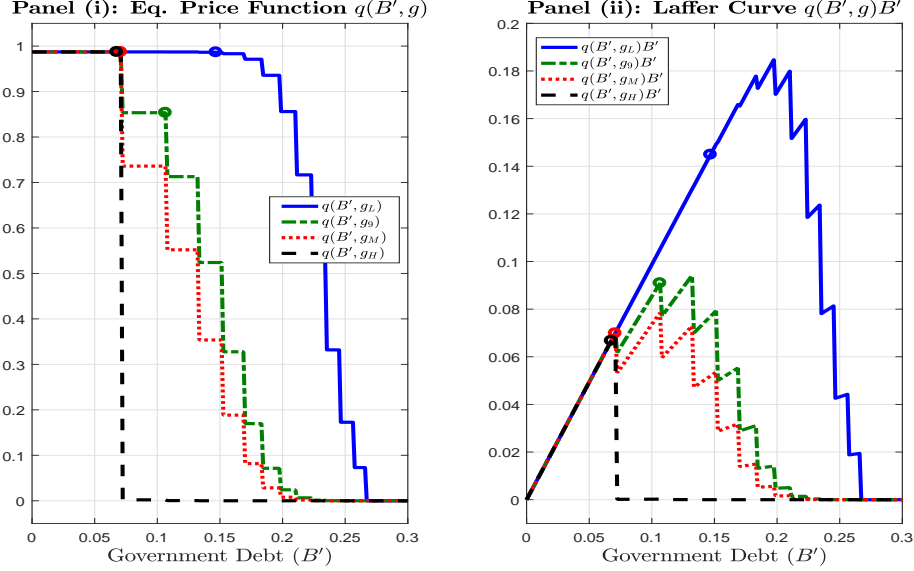
³⁰Since $\Gamma(b, y)$ is time- and state-contingent, we use an average wealth distribution computed by taking the average of each element of $\Gamma(b, y)$ over the full time series simulation excluding default episodes. See Appendix A-7 for further analysis of the differences between $\omega(b, y)$ and the average $\Gamma(b, y)$.

³¹See www.ecb.europa.eu/pub/pdf/scpsps/ecbsp18.en.pdf?d2911394a25c444cd8d3db4b77e8891a (Table 4.1, page 45).

percent, respectively. The ratio of the median to the top 10 percent of the distribution is 20.8 percent, 29.2 percent and 0 percent according to the data, $\omega(b, y)$, and $\bar{\Gamma}$, respectively. Hence, the average $\Gamma(b, y)$ overestimates (underestimates) significantly the fraction of wealth in hands of agents at the top (bottom) of the distribution relative to both the data and the ω function. The result that the wealth distribution does not match well the actual concentration of wealth is a well-known feature of standard heterogeneous-agents models in which uninsurable idiosyncratic risk is the only determinant of the wealth distribution. In our setup, the SMM calibration yields weights $\omega(b, y)$ with lower concentration at the top and a lower fraction of agents with little wealth in order to weaken distributional default incentives so as to approximate well Eurozone's mean spread.

Consider next the equilibrium pricing function of public debt. Panel (i) of Figure 9 shows the pricing function as a function of new debt issuance B' for four values of g . In addition to g_L , μ_g , and g_H , we include a curve for $g = g_9 = 0.193$, which is the ninth element in the Markov vector of realizations of g and is also the value observed at $t = -1$, just before the default, in Panel (ii) of Figure 3. In the curves for $g = \{g_L, g_M, g_H\}$, we marked with a circle the values implied by the optimal choice of B' that the government makes if the outstanding debt is $B = B_M$ (i.e., the values implied by the equilibrium decision rule $B'(B, g)$). In the curves for g_9 , the circles also denote values implied by the debt decision rule, but with $B = 0.11$, which is the value observed at $t = -1$ in the default event analysis. Hence, the circles in this case identify values implied by the optimal debt choice made at $t = -1$.

Figure 9: Pricing Function $q(B', g)$ and Debt Laffer Curve



Note: Circles on the curves with $g \in \{g_L, g_M, g_H\}$ denote values implied by the optimal choice of government debt at the corresponding value of g and with $B = B_M$. The circles for curves with $g = g_0$ (the ninth element in the Markov vector of g) denote values implied by the optimal debt choice when $B = 0.126$. This combination of outstanding debt and government expenditures is the one observed at $t = -1$ in Figure 3.

Since bond prices satisfy the same arbitrage condition of risk-neutral foreign investors as in EG external default models, the pricing functions have a similar shape as in those models. If B' is sufficiently low for default in the next period to have zero probability, q equals the risk-free price $1/(1 + \bar{r})$. Conversely, if B' is sufficiently high for default to be expected with probability 1, the bond market collapses and the price is zero. In between these two regions, q falls rapidly as B' rises, because the probability of default is higher the more debt is issued. Comparing across pricing functions, it is also clear that for debt that carries default risk, prices are lower at higher g , because the probability of default is also higher at higher g for given B' .³² Despite the similar shape of these pricing functions and those of EG models, the default decision that determines the default probability driving bond prices is determined in a very different way, with the government taking into account the distribution of gains from default across all domestic agents, including its domestic creditors.

Panel (ii) of Figure 9 plots the debt Laffer curves associated with the four pricing functions of Panel (i). These curves show how the resources the government obtains by issuing debt, $q(B', g)B'$, vary as B' changes. The government's optimal choice of B' is again marked

³²Notice this is a statement about how the realization g_t affects the probability of a default at $t+1$, whereas what we showed earlier is that, for sufficiently large B_{t+1} , the government optimally chooses to default at $t+1$ if g_{t+1} exceeds a threshold value. However, $p_t(B_{t+1}, g_t)$ still rises with g_t because the Markov process of g approximates an AR(1) process with 0.86 autocorrelation.

with circles in each curve. These Laffer curves increase linearly in most of the upward-sloping segment of the curves because debt is risk-free in those regions and hence q is constant. As debt rises enough to produce default risk, the curves quickly change slope and drop sharply, in line with the steep pricing functions of Panel (i). The Laffer curves shift down and to the left as g rises.

The optimal debt choices marked in Panels (i) and (ii) reflect the outcome of the government's optimization problem trading off the social costs and benefits of issuing debt. For low g (and B at the long-run average), debt is sold as a risk-free asset and the optimal amount of resources is raised at an internal solution along the upward-sloping segment of the Laffer curve. For average or high g , however, the government finds it optimal to generate the most resources it can by issuing debt (i.e., it chooses B' at the maximum of the Laffer curve), but the debt is still sold at zero default risk. Less debt is suboptimal, because it generates fewer resources and the Laffer curve is linearly increasing. More debt is suboptimal, because default risk rises sharply, making bond prices drop significantly and thus yielding much fewer resources. Hence, although in all three cases debt is sold at the risk-free price, the case with low g differs from those with average or high g because in the latter two the debt choice is effectively "constrained" by default risk. Thus, while in this model debt is issued risk free most of the time, the amount of debt that is issued can still be limited by the government's inability to commit. Weaker repayment incentives can result in states of nature in which less risk-free debt is offered.

The case with $g = g_9$ is interesting because the optimal debt is sold with default risk and is also less than what maximizes the Laffer curve, although it is close to it. The price is below the risk-free price but, in contrast with what the other three cases shown in the Figure portray, the price drop is not large enough to put the government in the decreasing segment of the Laffer curve. Moreover, this is an outcome actually observed along the model's equilibrium path, and particularly in the period just before the default of the event analysis in Figure 3. The fact that optimal debt is lower than the maximum value of the Laffer curve indicates that the redistribution attained by selling less debt at a higher price, but still smaller than the risk-free price, is socially preferable to higher amounts that can still be sold at a well-defined but lower price and yield more resources. The government defaults in the following period because now the redistribution attained by defaulting is preferable to that attained by repaying and issuing debt at the market prices of that period.

In Appendix A-6 we provide further analysis of the debt decision rule that allows us to generalize the previous results as follows: The optimal debt choice $B'(B, g)$ is nearly independent of B for $g \geq \mu_g$. This is because, at relatively high levels of g , the optimal debt is the maximum value of the Laffer curve regardless of the value of B and it does not

vary much because, as shown earlier, social and individual welfare gains of default are also nearly independent of B since the exogenous default cost is absent (for example, the optimal debt is 0.107 for g_M and 0.071 for g_H for most of the domain of B). In this interval of g , debt is risk-free but as explained earlier it is effectively “constrained” by the government’s inability to commit to repay. For $g < \mu_g$, the optimal debt rises with B and is always below the maximum of the Laffer curve. Hence, it is at these levels of g that the government can choose debt lower than the maximum value of the Laffer curve, and in some states the debt is exposed to default risk.

4.4 Sensitivity Analysis

To close this section we conduct a sensitivity analysis showing how the main quantitative results change as the values of the model’s key parameters are altered.

(a) Welfare Weights

Consider first the effects of changing the welfare weights in the social welfare function. To this end, we introduce a more general formulation of ω given by:

$$\omega(b, y) = \sum_{y_i \leq y} \pi^*(y_i) \left(1 - e^{-\frac{(b+z)}{\bar{\omega}}}\right). \quad (41)$$

As before, $\bar{\omega}$ remains a measure of creditor bias in the welfare weights, while z controls the weight the planner assigns to agents who do not hold debt (i.e. those hitting the borrowing constraint). This can be potentially important because these agents are the ones receiving the liquidity benefit of public debt, and the largest redistribution of resources when new debt is issued under repayment or when outstanding debt is wiped out under default. We also analyze the case where welfare weights $\omega(b, y)$ are set equal to the long-run average of the wealth distribution observed on the equilibrium path of a version of our model without default risk.³³ We denote this distribution by $\bar{\Gamma}^{rf}(b, y)$.

Table 6 reports the model’s long-run averages and averages before default events for the baseline calibration, three scenarios with different values of $\bar{\omega}$ and z (cases (A)-(C)), and the case where $\omega(b, y) = \bar{\Gamma}^{rf}(b, y)$ (case (D)). The table also includes three additional statistics that help explain the results. First, it includes the accumulated welfare weight for agents with bond holdings up to a given amount across all income levels, defined as $\Omega(b) = \sum_{y \in \mathcal{Y}} \omega(b, y)$. We consider agents with b up to 0, 0.0004, 0.045 and 0.30, which are chosen because in the calibrated social welfare function, they correspond to cumulative welfare weights of 0, 1, 50 and 99 percent respectively. Second, we use equation (31) to

³³Results using the average distribution of the model with default risk are qualitatively and quantitatively similar.

report the threshold bond holdings $\hat{b}(\mu_y, \bar{B}^D, \bar{g}^D)$ at which an agent with average income is indifferent between repayment and default when the aggregate states of B and g are at their averages conditional on the government choosing to default. Agents with $b \geq \hat{b}$ and the same average income (and at the same \bar{B}^D, \bar{g}^D) prefer repayment. Third, we report the fractions of agents that favor repayment according to the mean wealth distribution $\bar{\Gamma}(b, y)$, and the fraction that the government assesses as being better off with repayment with the welfare weights $\omega(b, y)$ (the cdf derived from $\Gamma(b, y)$ is denoted as $\gamma(b, y)$).

