Abstract

What would be the effect of imposing a 100 percent reserve requirement to depository institutions? This paper contends that reserves do not compete with loans on the asset side of bank’s balance sheets. Thus, they only affect liquidity provision by banks indirectly through their impact on the cost of loan and deposit creation. This cost could be driven to zero if, as the Eurosystem does, central banks remunerated required reserves at the same rate of their refinancing operations. The paper argues that the crucial constraint imposed by a fully backed banking system is collateral availability by depository institutions.

Keywords: firm financing, investment, debt maturity, credit spreads, debt dilution

Keywords: narrow banking, endogenous money, interbank market, bank solvency, liquidity, monetary policy

JEL codes: E4, E5, G21
Acknowledgments

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1 Introduction

This paper evaluates the benefits and costs of narrow banking, in particular the suggestion that depository institutions should maintain a reserve requirement of a 100 percent of their deposits. This proposition can be found in broader proposals of financial reform such as the Chicago plan (see Phillips [34] or Benes and Kumhof [3] for excellent surveys) or the Limited-Purpose Banking of Chamley et al. [8]. It is also reminiscent of the "run-free" financial system proposed by Cochrane [10]. Nowadays narrow banking has even enter into the legislative process. Switzerland is organizing a referendum to vote on this issue and the government of Iceland is seriously considering reforming its monetary system in this direction.\(^1\)

As pointed out by its advocates, the main advantage of having fully reserve-backed depository institutions would be the complete elimination of bank runs which should significantly contribute to financial stability.\(^2\) On the other hand, critics of full reserve banking remark the unbearable inefficiencies such system would bring to our economies. For example, Diamond and Dybvig [14] state that

“(…) 100% reserve banking is a dangerous proposal that would do substantial damage to the economy by reducing the overall amount of liquidity. Furthermore, the proposal is likely to be ineffective in increasing stability since it will be impossible to control the institutions that will enter in the vacuum left when banks can no longer create liquidity. Fortunately, the political realities make it unlikely that this radical and imprudent proposal will be adopted”.

Similar criticisms can be found in Wallace [41], Goodhart [24] or Kashyap et al. [29].

The arguments put forward by both, proponents and critics of narrow banking, can be understood with a simplified version of the model by Diamond and Dybvig [13].\(^3\) In that model, banks act as intermediaries between savers and investment opportunities (or borrowers). In so doing, banks obtain funds from savers in exchange for deposits. In principle, this deposit contract allows deposits to be fully convertible into the assets initially provided. Then, banks have the choice between a short-term, low-return, liquid investment and a long-term, high-return, illiquid investment. Clearly, as long as the bank invests some of the deposits in the long-term technology, it will not be able to serve deposit convertibility should a sufficiently large number of depositors decide to exercise that right before the long-term investment matures. The prospects that there may not be enough funds available support an inefficient equilibrium in which

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1 See The Telegraph [37] and [38].
2 Throughout the paper, the terms "depository institutions", "monetary financial institutions", "commercial banks", or, more generally, "banks" are treated indistinctly to mean any institution engaged in the business of receiving deposits which are insured, as defined, for instance, by section 3 of the Federal Deposit Insurance Act of the US.
3 See Freixas and Rochet [19], chapter 7.
it is in the interest of any individual depositor to withdraw his funds given
that everyone else is also running to withdraw their funds. This possibility of
a bank run, and the inefficiencies associated with this form of bank instability,
summarizes the main argument of supporters of 100 percent reserve systems. If
all assets were invested in the liquid technology, nobody will have incentives to
coordinate into a bank run.

In response to the previous argument, supporters of a fractional reserve
system emphasize that narrow banking will automatically separate financial
institutions into two types: one, offering money-like deposits but investing all
their funds in the low-return, short-term asset, and another one dedicated to the
high-return, long-term investment but unable to be used for payment purposes.
For these authors, the costs associated with permanently maintaining a sizable
fraction of the economy’s assets idle in an inferior investment, and/or without
taking advantage of the synergies produced when the same institution provides
with loans and deposits simultaneously, will be enormous. These costs could be
avoided with a system that lets banks produce money-like liabilities and decide
on their optimal asset portfolio, but that tackles the perils of bank runs with
cheaper, so they claim, alternatives such as deposit insurance or suspension of
convertibility.