Table 6: Sensitivity Analysis: Social Welfare Weights

Moment (%)	benchmark $\bar{\omega} = 0.065$ $z = 0$	(A) $\bar{\omega} = 0.065$ $z = 0.025$	(B) $\bar{\omega} = 0.055$ $z = 0$	(C) $\bar{\omega} = 0.055$ $z = 0.025$	(D) $\omega(b, y)$ $= \bar{\Gamma}^{rf}(b, y)$
<i>Long Run Averages</i>					
Gov. Debt B	7.87	5.71	6.61	4.93	3.76
Dom. Debt B^d	4.37	4.21	4.27	4.14	3.23
Foreign Debt \hat{B}	3.50	1.50	2.35	0.79	0.53
Default Frequency	1.21	2.26	2.10	3.52	4.39
Spreads	1.22	2.32	2.15	3.65	4.592
Transf τ	9.90	9.93	9.92	9.95	10.01
Frac. Hh's $b = 0$	65.58	67.42	67.69	67.33	69.64
$\bar{\alpha}(B, g)$	-0.814	-0.781	-0.862	-0.766	-0.768
<i>Averages Prior Default</i>					
Gov. Debt B	10.82	7.99	9.26	6.97	5.43
Dom. Debt B^d	4.87	4.79	4.95	4.76	4.28
Foreign Debt \hat{B}	5.95	3.20	4.32	2.21	1.15
Spreads	9.53	12.67	12.30	19.78	16.16
Def. Th. $\hat{b}(\mu_y)$	0.095	0.068	0.081	0.060	0.049
% Favor Repay $(1 - \omega(\tilde{b}(\mu_y), \mu_y))$	22.44	21.92	21.59	20.48	4.91
% Favor Repay $(1 - \gamma(\tilde{b}(\mu_y), \mu_y))$	4.48	5.19	4.97	5.51	5.53
<i>Cumulative Welfare Weights</i>					
$\Omega(b = 0)$	0.00	32.06	0.00	36.59	65.64
$\Omega(b = 0.0004)$	1.00	32.29	0.51	36.84	65.65
$\Omega(b = 0.0447)$	50.00	67.09	57.48	73.01	85.83
$\Omega(b = 0.3025)$	99.00	99.41	99.63	99.77	93.82

Note: All moments reported correspond to averages across periods outside default, except those labeled “Averages Prior Default” which correspond to the average of observations prior to a default event. The model is simulated 160 times for 10,000 periods and we drop the initial 2,000 periods.

Start with the effects of increasing z for a given $\bar{\omega}$: comparing the Benchmark Column with Column (A) with $z = 0.025$. The latter results in a welfare weight of 32.06 percent

for agents with $b = 0$ v. zero in the Benchmark. The cumulative weights of agents with b up to either 0.0004 or 0.045 also rise, to 32.29 and 67.09 percent, respectively versus 1 and 50 percent, respectively in the Benchmark. The value of \hat{b} drops from 0.095 to 0.068, and the fraction of agents that the government sees as gaining from repayment drops from 22.4 to 21.9 percent, while in fact the actual fraction of agents that favor repayment *rises* from 4.5 to 5.2 percent. These changes indicate stronger incentives to default with $z = 0.025$, which are explained by a similar argument as the one studied in the first exercise of Section 3: By assigning positive weight to agents with $b = 0$ (and in general higher weight to agents with lower b), the fraction of agents that the government assesses as gaining from a default is much closer to the corresponding fraction in the economy's wealth distribution, which reduces incentives to repay.

The stronger default incentives result in a lower long-run average of the debt ratio and higher mean spreads and default frequency. The averages of domestic and external debt also drop, but the ratio of domestic to external rises sharply, from 1.25 to 2.81. Qualitatively similar changes are observed in the averages of these statistics prior to defaults.

Reducing $\bar{\omega}$ by 15 percent relative to the baseline calibration, while keeping $z = 0$, also strengthens default incentives (compare Column (B) with Benchmark). Agents without bond holdings remain with a zero welfare weight, but the lower $\bar{\omega}$ increases the welfare weight of agents with relatively small b . The resulting increases in the cumulative welfare weights of agents with b up to 0.0004 and 0.045 are smaller than in the scenario with higher z , so although we get the same results qualitatively for the effects on some indicators of default incentives and the averages of debt and its composition, the effects are weaker quantitatively.

In the case in which we introduce both higher z and lower $\bar{\omega}$ (Column (C) versus the Benchmark), we obtain the strongest reduction in repayment incentives of the scenarios in the Table, and hence the effects on the indicators of default incentives, debt ratios, and spreads in the long-run and before defaults are the strongest. In Column (C), agents without bond holdings have a cumulative welfare weight of nearly 36.6 percent, and the weight of agents with b up to 0.045 increased from 50 to 73 percent.

The results from using the long-run average wealth distribution $\bar{\Gamma}^f(b, y)$ (case (D)) are qualitatively similar to those presented in case (A) but are quantitatively more pronounced. $\bar{\Gamma}^f(b, y)$ assigns more weight on low wealth households than the benchmark distribution $\omega(b, y)$. As the fraction in favor of repayment decreases, spreads rise and the level of government debt that can be sustained declines. The increase in default risk also reduces the domestic demand for government bonds but the decline is substantially smaller than that of total debt, resulting in a much higher domestic debt to total debt ratio than in the benchmark. The spike in spreads prior to default is considerably larger than in the other cases

analyzed.

In summary, despite the nontrivial changes in results across all the scenarios in Table 6, they show that the model still sustains debt exposed to default risk and at nontrivial spreads, even with ω functions that imply high welfare weights for agents with little or no debt holdings. Moreover, in all cases, the frequency of domestic default remains low and spreads remain high.

(b) *Preference Parameters and Income Process*

Table 7 presents results for scenarios with changes in β , σ , and σ_u . These three parameters are key determinants of precautionary savings, and hence they are important for driving the model’s equilibrium dynamics. Note that, since bond prices are determined by the risk-neutral arbitrage condition of foreign investors, these parameter changes affect bond prices and spreads only indirectly by affecting the government’s debt and default decisions and the implied default probability. In particular, changes in σ do not affect bond prices directly via domestic marginal rates of substitution in consumption, although this still matters as a determinant of domestic demand for debt.

Table 7: Sensitivity Analysis: Preference Parameters and Income Process

		β		σ		σ_u		
Moment (%)	benchmark	0.853	0.888	0.75	1.25	0 [†]	0.28	0.34
<i>Long Run Averages</i>								
Gov. Debt B	7.87	7.90	8.03	7.79	7.90	2.82	7.86	7.88
Dom. Debt B^d	4.37	2.45	7.53	1.05	10.07	0.00	2.85	5.92
Foreign Debt \hat{B}	3.50	5.46	0.50	6.74	-2.16	2.82	5.01	1.96
Def. Freq.	1.21	1.23	1.19	1.26	1.19	8.92	1.21	1.18
Spreads	1.22	1.24	1.21	1.278	1.202	9.793	1.224	1.199
Transf τ	9.896	9.895	9.897	9.896	9.896	10.35	9.896	9.896
Frac. Hh’s $b = 0$	65.58	82.99	60.07	88.75	55.67	99.99	81.30	63.28
$\bar{\alpha}(B, g)$	-0.814	-0.946	-0.698	-0.771	-0.877	-0.666	-0.803	-0.827
<i>Averages Prior Default</i>								
Gov. Debt B	10.82	10.84	10.74	10.24	10.78	3.73	10.80	10.82
Dom. Debt B^d	4.87	2.78	8.52	1.17	10.60	0.00	3.22	6.74
Foreign Debt \hat{B}	5.95	8.06	2.22	9.08	0.17	3.73	7.57	4.08
Spreads	9.530	9.495	9.631	9.299	9.563	14.76	9.562	9.530

Note: Benchmark model parameters are $\beta = 0.871$, $\sigma = 1$ and $\sigma_u = 0.31$. [†] This case has no idiosyncratic uncertainty and income for all agents is set to mean income μ_y . Also, the welfare function is adjusted to coincide with the observed distribution where all agents hold no bonds. All moments reported correspond to averages across periods outside default, except those labeled “Averages Prior Default” that correspond to the average of observations prior to a default event. The model is simulated 160 times for 10,000 periods and we drop the initial 2,000 periods.

The effects of preference parameter changes on B^d are standard from the incomplete-

markets theory: Increasing (reducing) incentives for self-insurance by rising (lowering) β , σ , or σ_u , increases (reduces) the long-run and before-default averages of domestic bond holdings. The effects on foreign debt are in the opposite direction, so the ratio of domestic to external debt rises (falls) as precautionary savings strengthens (weakens). With higher β , σ , or σ_u , domestic bond demand rises so much that almost all the public debt ends up being domestic (and, in the case of $\sigma = 1.25$, the country even becomes a net external creditor). The changes in total debt, on the other hand, display nonmonotonic patterns with respect to changes in β and σ_u : Debt is higher in the scenarios in which these parameters are higher or lower than their corresponding values in the Benchmark case.

Of particular interest is the case with $\sigma_u = 0$. This case corresponds to the case with a representative domestic agent since there is no income dispersion. In order to compute this case, we adjust the welfare weights since once idiosyncratic uncertainty is reduced to zero, the domestic demand for government bonds drop to zero.³⁴ The volatility of the aggregate process paired with government bond and default decision do not generate enough variation in aggregate disposable income to induce agents to accumulate assets for self-insurance. The change in the distribution of bond holdings and welfare weights increase default incentives and prevents the government from reaching the level of government debt observed under the benchmark parameterization.

Higher values of σ and σ_u reduce default incentives and yield lower spreads and default frequencies because the social welfare gain of default falls. The benefit of defaulting as a mechanism to substitute for redistribution that cannot happen through risk sharing and insurance decreases, while on the other hand the social value of debt for the provision of liquidity and the accumulation of precautionary savings rises. In the scenario with high β , in addition to the effects via domestic bond demand, a higher discounting of the future makes default more costly because the government values less the benefit of providing assets for self-insurance of future consumption against income shocks. While, as in the external default literature, government's incentive to borrow decreases at a higher discount factor, the previous mechanism dominates and results in higher debt when β is higher than in the benchmark.

In line with what we found for changes in the welfare weights, in all the scenarios reported in Table 7, the model sustains sizable debt ratios exposed to default risk, with default remaining a low-frequency event in all cases. The ratio of domestic to foreign debt is significantly more sensitive to all the parameter variations than the rest of the model's statistics.

(c) Income Tax Rate and Default Cost

Table 8 reports the effects of changes in the income tax rate (τ^y) and the exogenous

³⁴For the case where $\sigma_u = 0$, we use a welfare weight function with $\bar{\omega} \approx 0$ to be consistent with the observed distribution of government bonds along the equilibrium path.

default cost function ($\phi(g)$). For the latter, we use the following generalization of the cost function:

$$\phi(g) = \phi_1 \max\{0, (\hat{g} - g)^\psi\}.$$

Here, \hat{g} denotes the threshold realization of g below which the cost of default is incurred, and ψ controls the curvature of the cost function. In the baseline calibration, $\hat{g} = \mu_g$ and $\psi = 1/2$, and ϕ_1 was calibrated targeting Eurozone’s mean spread.

Comparing Tables 7 and 8, shows that higher (lower) τ^y has similar qualitative effects as lower (higher) σ_u . This is in part because higher (lower) τ^y reduces (increases) the variance of idiosyncratic disposable income, which is equal to $(1 - \tau^y)^2 \sigma_y^2$. In addition, as explained in Section 2, a higher (lower) income tax rate improves (worsens) the implicit cross-sectional sharing of idiosyncratic risk provided by government transfers. Hence, these results can also be viewed as indicative of the robustness of the model’s predictions to allowing the government to use means other than debt and default to redistribute resources across agents. The model’s baseline predictions with a 35 percent income tax are not altered much by lowering the tax to 20 percent or raising it to 45 percent, except for the allocation of debt holdings across foreign and domestic agents, with the share of the former being much higher at higher tax rates.

Table 8: Sensitivity Analysis: Income Taxes and Default Cost

		τ^y		ϕ_1		ψ		\hat{g}	
Moment (%)	benchmark	0.29	0.48	0.59	0.99	0.40	0.60	0.188	0.209
<i>Long Run Averages</i>									
Gov. Debt B	7.87	7.85	7.87	7.36	8.23	8.26	7.19	6.80	13.52
Dom. Debt B^d	4.37	7.41	2.33	4.22	5.48	4.41	4.24	4.22	4.44
Foreign Debt \hat{B}	3.50	0.45	5.54	3.14	2.75	3.85	2.96	2.58	9.09
Def. Freq.	1.21	1.21	1.18	0.42	4.10	1.30	0.31	0.06	2.64
Spreads	1.220	1.223	1.189	0.42	4.28	1.32	0.31	0.06	2.71
Transf τ	9.896	2.40	17.41	9.898	10.91	9.89	9.90	9.90	9.74
Frac. Hh’s $b = 0$	65.58	59.94	83.08	66.85	73.63	65.35	66.34	65.68	66.79
$\bar{\alpha}(B, g)$	-0.814	-0.897	-0.766	-0.636	-3.880	-1.046	-0.541	-0.213	-1.963
<i>Averages Prior Default</i>									
Gov. Debt B	10.82	10.78	10.82	9.40	9.97	12.02	9.03	11.03	18.62
Dom. Debt B^d	4.87	8.31	2.66	4.56	5.85	5.08	4.54	5.37	4.46
Foreign Debt \hat{B}	5.95	2.47	8.16	4.85	4.12	6.94	4.49	5.65	14.15
Spreads	9.530	9.560	9.524	4.207	10.33	10.42	2.98	8.46	11.34

Note: Benchmark model parameters are $\tau^y = 0.386$, $\phi_1 = 0.793$, $\psi = 1/2$ and $\hat{g} = 0.199$. All moments reported correspond to averages across periods outside default, except those labeled “Averages Prior Default” that correspond to the average of observations prior to a default event. The model is simulated 160 times for 10,000 periods and we drop the initial 2,000 periods.

Regarding the effects of changes in the parameters of $\phi(g)$, changes that increase the exogenous cost of default (higher ϕ_1 , lower ψ , or higher \hat{g}) weaken incentives to default and allow the government to sustain more debt on average. Everything else the same, weaker default incentives should reduce the probability of default and yield lower spreads, but since the weaker incentives also make it optimal for the government to issue more debt (note that the mean social welfare gain of default falls with the higher default costs), the equilibrium default probabilities for the higher debt are higher, resulting in higher spreads. Higher spreads induce an increase in domestic demand for debt. Average debt ratios in the years before defaults occur are also higher with the higher default costs (except in the case of $\phi_1 = 0.99$), and in the three cases the average spreads before defaults are higher.

These results are important because they show the extent to which the model's predictions hinge on the exogenous income cost of default. The value of the scale parameter ϕ_1 is relevant mainly for the spreads, while the other model moments are less affected. Still, even with a value reduced to three-quarters the size of that in the baseline calibration, the long-run mean spread is about 42 basis points and the average spread before defaults is 421 basis points. The threshold \hat{g} was shown earlier to be important for explaining the dispersion of individual default gains, the government's default incentives, and the association of periods of increasing debt with low realizations of g . Here we showed that lowering \hat{g} , so that the exogenous default cost is active for a narrower range of realizations of g , has a small effect on total debt and its domestic and external components. On the other hand, the average social welfare gains of default are significantly higher and spreads are sharply lower. The effects of increasing ψ are similar, since higher ψ lowers the marginal cost of a given reduction of g below the threshold, suggesting that lower values of \hat{g} could be traded for lower ψ without altering the results significantly. On the other hand, these results do show that the default cost parameters play an important role in the model's ability to match observed spreads and thus in sustaining debt that carries default risk.

As in the other sensitivity experiments, in all the results shown in Table 8 the model continues to sustain sizable ratios of total and domestic public debt exposed to default risk. Spreads are also non-trivial and default remains an infrequent event preceded by sudden, sharp increases in debt and spreads. The model's ability to produce sizable spreads, however, does depend on the exogenous default costs. In light of these findings, it is worth considering the model's predictions without exogenous default costs ($\phi_1 = 0$). This case yields a long-run mean debt ratio of 5.6 percent and a domestic debt of 4.3 percent, similar to the benchmark, but with a zero mean spread. Debt is optimally chosen to be risk-free as incentives to default weaken considerably, resulting in a social welfare gain of default that is still negative but higher than in the benchmark and close to zero, at -0.07 percent. Default does not become

generally optimal without exogenous default costs, as it is the case in the perfect-foresight analysis of Section 3, because the endogenous default costs due to the social value of debt are still present. The bond pricing function is too steep at debt levels that could be offered with positive spreads, which leads the government to prefer issuing risk-free debt. Hence, as noted earlier, the debt is risk-free but still the government's borrowing capacity is hampered by its inability to commit to repay.

(d) Partial Default

In our benchmark model, the government defaults on the totality of its debt obligations (a default rate $\varphi = 1$). In this section, we consider cases where the government can default only on an exogenous fraction of debt $\varphi \in \{0.90, 0.80, 0.50\}$ (cases (A), (B), and (C)) and also the case where the fraction of default is endogenous (i.e, the government chooses $\varphi \in [0, 1]$). These cases allow us to study not only the dynamics around defaults that are followed by a renegotiation and repayment of some fraction of the debt (where the haircut equals φ) but also the dynamics around defaults via changes in wealth taxes and/or inflation. The details of the solution are provided in the appendix. In short, for the cases where the fraction of default is exogenous, during the default period the government is forced to repay a fraction $0 < (1 - \varphi) < 1$ of its debt. For the case where the fraction of default is endogenous, we solve for an auxiliary problem to compute how much each household in the economy values a different value of φ (similar to the problem we solve to choose the optimal level of debt). We then integrate the individual indirect utility functions to obtain the associated aggregate welfare value for each φ and let the government choose the one that maximizes welfare.

Table 9 presents the results.

Table 9: Sensitivity Analysis: Partial Default

		(A)	(B)	(C)	(D)
	bench. $\varphi = 1.0$	Fixed Default Rate			Endogenous Default Rate
Moment (%)		$\varphi = 0.90$	$\varphi = 0.80$	$\varphi = 0.50$	
<i>Long Run Averages</i>					
Gov. Debt B	7.87	7.96	8.21	12.62	7.87
Dom. Debt B^d	4.37	4.28	4.30	4.68	4.37
Foreign Debt \hat{B}	3.50	3.68	3.91	7.93	3.50
Def. Freq.	1.21	1.10	1.05	1.87	1.21
Spreads	1.220	1.111	1.058	1.902	1.220
Transf τ	9.896	9.910	9.921	10.199	9.896
Frac. Hh's $b = 0$	65.58	67.17	66.43	59.00	65.58
$\bar{\alpha}(B, g)$	-0.814	-0.849	-0.870	-1.112	-0.814
<i>Averages Prior Default</i>					
Gov. Debt B	10.82	10.94	11.60	20.57	10.82
Dom. Debt B^d	4.87	4.85	4.92	6.12	4.87
Foreign Debt \hat{B}	5.95	6.09	6.67	14.45	5.95
Spreads	9.530	8.415	8.098	9.802	9.530
Recovery Rate $(1 - \varphi)$	0.00	10.00	20.00	50.00	0.00

Note: All moments reported correspond to averages across periods outside default, except those labeled “Averages Prior Default” that correspond to the average of observations prior to a default event. The model is simulated 160 times for 10,000 periods and we drop the initial 2,000 periods.

It is simpler to start by describing the differences between the benchmark and the case with endogenous default (case (D)). When the government is allowed to choose the fraction of default φ , in equilibrium, it always chooses $\varphi = 1$ (the average recovery rate equals 0%). There are regions in the government debt/government expenditures space where the optimal choice is below 1; however, this never happens during the simulation of the model. The changes off-the equilibrium affect bond prices and saving decisions but on-the-equilibrium path we find that the benchmark and the model with endogenous partial default behave identically.

Cases (A) through (C) correspond to cases where the government repays an exogenous fraction of the debt after a default. We observe that as the recovery rate $(1 - \varphi)$ increases, the bond schedule shifts up (i.e., for a given debt level and default probability interest rates decline), so the government faces better credit terms. This allows the government to borrow more and results in higher debt levels than in the benchmark. The increase in debt levels is not reflected in higher spreads until the change in debt is large enough. Cases (A) and (B) present spreads that are smaller than in the benchmark. Case (C) average debt and spreads are considerably higher than in the benchmark. The demand for domestic debt declines (increases) when spreads decrease (increase). Interestingly, the average value of repayment

increases with the repayment fraction. Even though debt level and the cost of servicing the debt increase, the value of repayment raises (i.e., $\bar{\alpha}(B, g)$ declines) with the fraction of default. As the value of φ decreases the difference between full repayment and partial default declines, reducing the short term benefits of defaults.