Despite its initial role as bank runs backstops, deposit insurance and sus-
pension of convertibility have also been criticized in the literature. Kareken and
Wallace [28] or Freeman [18] show how deposit insurance creates a moral hazard
problem for banks in that they end up taking too much risk. This prediction
has been corroborated empirically by Anginer et al. [1] or Demirgüç-Kunt and
Detragiache [12]. Furthermore, Wallace [40] illustrates how deposit insurance
is preferred to suspension of convertibility if the fraction of agents with a liq-
uidity shock is random. This is because deposit insurance allows for contingent
allocations. Engineer [16] demonstrates that suspension of convertibility does
not prevent bank runs in longer horizon models. Samartin [35], however, indi-
cates that the choice between deposit insurance and suspension of convertibility
depends on the level of risk aversion, the intertemporal discount factor or the
existence of moral hazard. Finally, Bruche and Suarez [6] point out that the
presence of deposit insurance can contribute to a money market freeze in the
event of increased counterparty risk.

Needless to say, the seminal paper by Diamond and Dybvig [13] and all the
subsequent work based on this type of analysis have fundamentally improved
our understanding of the role financial intermediaries play in our economies.
However, this traditional view of modelling commercial banks as middlemen,
obtaining assets from savers that are then channeled to borrowers, misses several
important features of depository institutions we find in actual financial systems.
This paper contends that a model incorporating a realistic description of the
functioning of a modern monetary system produces a set of results that are
at odds with those of traditional models, at least regarding the role required
reserves play and their impact on economic activity and welfare. In particular,
the model developed in this paper implies that (i) reserve requirements do not
directly affect liquidity provision by depository institutions to the non financial
sector, (ii) reserve requirements only have an indirect effect on loan and deposit creation through its impact on the borrowing-deposit rate spread, and (iii) this indirect effect can be driven to zero if required reserves are remunerated at the refinancing rate set by the central bank. According to the model, an important consequence of these results is that 100 percent reserve requirements does not automatically imply the separation of financial institutions mentioned above. Commercial banks could go on with their long term lending in a fully reserve-backed system as they do now with fractional reserves and take advantage of any synergies arising from combining deposit and loan production.

Although reserve requirements may not necessarily influence liquidity provision by banks, I also show that collateral regulations by central banks could effectively constrain the implementation of a fully-backed deposit system. Below I provide some insights to go around this constraint.

To elaborate the argument, section 2 describes the main features of modern depository institutions incorporated in the model below. Section 3 presents the model. The main point of the paper can be made with a partial equilibrium model for the liquidity management problem of a bank. However, I would also like to measure the effects of imposing 100 percent reserve requirements when the conditions that isolate liquidity provision by banks from these requirements are not met. For that, the second part of section 3 closes the model incorporating a solvency problem for banks. Section 4 characterizes the equilibrium and tests the robustness of the results of the model by discussing several extensions including the existence of collateral regulations by central banks. Section 5 shows numerical simulations to quantitatively measure the effects of different reserve requirement schemes. Finally, section 6 concludes and discusses policy implications.

2 Describing a modern monetary system

This section reviews some of the features of modern depository institutions not included in traditional models of banking that I believe are crucial to understand their contribution in our economies. To help fixing the main idea, I start with a simple example in which I isolate a string of financial operations and follow their accounting.