5 Conclusions

This paper aims to explain domestic sovereign defaults. The paper proposes a model of heterogeneous agents and incomplete asset markets in which a social planner who values the welfare of all domestic agents, including its creditors, makes optimal plans for debt issuance and default. The planner makes these plans seeking to redistribute optimally resources across agents and through time by balancing distributional incentives to default with endogenous default costs due to the social benefits of debt for self-insurance, liquidity-provision and risk-sharing, and an exogenous income cost of default. A rich feedback mechanism links debt issuance and default choices, government bond prices, the agents' optimal plans, and the dynamics of the distribution of bonds across agents.

A quantitative analysis based on a baseline calibration to data for the Eurozone and several scenarios with parameter variations yields this key finding: The model sustains sizable public debt ratios exposed to default risk with default as an infrequent event. In most periods, debt is sold as a risk-free asset, but the amount of debt is still constrained by the government's inability to commit to repay. The model was calibrated to match averages of the ratio of domestic to total debt, sovereign spreads (versus Germany), and the total public debt ratio. With this calibration, the model matches two key facts documented by Reinhart and Rogoff [42]: Domestic defaults are infrequent (with 1.2 percent frequency in the model) and defaults occur with relatively low external debt (external debt is roughly two-fifths of the total debt). In addition, pre-default dynamics match typical debt crisis observations. Debt, spreads, and the ratio of foreign to domestic debt rise sharply and suddenly in the two years before a default. The debt ratio grows 38 percent above its long-run average and spreads reach 950 basis points. The model is also consistent with key cyclical moments observed in the data, particularly correlations of spreads with disposable income and government expenditures.

The findings of this paper make three main contributions to the literature. First, they address Reinhart and Rogoff's "forgotten history of domestic debt" by providing a framework that explains outright defaults on domestic public debt holders. Second, debt and default dynamics are not driven by the value of consumption smoothing for a representative agent, as is typical in external default models, but by a rich feedback mechanism in which the social welfare gain of default incorporates the welfare of both domestic bond holders and

non-bond holders, and debt has social value for self-insurance, liquidity, and risk-sharing in a heterogeneous-agents economy. Third, realistic debt, default, and spread dynamics are obtained, relying in part on endogenous default costs due to the social value of debt and without exclusion from credit markets beyond the default period, while external default models often rely heavily on exogenous default costs and credit-market exclusions of stochastic length.

The literature on domestic sovereign default is at an early stage. Some areas that would be important to consider for future research include considering partial or de facto akin to inflation or currency depreciation, adding a richer structure of saving vehicles including real and financial assets, complementing debt and default choices with an optimal choice of distortionary taxes, and adding secondary debt markets.

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Appendix to Optimal Domestic (and External) Sovereign Default

by

Pablo D’Erasmus and Enrique G. Mendoza

This Appendix is divided in seven sections. The first section presents a Table with summary indicators of the fiscal situation of the main Eurozone countries in 2011. The second section contains a detailed description of the data sources and transformations for the various macro variables used in the analysis. The third section describes the solution method used to solve for the model’s Recursive Markov Equilibrium. The fourth section offer additional details on the default event analysis. The fifth section offers an analysis of the model’s time-series dynamics between two representative default events. The sixth provides further analysis of the recursive equilibrium functions, particularly the individual welfare gains of default and the optimal debt decision rule. The final section contains a more detailed comparison of the welfare weights versus the average wealth distribution, looking at marginal distributions over different income levels.

A-1 Eurozone Fiscal Situation in 2011

Table 10: Eurozone Fiscal Situation in 2011

Moment in (%)	Gov. Debt	Gov. Debt Held by Residents	Gov. Exp.	Gov. Rev.	Primary Balance	Sov. Spreads
France	62.73	46.17	24.48	50.60	-2.51	0.71
Germany	51.49	44.47	19.27	44.50	1.69	0.00
Greece	133.10	29.68	17.38	42.40	-2.43	13.14
Ireland	64.97	45.35	18.38	34.90	-9.85	6.99
Italy	100.23	64.33	20.42	46.20	1.22	2.81
Portugal	75.84	37.36	20.05	45.00	-0.29	7.63
Spain	45.60	66.00	20.95	35.70	-7.04	2.83
Austria	52.92	38.03	19.78	48.55	-0.45	0.71
Belgium	83.58	54.73	23.77	50.31	-0.91	1.62
Finland	-48.79	23.90	23.62	53.34	-1.02	0.40
Netherlands	37.19	44.76	25.99	42.68	-3.04	0.38
Avg	59.90	44.98	21.28	44.93	-2.24	3.38
Median	62.73	44.76	20.42	45.00	-1.02	1.62
Avg (GDP w)	62.93	50.14	21.45	45.25	-1.20	2.37

Note: Author’s calculations based on OECD Statistics, Eurostat and European Central Bank (ECB). “Gov. Debt” corresponds to total general government net financial liabilities as a fraction of GDP; “Gov. Debt Held by Residents” refers to fraction of gross government debt held by domestic non-financial corporations, financial institutions, other government sectors, households and non-profit institutions; “Gov. Exp.” is general government final consumption as a fraction of GDP; “Gov. Rev.” corresponds to general government revenues as a fraction of GDP. “Prim. Balance” corresponds to the primary balance (total expenditures net of interest payments minus total revenue) as a fraction of GDP; and “Sov Spreads” correspond to the difference between interest rates of the given country and Germany (for bonds of similar maturity). For a given country i , spreads are computed as $\frac{(1+r^i)}{(1+r^{Ger})} - 1$. See Appendix A-2 for a detailed explanation of variables and sources.

A-2 Data Description and Sources

This Appendix describes the variables we gathered from the data and the sources. Most data cover the 1981-2015 period, but for some variables the sample starts in 2002. Most of the moments used for the calibration correspond to GDP-weighted averages of country specific moments. The countries we use for this calibration are Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Spain. We calculate the weights using real GDP data from 2007 (the year prior to the start of the crisis). The weights for Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Spain are 0.031, 0.038, 0.021, 0.212, 0.273, 0.026, 0.019, 0.177, 0.067, 0.019, and 0.117,

respectively.

The details are as follows:

1. Government debt: total general government net financial liabilities as a fraction of GDP, from OECD Statistics for the period 1981–2015.³⁵
2. Fraction of government debt held by residents (also referenced in the paper as fraction of domestic debt): corresponds to fraction of general government gross debt held by domestic investors in the IMF dataset put together by Arslanalp and Tsuda [10]. We extended the data when necessary to complete the 1981–2015 sample using information from OECD Statistics on the fraction of marketable debt held by residents as a fraction of total marketable debt. The correlation between both series when they overlap is equal to 0.84.
3. Government expenditures: general government final consumption as a fraction of GDP from World Development Indicators for the period 1981-2015.
4. Government revenue: Total general government revenue as a fraction of GDP from OECD statistics for the period 1981-2015.
5. Sovereign spreads: constructed using EMU convergence criterion bond yields from Eurostat for the period 2002-2015. For a given country i , spreads are computed as $\frac{(1+r^i)}{(1+r^{Ger})} - 1$, where r^{Ger} is the yield on German bonds. Data before 2002, prior to the introduction of the euro, are excluded because spreads were heavily influenced by currency risk, and not just sovereign risk. The GDP-weighted average in this case re-normalizes weights because the average is computed without Germany (the country use as reference for the risk-free rate).
6. Cross sectional variance of log-wages (needed to calibrate the income process) obtained from the cross-sectional variance of residual log-earnings in Germany, Italy and Spain as reported by Fuchs-Schundeln, Krueger and Sommer [24] (Germany), Japelli and Pistaferri [28] (Italy), and Pijoan-Mas and Sanchez Marcos [40] (Spain).
7. Income net of fixed investment (μ_y): constructed as GDP minus gross capital formation (formerly gross domestic investment) as a ratio of GDP, from World Development Indicators for the period 1981-2015.

³⁵At present, and as opposed to other countries in our sample, the financial assets of Finland’s private pension system are included in the balance sheet of the general government. For this reason, its net financial liabilities are negative.

8. Maturity adjusted debt ratio: computed using the Macaulay duration rate. The Macaulay duration for a consol is $D = \frac{1+r^*}{r^*+\delta}$, where r^* is the consol's constant annual yield. Denoting the observed outstanding debt as \overline{B} and the equivalent one-period debt at the beginning of the period (i.e. the maturity-adjusted debt) as B , we use δ to express \overline{B} as the present value of outstanding coupon claims $\overline{B} = \sum_{s=1}^{\infty} \frac{B(1-\delta)^{s-1}}{(1+r^*)^{s-1}}$, which then reduces to the expression noted in the text:

$$\overline{B} = \frac{B(1+r^*)}{(r^*+\delta)}.$$

Duration is calibrated to average term to maturity of central government debt. Source: OECD statistics for the period 2002-2010. OECD stopped updating this dataset after 2010.

9. Tax revenue: defined to include only effective labor taxes levied on individuals, accruing to both individual labor income and consumption taxes, and excluding all forms of capital income taxation. Consumption tax revenues and the split of labor and capital components of individual income taxes are obtained using the effective tax rates constructed by Mendoza, Tesar, and Zang [36]) using OECD data for the period 1995-2015.
10. Government transfers: measured as a residual using the government budget constraint. Hence, transfers are equal to transfer and entitlement payments, plus other outlays (total outlays minus current expenditures, debt service and transfers), minus tax revenue other than effective labor taxes, plus the difference between net lending in the general government national accounts and the change in reported net general government financial liabilities. Data from OECD Statistics for the period 1995-2015.
11. Household disposable income: corresponds to gross household disposable income at constant 2010 prices from OECD Statistics (downloaded from Bloomberg) for the period 1981-2015.
12. Trade balance: external balance on goods and services as a fraction of GDP, from World Development Indicators for the period 1981-2015.

Table 11: Country Specific Moments (averages)

	GDP-weights	G/Y	μ_y	ρ_g	σ_e	B^d/B
France	0.212	22.60	77.96	0.87	0.019	56.45
Germany	0.273	19.24	77.98	0.80	0.021	50.43
Greece	0.026	18.79	76.49	0.88	0.045	25.47
Ireland	0.019	17.72	78.02	0.93	0.061	40.09
Italy	0.177	18.85	79.27	0.82	0.024	62.50
Portugal	0.019	17.46	75.56	0.94	0.033	35.34
Spain	0.117	17.27	75.77	0.94	0.026	73.23
Austria	0.031	18.95	75.07	0.85	0.019	54.30
Belgium	0.038	22.19	77.76	0.92	0.023	52.43
Finland	0.021	21.52	75.98	0.89	0.038	23.33
Netherlands	0.067	22.99	78.27	0.94	0.030	58.39
GDP-weighted avg		19.98	77.74	0.86	0.024	55.53
	Spreads [†]	B/Y^*	D	$(B/Y)/D$	Tax Rev	τ
France	0.33	31.18	6.52	4.78	33.57	10.02
Germany	0.00	38.47	6.11	6.29	30.33	9.82
Greece	4.88	90.07	7.13	12.63	25.70	10.55
Ireland	1.55	33.04	5.62	5.88	22.78	5.61
Italy	1.21	90.40	6.59	13.72	27.95	4.96
Portugal	2.21	60.52	5.24	11.54	24.41	4.83
Spain	1.15	37.25	6.42	5.80	25.18	6.11
Austria	0.34	43.84	7.46	5.87	35.14	14.23
Belgium	0.53	101.60	6.29	16.14	33.10	4.28
Finland	0.22	-35.31	4.06	-8.71	36.01	12.17
Netherlands	0.21	31.20	6.38	4.89	30.53	6.27
GDP-weighted avg	0.92	48.23	6.35	7.45	30.00	8.15

Note: [†] GDP-weighted spreads use GDP-weights re-normalized for a sample that excludes Germany. * At present, and as opposed to other countries in our sample, the financial assets of Finland's private pension system are included in the balance sheet of the general government. For this reason, its net financial liabilities are negative.

Table 12: Country Specific Moments (Peak-Crisis)

	GDP-weights	B/Y^*	D	$(B/Y)/D$	B^d/B	Tax Rev	τ	G/Y	Spreads [†]
France	0.21	67.44	6.60	10.22	47.59	32.64	15.13	23.93	1.04
Germany	0.27	49.34	5.89	8.38	51.46	29.69	13.86	19.56	0.00
Greece	0.03	101.78	7.10	14.34	29.69	22.80	24.60	23.31	21.00
Ireland	0.02	79.28	4.31	18.40	45.35	21.13	22.62	20.19	6.99
Italy	0.18	111.77	6.80	16.44	67.65	28.19	20.54	20.63	3.99
Portugal	0.02	90.55	5.77	15.69	40.40	23.74	25.26	21.43	9.05
Spain	0.12	59.12	6.40	9.24	73.74	22.93	15.88	20.52	4.35
Austria	0.03	57.90	8.30	6.98	38.30	34.33	19.16	20.58	0.87
Belgium	0.04	92.22	5.94	15.53	55.88	31.99	14.48	24.26	1.62
Finland	0.02	48.79	3.90	12.51	25.41	34.37	28.06	20.70	0.52
Netherlands	0.07	39.69	6.60	6.01	47.32	30.70	15.02	22.06	0.47
GDP-weighted avg		69.36	6.34	10.94	54.15	29.20	16.78	21.34	3.34

Note: [†] GDP-weighted spreads use GDP-weights re-normalized for a sample that excludes Germany. Peak-Crisis duration refers to the minimum value during 2008-2010. * At present, and as opposed to other countries in our sample, the financial assets of Finland's private pension system are included in the balance sheet of the general government. For this reason, its net financial liabilities are negative.

Table 13: Country Specific Moments (Business Cycle Correlations)

	France	Ger.	Greece	Ireland	Italy	Port.	Spain	Austria	Belgium	Finland	Neth.	Avg.	Min	Max
Standard Dev.														
Consumption	0.82	1.12	1.18	1.26	0.98	0.99	0.83	0.36	0.47	0.69	0.37	0.89	0.36	1.26
TB/GDP	0.55	1.05	0.56	0.90	0.55	0.67	0.57	0.51	0.55	0.48	0.29	0.68	0.29	1.05
Spreads	0.16	n.a.	1.85	0.40	0.47	0.92	0.40	0.14	0.21	0.05	0.04	0.35	0.04	1.85
B/GDP	3.42	3.49	4.07	0.06	2.60	2.73	1.60	2.22	2.50	3.57	1.39	2.82	0.06	4.07
B^d/GDP	2.87	2.14	1.11	0.03	2.54	1.13	1.61	1.87	3.14	0.74	1.09	2.15	0.03	3.14
Correl($x, hmdi$)														
Consumption	0.76	0.56	n.a.	0.32	0.67	0.82	0.46	0.48	0.45	0.49	0.51	0.60	0.32	0.82
TB/GDP	-0.27	-0.01	n.a.	0.36	-0.50	-0.74	-0.33	0.11	-0.09	-0.48	0.08	-0.21	-0.74	0.36
Spreads	-0.24	n.a.	n.a.	-0.23	-0.60	-0.52	-0.61	-0.04	-0.25	-0.20	0.39	-0.33	-0.61	0.39
B/GDP	-0.24	-0.25	n.a.	-0.58	-0.47	-0.18	-0.21	-0.04	-0.18	-0.24	-0.04	-0.26	-0.58	-0.04
B^d/GDP	-0.32	-0.15	n.a.	-0.57	-0.53	0.32	-0.36	0.10	-0.57	0.23	0.02	-0.27	-0.57	0.32
Correl($x, g/GDP$)														
Consumption	-0.69	-0.31	0.02	-0.47	-0.17	0.18	-0.19	-0.13	-0.15	-0.41	-0.13	-0.31	-0.69	0.18
TB/GDP	0.40	-0.11	0.01	-0.20	-0.11	-0.07	0.27	-0.11	0.15	-0.19	-0.44	0.03	-0.44	0.40
Spreads	0.11	n.a.	0.07	-0.14	-0.21	-0.45	-0.23	0.27	0.20	0.45	-0.34	-0.07	-0.45	0.45
B/GDP	0.47	0.47	0.33	0.31	0.46	0.23	0.20	0.48	0.66	0.01	0.09	0.40	0.01	0.66
B^d/GDP	0.24	0.71	0.25	0.02	0.32	0.33	-0.19	0.38	0.44	0.20	-0.03	0.32	-0.19	0.71

Note: TB denotes trade-balance. $hmdi$ denotes household disposable income. In the model, $hmdi = (1 - \tau^y)Y + \tau$ and $TB = Y - C - g \cdot hmdi$ and C are logged and HP filtered with the smoothing parameter set to 6.25 (annual data). GDP ratios are also HP filtered with the same smoothing parameter. Standard deviations are ratios to the standard deviations of $hmdi$ (except for Greece where $hmdi$ data is not available and we provide the ratio to the standard deviation of GDP). GDP-weighted moments for spreads use GDP-weights re-normalized for a sample that excludes Germany.

A-3 Computational Algorithm

This Appendix describes the algorithm we constructed to solve for the model's CRME and RME. The algorithm performs a global solution using value function iteration. We ap-

proximate the solution of the infinite horizon economy by solving for the equilibrium of a finite-horizon version of the model for which the finite number of periods (T) is set to a number large enough such that the distance between value functions, government policies and bond prices in the first and second periods are the same up to a convergence criterion. The corresponding first-period functions are then treated as representative of the solution of the infinite-horizon economy.

The algorithm has a backward-recursive structure with the following steps:

1. Define a discrete state space of values for the aggregate states $\{B, g\}$ and individual states $\{b, y\}$
2. Solve for date- T recursive functions for each $\{b, y\}$ and $\{B, g\}$:
 - Government debt choice: $B'_T(B, g) = 0$, because T is the final period of the economy.
 - Price Debt: $q_T(B', g) = 0$, also because T is the final period.
 - The lump-sum tax under repayment follows from the government budget constraint:

$$\tau_T(B', B, g) = B + g - \tau^y Y.$$

- Using the agents' budget constraint under repayment, we obtain the agents' value function for arbitrary debt choice (note that at T it is actually independent of \tilde{B} since $q_T(B', g) = 0$)

$$\tilde{V}_T^{d=0}(\tilde{B}, y, b, B, g) = u((1 - \tau^y)y + b - g - B + \tau^y Y)$$

- The agents' value functions under repayment and default can then be solved for as:

$$V_T^{d=0}(y, b, B, g) = \tilde{V}_T^{d=0}(0, y, b, B, g).$$

$$V_T^{d=1}(y, g) = u((1 - \tau^y)y(1 - \phi(g)) - g + \tau^y Y).$$

- Given the above, the social welfare functions under repayment and default are:

$$W_T^{d=0}(B, g) = \int_{Y \times B} V_T^{d=0}(y, b, B, g) d\omega(b, y)$$

$$W_T^{d=1}(g) = \int_{Y \times B} V_T^{d=1}(y, g) d\omega(b, y).$$

- The default decision rule can then be obtained as:

$$d_T(B, g) = \arg \max_{d=\{0,1\}} \{W_T^{d=0}(B, g), W_T^{d=1}(g)\}.$$

- The agents' ex-ante value function (before the default decision is made) is:

$$V_T(y, b, B, g) = (1 - d_T)V_T^{d=0}(y, b, B, g) + d_TV_T^{d=1}(y, g).$$

3. Obtain the solution for periods $t = T - 1, \dots, 1$.

(a) Set $t = T - 1$.