Imagine I am about to make you a payment of A dollars in exchange for the provision of a service or for buying a particular good or asset. Furthermore, imagine I lack these funds so, to obtain them, I have to ask for a loan to my bank. In real life and unlike it is represented in traditional models of banking, if the loan is approved, my bank does not search for existing deposits to channel them to me. Instead, what my bank does is just to create these deposits on the spot, out of the thin air, at the stroke of a computer key. This deposit creation power is the distinguishing characteristic of depository institutions. Thus, at the time the loan is provided, my bank will have the same amount A in their

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4The accounting conventions that allow banks to create money out of nothing are described in Werner [42] and [43].
asset side as a loan as it has in its liability side as a deposit. These two entries will both be under my name. Now, by the time I want to dispose of my deposits to make you the promised payment, two things may happen. If you have an account in the same bank as me, our bank just renames my deposits under your name and the payment is made. However, if you have an account in a different bank, funds need to be transferred to that depository institution. These funds are usually reserves, that is, current accounts of depository institutions at the central bank. In general, these reserves are obtained through a loan from the central bank. Thus, imagine my bank asks for a loan equal to $A$, the amount to be transferred. Then, before the payment is made, my bank will have $A$ in reserves plus $A$ in loans on the asset side and $A$ in deposits plus $A$ in borrowing from the central bank on its liability side. Next, as the payment is made, an amount $A$ of deposits and reserves are transferred from my bank to your bank. At this point, my bank will have my loan $A$ in its assets, and the borrowing $A$ from the central bank in its liabilities. Your bank will have $A$ in reserves in its asset side and your deposit of $A$ in its liabilities. After that, you could use your deposits to make further payments to other agents and these deposits together with the reserves associated with them will be circulating in the economy. This way, the value in deposits each loan creates can potentially move around from agent to agent as long as the original loan does not mature.\footnote{Notice this circulation of deposits should also include deposit competition by which one bank tries to attract other banks' deposits in order to "fund" their loans.}

This example highlights a set of features of modern monetary and payment systems that are at odds with the representation provided in traditional models of banking. First, as a matter of practice, commercial banks create money, in the form of bank deposits, when making new loans. This is how the bulk of deposits were set to make payments is originated.\footnote{See McLeay et al. [32].} If you could trace back the life of a deposit someone has recently transferred to you, invariably it was born with a loan to someone somewhere in the past. This view in which money is created through credit is shared both by "orthodox" academicians (see Goodhart [24]), as well as central bankers (see, among others McLeay et al. [32], from the Bank of England, Holmes [26], from the Federal Reserve Bank of New York, or Constancio [11] from the ECB) and market practitioners (see Sheard [36]). Importantly, this observation calls for the abandonment of the traditional view of banking as it is loans, and the incentives to create credit, what originates deposits and not the other way around, as this traditional view contends.

The second challenge of the traditional view of banking is the role reserves play in the process of money creation. In the traditional view, reserves are just deposited assets that are left idle or invested in an inferior technology that allows full recovery at anytime. In reality, reserves, in the form of current accounts at the central bank, are a completely different object than customer’s deposits at commercial banks or the loans these banks provide to their borrowers. Reserves are produced by the central bank, while loans and deposits are produced by commercial banks. Banks maintain reserves for two reasons. As seen above, the first reason is to satisfy depositor’s payments demand. Whenever a client
wishes to make a payment to be done at another bank, this payment is usually
done with reserves. The second reason is to satisfy reserve requirements where-
ever these requirements are in place. Thus, reserve demand is driven both by
regulation as well as by the netting of payments derived from the loan and de-
posit creation to finance economic activity. Reserve supply, on the other hand, is
characterized by the monetary policy stance of the central bank. This monetary
policy stance is typically defined as a target on very short rates (i.e. overnight)
in money markets. In implementing its monetary policy, monetary authorities
are usually ready to supply, at the target rate, as much reserves as depository
institutions demand. When the monetary policy stance changes, the central
bank modifies its interest rate target but still "reads" the amount of reserves
needed to support that new target from demand by commercial banks.