(b) Obtain the default probability for all $\{B', g\}$ as:

$$p_t(B', g) = \sum_{g'} d_{t+1}(B', g')F(g', g).$$

(c) Solve for the pricing function $q_t(B', g)$:

$$q_t(B', g) = \frac{1 - p_t(B', g)}{1 + r}.$$

(d) Given the above, the lump-sum tax under repayment for an initial (B, g) pair and a given B' is:

$$\tau_t(B', B, g) = B + g - q_t(B', g)B' - \tau^y Y.$$

(e) Solve the agents' optimization problem for each agent with bonds and income b, y and each triple $\{\tilde{B}, B, g\}$:

$$\tilde{V}_t^{d=0}(\tilde{B}, y, b, B, g) = \max_{b'} u(c) + \beta E_{g'}[V_{t+1}(b', y', \tilde{B}, g')]$$

s.t.

$$c = (1 - \tau^y)y + b - q_t(\tilde{B}, g)b' - \tau_t(\tilde{B}, B, g).$$

(f) Given the solution to the above problem, solve for the optimal debt choice of the government:

$$B'_t(B, g) = \arg \max_{\tilde{B}} \int \tilde{V}_t^{d=0}(\tilde{B}, y, b, B, g) d\omega(b, y).$$

(g) The agents' continuation value under repayment is:

$$V_t^{d=0}(y, b, B, g) = \tilde{V}_t^{d=0}(B'_t(B, g), y, b, B, g).$$

(h) The agents' continuation value under default is:

$$V_t^{d=1}(y, g) = u((1 - \tau^y)y(1 - \phi(g)) - g + \tau^y Y) + \beta E_{g'}[V_{t+1}^{d=0}(y', 0, 0, g')]$$

(i) Given the above, the social welfare functions under repayment and default are:

$$W_t^{d=0}(B, g) = \int_{Y \times B} V_t^{d=0}(y, b, B, g) d\omega(b, y)$$

$$W_t^{d=1}(g) = \int_{Y \times B} V_t^{d=1}(y, g) d\omega(b, y).$$

(j) Compute the government's default decision as:

$$d_t(B, g) = \arg \max_{d=\{0,1\}} \{W_t^{d=0}(B, g), W_t^{d=1}(g)\}.$$

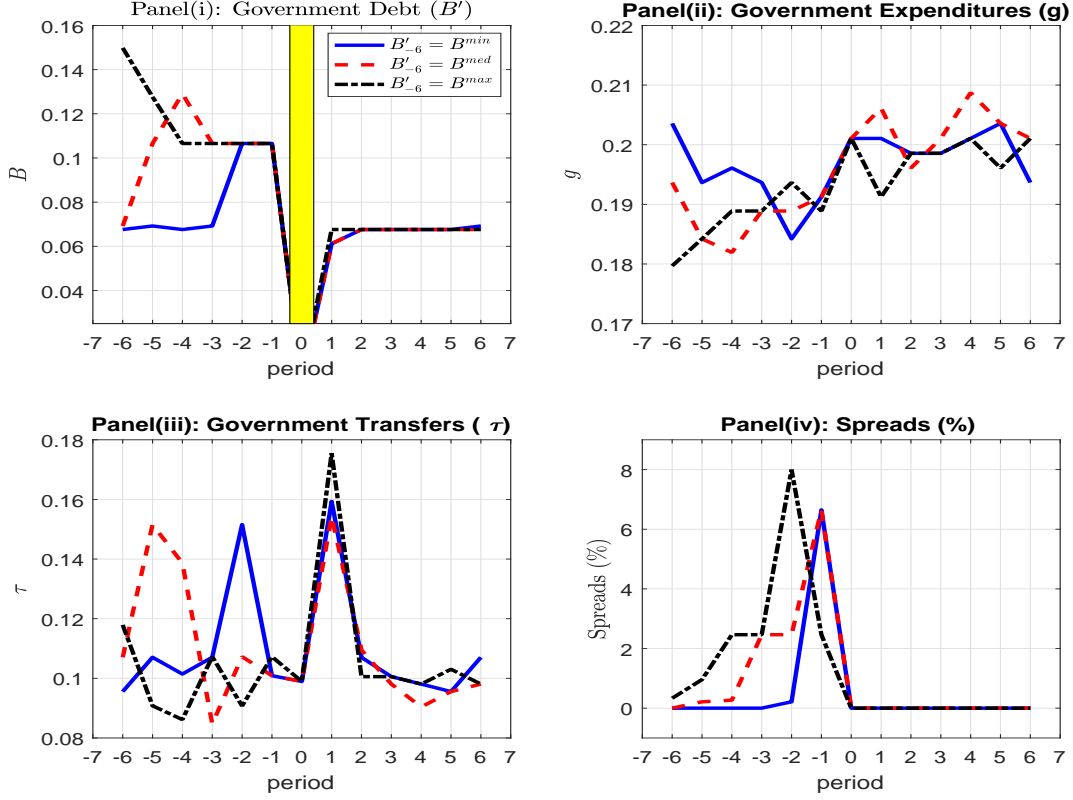
(k) If $t > 1$, set $t = t - 1$ and return to point 3b. If $t = 1$ continue.

4. Check whether value functions, government decision rules, and bond prices in periods $t = 1$ and $t = 2$ satisfy a convergence criterion. If they do, the functions in period $t = 1$ are the solution of the RME and the algorithm stops. If the convergence criterion fails, increase T and return to Step 2.

A-4 Default Event Analysis Extended

Figure 10 presents the evolution of debt, government expenditures, transfers, and spreads across three different default events: one with the maximum level of debt at the beginning of the default event window (denoted by $B_6 = B^{max}$), other with median level of debt in period $t = -6$ (denoted by $B_6 = B^{med}$ is the same event presented in Figure 3 in the body of the paper), and one with the lowest debt level observed at the beginning of the default window (denoted by $B_5 = B^{min}$).

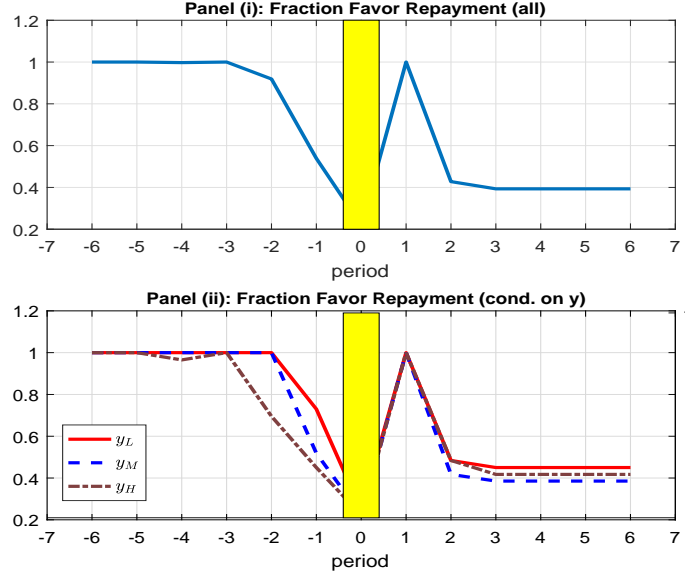
Figure 10: Default Event Analysis



We observe the same pattern across default events. As government expenditures decrease, the government has more room to redistribute and that results in an increase in the debt level and lump-sum transfer.

Figure 11 shows event windows for the government's perceived fraction of agents who prefer repayment (i.e., the fraction of agents for whom $\alpha(b, y, B, g) < 0$ obtained by aggregating using the social welfare weights $\omega(b, y)$), again using medians across each of the 121 defaults events for each of the 13 periods in the windows. Panel (i) aggregates across all (b, y) and Panel (ii) splits the results into low, mean and high income levels.

Figure 11: Preferences over Repayment



Panel (i) shows that the perceived fraction of agents that prefer repayment remains close to 100 percent until 3 years before the default. It then declines in periods $t = -3, -2, -1$, when Figure 3 shows that default risk rise. Since debt is stable prior to the default, these movements reflect mainly the effects of changes in government expenditures and transfers (a reduction in government redistribution). Then in year 0, the increase in g is sufficient to make default optimal even though debt did not increase in the previous two years.

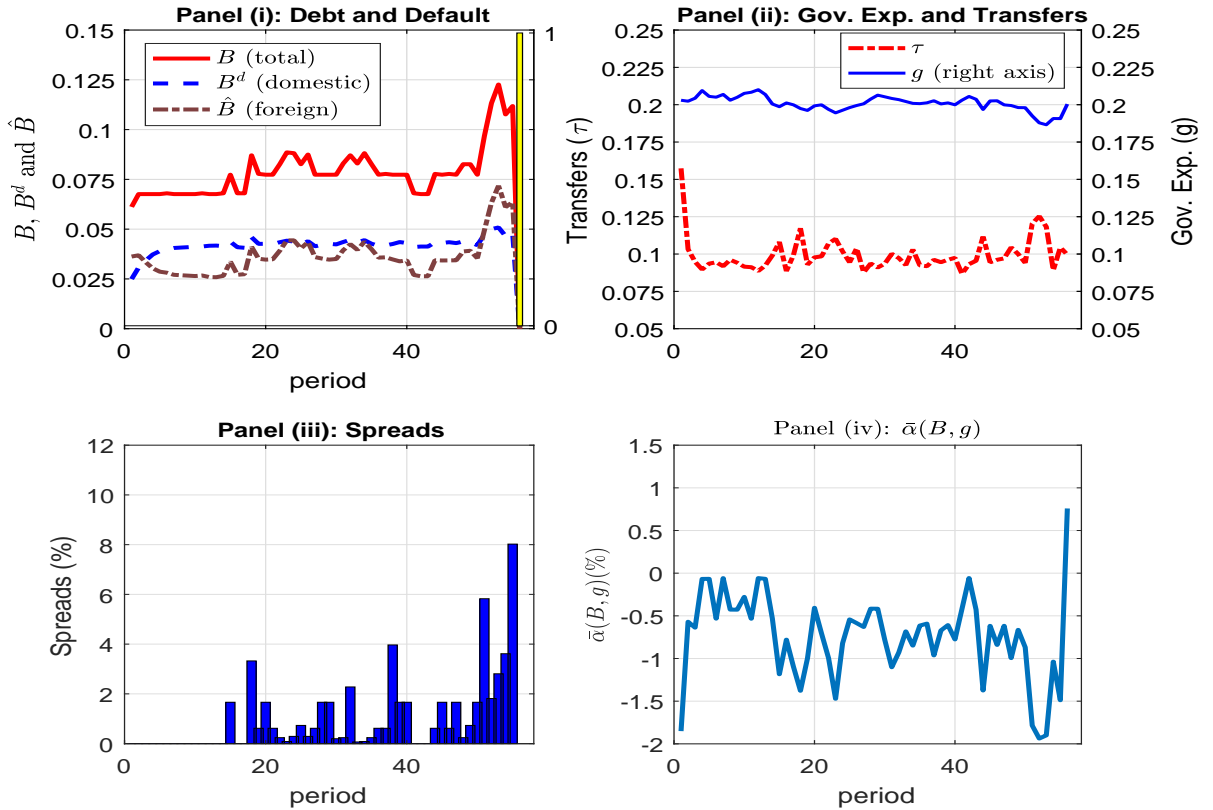
Panel (ii) shows interesting dynamics in the perceived fractions of agents who prefer repayment across income levels. The fraction is highest for low-income agents who value lump-sum transfers and the liquidity benefits of debt the most. The fraction of low-income agents who favor repayment drops only in years $t = -2, -1$. The fraction of mid-income and high-income agents who prefer repayment follows a similar pattern, but the decline starts a year earlier in the case of high income households. Mid-income and high-income agents value the liquidity services of debt but rely less on lump-sum transfers that can be sustained with debt. Interestingly, the fraction of agents who favor repayment is above zero in all years before and after the default and for all income levels. This is because there are sufficiently wealthy individuals with very low income that still favor repayment.