An important conclusion can be drawn from the description in the previous
paragraphs. In modern monetary systems there is no sense in which holding
central bank reserves represents maintaining idle financing capacity or investing
society’s assets in an inferior technology. Reserves are not competing with
other financial investment in the asset side of bank’s balance sheets.\(^7\) As seen
above, in normal times reserves are, in fact, a consequence of banks looking for
productive loan opportunities and the deposit creation this provision of loans
generates. Even without reserve requirements, banks need to hold them for
payment settlement purposes. In the example above, maintaining a 100 percent
reserve ratio (so that the bank kept an equal amount of reserves and deposits)
was not problematic because it made it impossible for the bank to provide
loans. The problem appears because these reserves had to be borrowed from
the central bank at a cost. Thus, reserve requirements impose a tax in the
banking business of loan and deposit provision. This tax should appear as
a wedge in the lending-deposit rate spread and affect the efficiency of bank
intermediation between borrowers and depositors. However, these costs could
be driven to zero if, as the Eurosystem does, required reserves are remunerated
at the same rate banks have to pay to obtain them in the first place. In such a
case, reserve requirements could be set to any level, including a 100 percent
of deposits, without this cost channel affecting liquidity creation or how much
the nonfinancial sector is financed through banks.

Another way to state the previous conclusion is that neither deposits nor
reserves holdings should constrain loan production by commercial banks. De-
posits are automatically produced when creating new loans. On the other hand,
as explained above, reserve demand is determined by reserve requirements and
the netting of payments both of which depend on loan and deposit creation.
Thus, it is not clear whether forcing banks to hold more reserves will directly
dry out the overall amount of liquidity the nonfinancial sector uses to finance
real economic activity. Reserves and deposits are two different layers of liquidity,
overlapped to each other, and used by different economic agents (banks and the
nonfinancial sector, respectively). To see how imposing restrictions on one layer
affects the other calls for including in our models the relation between these two

\(^7\)See Keister [30].
At this point, we could now ask: if official reserves and existing deposits do not constrain bank lending and money creation, what does? Banks face several constraints, some exogenous to the banking system and some endogenous. The most obvious exogenous constraint is capital requirements. Banks have to maintain a fraction of their risky assets in the form of capital. As capital is expensive to collect, the potential to create new loans (and the money associated with them) is impaired by the amount of capital banks hold. Another limit is loan demand. To create new money, someone has to agree to take a loan. Furthermore, banks themselves constrain their lending behavior as they look for profitable opportunities, at a reasonable level of risk, where to place their loans. Finally, all these constraints are also affected by monetary policy as it influences the opportunity cost of money together with the amount of liquidity with which to fund these loans and how the interbank market distributes it among depository institutions. All these dimensions in which endogenous money is created will be present in the model below.

Regarding monetary policy, and as pointed out before, one important element in the whole money creation process, and in the relevance on reserve requirements in particular, is whether reserve requirements are remunerated or not, as well as the level of its remuneration. Goodfriend [22], Keister et al. [31], and the literature on corridor systems (see, for instance, Perez Quiros and Rodriguez Mendizabal [33]), among others, have analyzed the effect of paying interest on reserves and how this monetary policy instrument can be used for the simultaneous control of prices and quantities in the market for daily funds. In the model presented here, the remuneration of required reserves (in fact, the difference between the rate of the central bank’s open market operations and the interest paid back to banks on required reserves) affects reserve demand in a trivial way and can not be used to separate the control of interest rates from that of quantities. The model shows that it is the remuneration of excess reserves what matters for this separation. On the other hand, the remuneration of required reserves eliminates the distortionary taxation on liquidity creation by depository institutions, as Friedman [20] advocated, which represents a factor altering the efficient supply of liquidity to the nonfinancial sector.

Another important item regarding how monetary policy affects money creation by banks is the collateral regulations of central banks. Typically, when banks borrow reserves from the central bank, they have to pledge enough assets as guarantees for that loan. Given existing regulations in developed countries, the overall amount of assets that are eligible to serve as collateral in the re-

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8 An important element to the description in the main text is the possible connections between the amount of reserves and the amount of deposits. These connections are exemplified by the Treasury accounts at the central bank and cash holdings by the nonfinancial sector. For example, as we withdraw cash from ATMs, banks use their reserves to get the banknotes needed to replenish their cash machines. In normal times, however, these movements are not significant and the central bank usually accommodates them to restore the levels of reserves held previously by banks. In any case, to analyze the implications of a hypothetical situation in which the nonfinancial sector massively transform deposits into cash one needs a model that distinguishes between deposits and reserves as different objects as it is done here.
financing operations of central banks is, to a large extent, exogenous to the banking system.\footnote{Nowadays, this statement could be refined. One of the consequences of the 2007-2008 financial crisis has been the general expansion of the lists of eligible collateral for most central banks which include debt instruments issued by credit institutions, asset-backed securities or non-securities such as loans. See BIS [15] or ECB [15] for a description of these modifications observed across different central banks.} Thus, existing eligible assets could effectively constraint the amount of reserves to be supplied to the banking system and, therefore, the amount of deposits that these reserves could back up.

The endogenous view of the process of money creation developed in this paper is slowly entering mainstream economics. For example, Disyatat [15] uses this approach when revisiting the bank lending channel of monetary policy. His model, however, is partial equilibrium. Goodfriend and McCallum [23] also include some form of endogenous deposit creation by banks. Their model, however, assumes binding reserve requirements as a constraint on the deposit production by banks. As explained above, under the institutional arrangements we observe in reality, this does not seem to be the case. Furthermore, the models in Jakab and Kumhof [27] as well as Bianchi and Bigio [5] are closer to the research in this paper but there are several differences. Developing fully dynamic models, both these papers cannot explicitly find solutions and ought to rely on simulations instead. Here, I take advantage of the model being static and am able to present explicit solutions. Jakab and Kumhof [27] compare the response to various shocks under the traditional view and the endogenous money creation view (what they call "financing through money creation") and conclude the way we model bank intermediation may have significant quantitative effects. These authors argue that reserves do not constrain loan behavior by commercial banks and so, they abstract from reserve demand and interbank markets. As in this paper, Bianchi and Bigio [5] also include deposit creation through loan production together with an interbank market and monetary policy instruments such as open market operations, reserve requirements and standing facilities. In their model, however, banks produce deposits and loans in real terms. Finally, Chari and Phelan [9] and Benes and Kumhof [3] use this endogenous view to explicitly evaluate the virtues of fractional reserve banking and/or fully reserve-backed deposits. Unlike my model, Chari and Phelan [9], as Goodfriend and McCallum [23], assume binding reserve requirements as a constraint on the deposit production by banks. On the other hand, Benes and Kumhof [3], as Jakab and Kumhof [27], do not solve the model explicitly and also abstract from reserve demand and interbank markets.

Compared with this recent literature, my model includes a seemingly realistic representation of payment systems which explicitly encompasses monetary policy instruments (as summarized by open market operations, standing facilities, reserve requirements, and reserve remuneration), bidding behavior by banks at the open market operations, and net positions in the interbank market and links these items to the funding of decisions on real economic activity. This connection is done by solving the solvency and liquidity problems of banks in their financing of the nonfinancial sector.
3 A model of modern depository institutions

This static model is characterized by an economy consisting of a continuum of identical islands with measure 1. Each island is represented by a circle with circumference length equal to 1. On each point of the circle there is a location which includes a measure 1 of risk averse agents. Thus, the total population of the economy is equal to 1. Each island includes a market in which agents need to make payments to buy some good or service. For the time being, I assume that agents find whatever they demand in the island they live in. Thus, no payment between agents leaves each island. I make this assumption to simplify the exposition of the model but, as shown in the robustness section below, it is completely inconsequential for the main results of the paper.

The problem these agent face is that they lack the funds to make these payments. The role of banks in this model is to create deposits that serve as medium of exchange. The following assumptions introduce some structure in the banking sector.

Assumption 1. Each island is equally divided into $\beta \geq 2$ regions of contiguous locations. Each region is served by a different bank.