A-5 Dynamics Between Default Events

In the text, we illustrated the time series dynamics of the model using an event analysis with 13-year event windows centered on default events. In this appendix, we follow an alternative

approach by studying time series dynamics across two default events. Figure 12 shows the time-series dynamics between two defaults that are separated by a number of years equal to the mode duration of the non default or repayment period in the simulated data set, which is 57 years (the mode of the distribution of periods between default events). This long mode repayment period is in line with the result that defaults occur with a long-run frequency of only 1.2 percent. The figure is divided in the same four panels as the event analysis plots in the text. Panel (i) shows total government bonds (B) and their aggregate domestic and foreign holdings (B^d and \hat{B} respectively). Panel (ii) shows g and transfers (τ). Panel (iii) shows the bond spreads and Panel (iv), displays the social welfare gain of default $\bar{\alpha}$ (in %). These charts start just after the first of the two defaults occurred, and end right when the next default occurs, 57 years later.

Figure 12: Time-Series Dynamics between Default Events



Panel (i) of Figure 12 shows that public debt grows rapidly after the initial default but stays close to its mean (the value that maximizes the “Debt Laffer” curve) for a large portion of the sample, and then (around period 50) starts to grow at a faster pace, until it reaches about 12.5 percent of GDP and the second default occurs. In line with what we found in the event analysis, the initial rise in debt occurs with declining g , which makes default

more costly due to the exogenous income cost of default, thus strengthening repayment incentives and allowing the government to sustain more debt. Also in line with what the event analysis showed, taxes are generally lower than government purchases when the debt is rising, generating a primary deficit (see Panel (ii)). Spreads are generally small (Panel (iii)), and the social welfare gain of default is negative and relatively large (Panel (iv)).

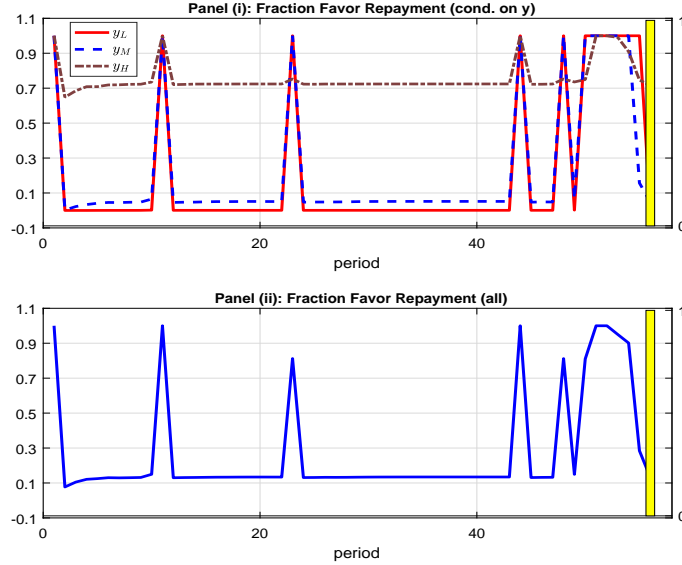
Panel (i) also shows that in the early years after the initial default, when the supply of public debt is increasing, domestic demand for risk-free assets is also rising, as the government is lowering taxes (which increases disposable income) and agents with relatively high-income realizations seek to replenish their buffer stock of savings. Domestic debt remains a higher fraction of total debt in most periods, as well as on average over the 57 years plotted. The ratio of domestic to external debt holdings, however, fluctuates, being smaller in the initial and final years than in the prolonged period in between.

In the last 10 years before the second default, domestic demand for risk-free assets increases but not as fast as total debt, which implies that the bulk of the new debt is placed abroad. With this creditor mix, and since foreign creditors do not enter in the social welfare function, default risk and spreads increase significantly. This pattern of spreads shifting suddenly from, on average, 1 percent to high levels is qualitatively consistent with standard predictions of external default models and with the stylized facts of debt crises. Still, default does not occur because the social welfare gain of default remains negative, until the 57th year arrives and the realization of g is sufficiently high to make default optimal at the existing outstanding debt since the relatively high level of debt in combination with the increase in expenditures forces the government to reduce lump-sum transfers.

The dynamics of the social gain of default in panel (iv) also capture the previous result showing that, even though the welfare weights given by $\omega(b, y)$ are exogenous, the heterogeneity of agents plays a central role. The fraction of agents that the planner sees as benefiting from a default changes endogenously over time as debt, taxes, and spreads change, and the associated changes in the dispersion of individual gains of default affect the social welfare function, the default decision, and spreads.

We examine next the evolution of the fraction of agents in the economy who value repayment (i.e., those with $\alpha(b, y, B, g) < 0$ in the actual wealth distribution $\Gamma_t(b, y)$). Figure 13 plots the evolution of this fraction for three income levels in Panel (i) and across all (b, y) in Panel (ii).

Figure 13: Preferences over Repayment



With sufficiently large fraction of agents close to the borrowing limit, the fraction of agents who favors repayment remains relatively low for most of the period. In fact, only close to the default event, the fraction that favors repayment reaches 1 for more than one period. This is due to the fact that as g declines the government issues more debt and increases transfers. As time goes by, the government starts to reduce the level of debt but a new g shock (period 55) results in a reduction in the fraction of agents in favor of repayment, since the government does not have room for further redistribution via debt at a relatively high initial debt and needs to cut transfers to pay, which induces a government default.

In line with the discussion of default payoffs in the text, the fraction of low-income agents who prefer repayment increases faster than the fraction of high-income agents who prefer repayment when confronted with government spending shocks. Interestingly, the fraction of agents with all levels of income, including the lowest, who favor repayment remains positive throughout. This is because, as we also noted in the text, there are sufficiently wealthy individuals with very low income that still favor repayment.

A-6 Details on Recursive Equilibrium Functions

This section of the Appendix provides further details on some of the implications of the recursive equilibrium functions. First we give a broader perspective on the cross-sectional properties of the individual welfare gains of default, which were examined in the paper using two-dimensional charts. Here we show that those properties are more general using intensity

plots to illustrate three-dimensional variations. Figure 14 shows two intensity plots of how $\alpha(b, y, B, g)$ varies over b and y with $g = \mu_g$. Panel (i) is for $B = B_L$ and Panel (ii) is for $B = B_H$.

The intuition for the features of these plots follows from the discussion of the threshold wealth that separates favoring repayment from favoring default, $\hat{b}(y, B, g)$, near the end of Section 2 in the main text. Comparing across panels (i) and (ii), $\alpha(b, y, B, g)$ is higher with the higher B for a given (b, y) pair, because $\hat{b}(y, B, g)$ is increasing in B . Consider next the variations along the b dimension. With $g = \mu_g$, only agents with very low b prefer default at both values of B . These agents benefit from the lower taxes associated with default and suffer negligible wealth losses. As b rises agents value increasingly more repayment for the opposite reason.

Explaining the variations along the y dimension is less straightforward, because both the repayment and default payoffs depend on y . $V^{d=1}(y, g)$ is increasing in y . $V^{d=0}(b, y, B, g)$ is increasing in “total resources,” $y + b$, but is non-monotonic on b and y individually. In particular, while for a given b , $\alpha(b, y, B, g)$ is generally increasing in y , it decreases in y for high B and very low b . The reason for this follows from the discussion around Figure 4 in the paper.

Figure 14: $\alpha(b, y, B, g)$ (for different B at $g = \mu_g$)

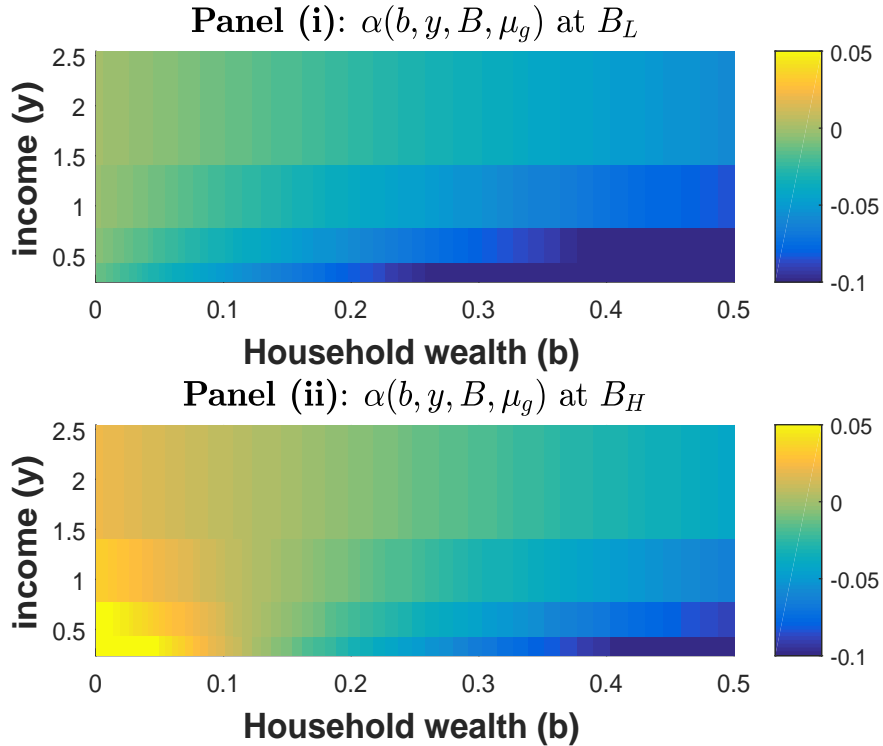
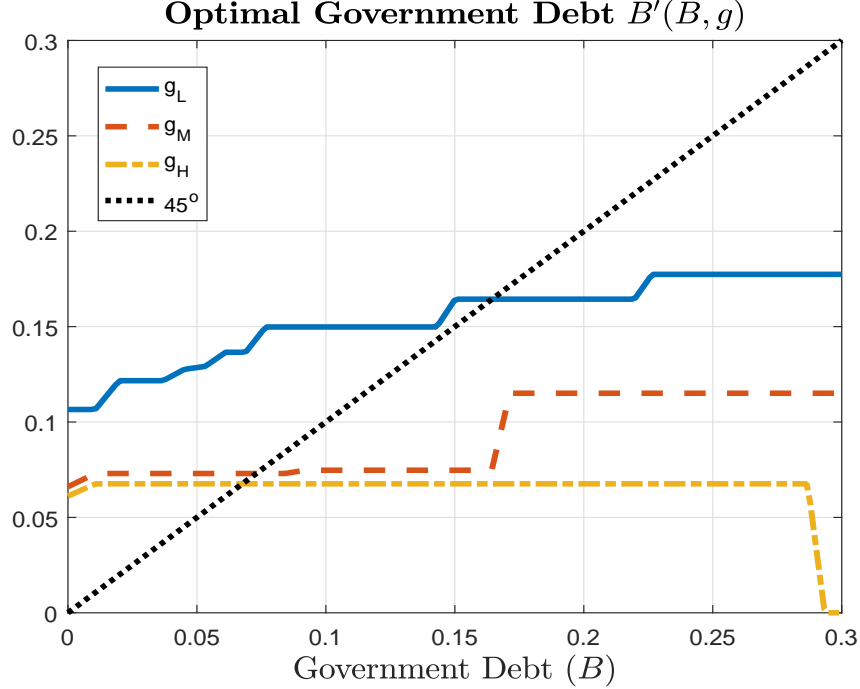


Figure 15 shows that for high or average g , the optimal debt choice is independent of B . In both cases, the government chooses the amount of debt that maximizes the Laffer curve regardless of the value of B (0.0106 for g_M and 0.0708 for g_H). Debt is risk-free but effectively “constrained” by the inability to commit to repay. For low g , the optimal debt rises with B and is always below the maximum of the Laffer curve (0.139).