Assumption 1 means that each bank serves a measure $\delta = 1/\beta \leq 1/2$ of contiguous locations. The way in which a particular bank serves the locations in its region is as follows. Whenever an agent wants to make a payment, he will ask for a loan to his local bank. Then, after the loan is made, the buyer will order the bank to transfer these funds to the account of the seller. Notice Assumption 1 introduces market segmentation in banking services. Agents can only ask for a loan to and hold deposits from the single bank servicing their location.

Next, I include two assumptions to incorporate liquidity and solvency risks in the allocation problems of banks.

Assumption 2. The fraction of loans repayed to a bank is a random variable $x$ with probability distribution $\Xi(x)$ and density $\xi(x)$. This random variable is identical and independently distributed across banks in the economy.

The realization of $x$ is private information of each bank.

This assumption implies that banks face solvency risks. If the realization of $x$ is low enough, the bank becomes insolvent in that the value of its assets will be below that of its liabilities. Because this fraction of solvent loans provided by the bank is private information, there will be no insurance market among banks to insure this risk away. The way the economy deals with the solvency risk faced by banks is twofold. On the one hand, banks accumulate capital provided by investors. As owners of the bank, investors will have access to the information on the nonperforming loan rate (the realization of $1-x$) of the bank. The existence of capital will decrease but not completely eliminate the possibility of bank failure so that depositors (sellers) still risk loosing their deposits. The prospects of a bank being insolvent could trigger a bank run if deposits are transferable into cash. Notice this bank run is not necessarily connected to illiquidity of the
bank (that would depend on the liquidity provisions of the central bank, to be explained below) but on the possibility that the bank becomes insolvent. Thus, the second way to deal with banks’ solvency risk, the one faced by depositors, is for the economy to create a deposit insurance scheme.

The way the deposit insurance scheme works is as follows. The bank has now two types of liabilities. On the one hand, there is capital in the hands of investors with information about the performance of the loans made by the bank. On the other hand, there are deposits owned by agents without that information. In this regard, I assume next the existence of a costly monitoring technology to audit banks.

Assumption 3. Each individual depositor could learn the realization of \( \varepsilon \) for a particular bank by spending a fraction \( \phi \) of the total value of loans of the bank. This cost is paid for with cash withdrawn from deposits and is used to pay monitors.

This setup resembles the costly state verification framework of Gale and Hellwig [21], Townsend [39] and Williamson [44]. The information asymmetry between banks and depositors creates a moral hazard problem as banks have incentives to misreport the fraction of loans defaulting and to claim not to have enough resources to pay back depositors. The optimal financial contract should be structured so as to induce the bankers to truthfully report the realization of \( \varepsilon \). These authors show that in this setup the optimal contract has the form of risky debt. This contract is characterized by a threshold level for \( \varepsilon \), call it \( \underline{\varepsilon} \), that separates its support \([0, 1]\) in two sections. In section \((\underline{\varepsilon}, 1]\), there is no monitoring and banks make a constant repayment to depositors, independent of the realization of \( \varepsilon \). In section \([0, \underline{\varepsilon}]\), there is monitoring, the realization of \( \varepsilon \) is learnt by depositors, and these depositors appropriate all assets of the bank net of monitoring costs. Clearly, having all depositors sign for this contract duplicates monitoring costs unnecessarily. Thus, a deposit insurance (DI) scheme is introduced in the economy that centralizes all bank monitoring activities. This DI insures the face value of deposits (not the interest) and is the one negotiating the terms of the deposit contracts with the banks. The details of the relation between the DI and the depositors will be spelled out below. Notice the optimal contract is risky debt in the relation between banks and the DI but has the form of insured deposits from the point of view of depositors.

Finally, I make the last assumption:

Assumption 4. Apart from the transfer of deposits from buyers to sellers, banks face a random net outflow of funds (deposits and reserves), equal to a random fraction \( \varepsilon \in (-\infty, \infty) \) of deposits. This shock is distributed according with probability distribution \( \Psi(\varepsilon) \) and corresponding density \( \psi(\varepsilon) \). This random variable is identical and independently distributed across banks in the economy with mean 0 and standard distribution \( \sigma_{\varepsilon} \).