Figure 15: Optimal Debt $B'(B, g)$



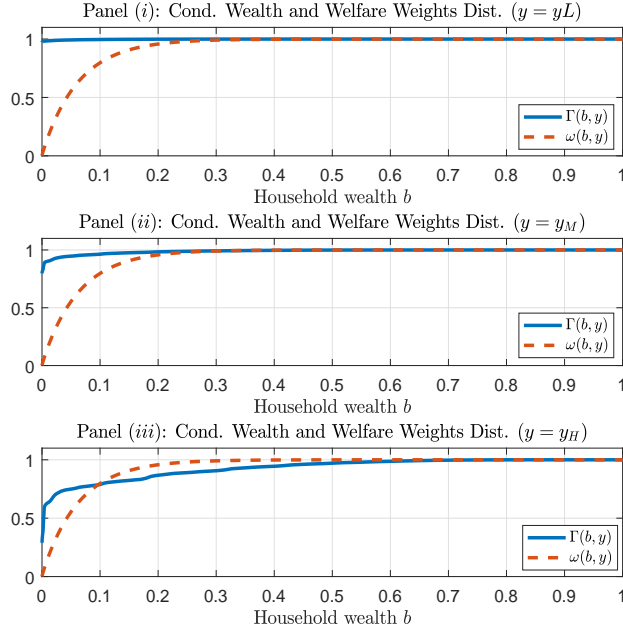
A-7 Welfare Weights versus Wealth Distribution

Figure 16 compares the weights of the social welfare function $\omega(b, y)$ with the distribution of wealth in the economy $\Gamma(b, y)$. The comparison is useful because, as explained in Section 3 of the main text, the distributional incentives to default are weaker the higher the relative weight of bond holders creditors in $\omega(b, y)$ v. $\Gamma(b, y)$. Since $\Gamma(b, y)$ is time- and state-contingent, we show the average $\bar{\Gamma}$ over the full time series simulation excluding default episodes. The plots show conditional distributions as functions of b for low, average, and high values of y in Panels (i), (ii), and (iii), respectively.

This figure shows the extent to which the fraction of agents with low b in the model economy exceeds their welfare weights. The differences are driven solely by differences in b because, by construction, $\bar{\Gamma}$ and ω have the same income distribution conditional on wealth ($\omega(b, y)$ was calibrated using $\pi^*(y)$ along the y dimension). Panels (i) and (ii) show that the majority of agents with income at the mean or lower are at the borrowing constraint or

close to it (i.e., their bond holdings are zero or nearly 0), while bond holdings need to be equal to 0.30 and 0.10 to obtain the same fraction of agents using $\omega(b, y)$ for low income and mean income, respectively. For agents with high income, Panel (iii) shows that the fraction of agents with $b < 0.1$ is about the same under both distributions.

Figure 16: “Average” Wealth Distribution $\bar{\Gamma}(b, y)$ and Welfare Weights $\omega(b, y)$



A-8 Algorithm Endogenous Partial Default

This Appendix describes the algorithm we constructed to solve for the model’s CRME and RME when there is endogenous partial default. The algorithm performs a global solution using value function iteration. We approximate the solution of the infinite horizon economy by solving for the equilibrium of a finite-horizon version of the model for which the finite number of periods (T) is set to a number large enough such that the distance between value functions, government policies and bond prices in the first and second periods are the same up to a convergence criterion. The corresponding first-period functions are then treated as representative of the solution of the infinite-horizon economy.

1. Problem in iteration T , for each $\{b, y\}$ and $\{B', B, g\}$:

- Government Debt choice: $B'_T(B, g) = 0$.
- Price Debt: $q_T(B', g) = 0$.

- Tax no default:

$$\tau_T^{d=0}(B', B, g) = B + g - \tau^y Y$$

- Define negative consumption flag:

$$\text{flag}_t^{c<0}(B', B, g) = I_{\{(1-\tau^y)y_1+b_1-\tau_T(B', B, g) \leq 0\}}$$

- Define household value (note that at T it does not depend on \tilde{B} since $q_T(B', g) = 0$)

$$\tilde{V}_T^{d=0}(\tilde{B}, y, b, B, g) = u((1 - \tau^y)y + b - g - B + \tau^y Y)$$

- If $\text{flag}_t^{c<0}(\tilde{B}, B, g) = 1$ set $\tilde{V}_T^{d=0}(\tilde{B}, :, :, B, g) = -\infty$

- In period T , government debt choice $B' = 0$.
- Household value in $d = 0$: $V_T^{d=0}(y, b, B, g) = \tilde{V}_T^{d=0}(0, y, b, B, g)$
- Tax under default:

$$\tau_T^{d=1}(B, g, \varphi) = (1 - \varphi)B + g - \tau^y Y$$

- Value in default

$$\tilde{V}_T^{d=1}(y, b, B, g, \varphi) = u((1 - \tau^y)y(1 - \phi(g)) - g + [1 - \varphi](b - B) + \tau^y Y)$$

- In period T , government $\varphi = 1$, $V_T^{d=1}(y, b, B, g) = \tilde{V}_T^{d=1}(y, b, B, g, 1)$
- Welfare values for default decision

$$W_T^{d=0}(B, g) = \int_{Y \times B} V_T^{d=0}(y, b, B, g) d\omega(b, y)$$

$$W_T^{d=1}(B, g) = \int_{Y \times B} V_T^{d=1}(y, b, B, g) d\omega(b, y)$$

- Default decision (note that $d = 1$ implies a given φ)

$$d_T(B, g) = \arg \max_{d \in \{0,1\}} \{W_T^{d=0}(B, g), W_T^{d=1}(g)\}$$

- Let

$$V_T(y, b, B, g) = (1 - d_T)V_T^{d=0}(y, b, B, g) + d_TV_T^{d=1}(y, b, B, g)$$

2. Problem in iteration $t = T - 1, \dots, 1$

- Solve for price function $q_t(B', g)$.

- Define fraction defaulted (default probability together with fraction defaulted)

$$p_t(B', g) = \sum_{g'} d_{t+1}(B', g') \varphi_{t+1}(B', g') F(g', g)$$

- price is

$$q_t(B', g) = \frac{1 - p_t(B', g)}{1 + r}$$

- Define Tax Function in $d = 0$ state

$$\tau_t^{d=0}(B', B, g) = B + g - q_t(B', g)B' - \tau^y Y$$

- Create flag for negative consumption: Combinations of B' , B and g that imply negative consumption for y_1 , b_1 when choosing b'_1

$$\text{flag}_t^{c < 0}(B', B, g) = I_{\{(1-\tau^y)y_1 + b_1 - q_t(B', g)B' - \tau_t(B', B, g) \leq 0\}}$$

- Solve problem household for b, y and $\{\tilde{B}, B, g\}$

$$\tilde{V}_t^{d=0}(\tilde{B}, y, b, B, g) = \max_{b'} u(c) + \beta E_{g'}[V_{t+1}(b', y', \tilde{B}, g')]$$

s.t.

$$c = (1 - \tau^y)y + b - q_t(\tilde{B}, g)b' - \tau_t(\tilde{B}, B, g)$$

- Optimal Debt choice

$$B'_t(B, g) = \arg \max_{\tilde{B}} \int \tilde{V}_t^{d=0}(\tilde{B}, y, b, B, g) d\omega(b, y)$$

- Define continuation value for the households under no default

$$V_t^{d=0}(y, b, B, g) = \tilde{V}_t^{d=0}(B'_t(B, g), y, b, B, g)$$

- Tax under default:

$$\tau_t^{d=1}(B, g, \varphi) = (1 - \varphi)B + g - \tau^y Y$$

- Value in default

$$\tilde{V}_t^{d=1}(y, b, B, g, \varphi) = u((1 - \tau^y)y(1 - \phi(g)) - \tau_t^{d=1}(B, g, \varphi)) + \beta E_{g'}[V_{t+1}(0, y', 0, g')]$$

- Optimal Fraction of Default Choice

$$\varphi_t(B, g) = \arg \max_{\varphi} \int \tilde{V}_t^{d=1}(y, b, B, g, \varphi) d\omega(b, y)$$

- Define value in default

$$V_t^{d=1}(y, b, B, g) = \tilde{V}_t^{d=1}(y, b, B, g, \varphi_t(B, g))$$

- Auxiliary functions:

$$\hat{p}_t(B, g) = \sum_{g'} d_{t+1}(B'_{t+1}(B, g), g') \varphi_{t+1}(B'_{t+1}(B, g), g') F(g', g)$$

$$\hat{q}_t(B, g) = \frac{1 - \hat{p}_t(B, g)}{1 + r}$$

$$\hat{\tau}_t(B, g) = B + g - \hat{q}_t(B, g) B'_{t+1}(B, g) - \tau^y Y$$

$$\hat{\text{flag}}_t^{c<0}(B', B, g) = I_{\{(1-\tau^y)y_1 + b_1 - \hat{q}_t(B, g) B'_{t+1}(B, g) - \hat{\tau}_t(B, g) \leq 0\}}$$

- Government values

$$W_t^{d=0}(B, g) = \int_{Y \times B} V_t^{d=0}(y, b, B, g) d\omega(b, y)$$

$$W_t^{d=1}(B, g) = \int_{Y \times B} V_t^{d=1}(y, b, B, g) d\omega(b, y)$$

- Government default decision

$$d_t(B, g) = \arg \max_{d=\{0,1\}} \{W_t^{d=0}(B, g), W_t^{d=1}(B, g)\}$$

3. After done with solution to periods $t = T - 1, \dots, 1$ check whether value functions, government policies and bond prices in periods 1 and 2 are sufficiently close. If they are, you are done. If not, increase T and restart.