Assumption 4 incorporates liquidity risks to the problem of banks and provides an active role for the interbank market. Banks are transferring deposits among
themselves as buyers are paying sellers for their purchases. Banks settle those transfers by paying among themselves the net of the deposit flows. At this point notice that the realization of $z$ still is private information of banks. This means loans to buyers are not a valid asset to be used as general means of payment among banks. In settling their accounts, a bank can always transfer bad loans and claim the receiver of the payment to have bad luck. Furthermore, I assume the initial assets provided by investors to be illiquid too (they could be invested in physical capital or some intangible). Thus, banks need a homogeneous, general acceptable means of payments to be used among themselves. This is the role reserves play in the model. Banks obtain these reserves through a loan from the central bank and settle with them their payment accounts. The liquidity shock of Assumption 4 makes the final reserve position of banks to be random and opens up the possibility of further trades in reserves through the interbank market. This liquidity shock could be related to deposit competition, float, or any other random event affecting the flow of funds between banks.

Along all this process, banks have also to satisfy reserve capital requirements imposed by the central bank. These requirements will be specified below.

### 3.1 Timing and balance sheets

To see the whole flow of funds, take a general bank with size $\delta$, meaning serving a measure $\delta$ of contiguous locations. At the very beginning of period 0 this bank has no resources. Then, first, investors provide outside illiquid nominal assets in the form of equity. Let $Q$ be the amount of equity per location this bank receives from investors. Thus, the initial balance sheet of this bank will read

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta Q$</td>
<td>$\delta Q$</td>
</tr>
<tr>
<td>Equity</td>
<td>Equity</td>
</tr>
</tbody>
</table>

Once this investment is made, the bank provides loans, $L$, to buyers on each of the locations it serves. When providing these loans, the bank makes a double entry in its books. On the asset side, the bank annotates the right associated with the loan taken by the buyer. On the other hand, deposits (i.e., means of payments) are created, and the liability side reflects the right of entrepreneurs (obligation for the bank) to dispose of those deposits to make payments. Thus, the balance sheet of the bank at the time loans are granted on period 0 is:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta Q$</td>
<td>$\delta L$</td>
</tr>
<tr>
<td>Deposits (buyers)</td>
<td>Equity</td>
</tr>
<tr>
<td>Loans (buyers)</td>
<td>$\delta Q$</td>
</tr>
</tbody>
</table>

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10 As it will be clear below, all locations start identical at the time agents, banks and investors make their decisions. For some variables it is more convenient to work with the equivalent per location.
These loans are used by buyers to pay sellers for their purchases. This means buyers order transferring the property of these deposits to sellers. To face the associated liquidity needs, the bank seeks liquidity at an open market operation (OMO) at the central bank. Let $M$ be the allotment of such an operation. At this point, and contrary to what happens in reality, I assume this operation is done without collateral. Below I will discuss collateral issues as they represent the main drawback regarding implementation of 100 percent reserve requirements. Thus, the balance sheet will be

**Balance sheet of bank when the open market operation takes place at period 0**

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves $M$</td>
<td>Loan from OMO $M$</td>
</tr>
<tr>
<td>Assets $\delta Q$</td>
<td>Deposits (buyers) $\delta L$</td>
</tr>
<tr>
<td>Loans (buyers) $\delta L$</td>
<td>Equity $\delta Q$</td>
</tr>
</tbody>
</table>

After the OMO, only the net flow of funds between banks, $FF(\varepsilon)$, exchanges hands in the form of changes in deposits and reserves. This net flow of funds will depend on the realization of the liquidity shock $\varepsilon$. The balance sheet now reads

**Balance sheet of bank when transfers are made at period 0**

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves $M + FF(\varepsilon)$</td>
<td>Loan from OMO $M$</td>
</tr>
<tr>
<td>Assets $\delta Q$</td>
<td>Deposits (sellers) $\delta L + FF(\varepsilon)$</td>
</tr>
<tr>
<td>Loans (buyers) $\delta L$</td>
<td>Equity $\delta Q$</td>
</tr>
</tbody>
</table>

Once liquidity uncertainty is resolved, banks know whether they have an excess or deficit of reserves and access the interbank market to compensate it. Let $I(\varepsilon)$ be the interbank lending of the bank (borrowing if negative), which will depend upon the realization of the liquidity shock, $\varepsilon$. Thus, the balance sheet of the bank right before the solvency shock $\varepsilon$ hits is:

**Balance sheet of bank when interbank market opens at period 0**

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves $M - I(\varepsilon) + FF(\varepsilon)$</td>
<td>Loan from OMO $M$</td>
</tr>
<tr>
<td>Assets $\delta Q$</td>
<td>Deposits (sellers) $\delta L + FF(\varepsilon)$</td>
</tr>
<tr>
<td>Loans (interb.) $I(\varepsilon)$</td>
<td>Equity $\delta Q$</td>
</tr>
<tr>
<td>Loans (buyers) $\delta L$</td>
<td></td>
</tr>
</tbody>
</table>

It is at this time that the solvency shock is resolved which makes a random fraction $1 - x$ of loans in each location to return nothing. Depending on the locations each bank serves, these defaults may make some banks go bankrupt. Auditors audit bankrupt banks while solvent banks pay returns on deposits and equity. Because there is no aggregate uncertainty, the fraction of insolvent banks at the end is known at all times. Thus, since the beginning of this timing, agents know the amount of deposits to be used to pay for auditors.
Notice $\delta L$ appears both in the asset and liability sides of the balance sheet not because the bank is intermediating between depositors (sellers) and borrowers (buyers). In fact, when the loans were created, buyers appeared on both sides of the bank’s balance. It is the loan creation to make payments what causes different agents to be on both sides of the bank’s balance sheet. It is in this sense that loans create deposits and not the other way around. Also notice interbank loans and loans to buyers are very different objects. Interbank loans are done with reserves, which are supplied by the central bank. Thus, an interbank loan does not change the size of the balance sheet of the bank. It just increases an asset category (loans in the interbank market) at the expense of the reduction in another category (reserves). In contrast, loans to agents are produced by creating deposits and, therefore, change the size of the balance sheet by increasing both an asset (loans to buyers) and a liability category (deposits). Finally, notice reserve holdings are used to face liquidity risks from payments while equity is used to deal with solvency risks. These two risks are related through the total amount of loans, $\delta L$, which is a decision of banks. In solving these problems, the bank will first decide on the amount of loans, $L$, considering these risks, and then will work out its implications for reserve demand. Because all banks are ex ante identical, they will all demand the same amount of reserves. After the liquidity shock realizes, some banks will be net recipients of funds and will make them available in the interbank market while other banks will face net outflows of funds and will demand them in the interbank market.

3.2 The liquidity problem of banks

3.2.1 Net worth

As seen above, there are a number $\beta$ of exante identical banks evenly distributed around each island. By the time one of these banks starts functioning, it has collected a total of $\delta Q$ of assets from investors in the form of equity. Banks take these funding choices as given. At this point, the bank makes a total of $\delta L$ loans to the buyers on the locations that bank serves. As seen above, a random fraction $x$ of these loans will be payed back to the bank. Absent any other uncertainty, conditional on the realization of $x$, the expost net worth of a solvent bank, would be

$$x(1 + i^l)\delta L + \delta Q - (1 + i^d)\delta L \geq 0,$$

where $i^l$ is the interest rate on loans, and $i^d$ is the interest rate on deposits.

Providing loans and deposits on the amount $\delta L$ has implications for liquidity, though, which will also affect expost net worth. These loans $\delta L$ are initially held by buyers as deposits to be transferred to the sellers they buy stuff from. If all sellers lived in the very same section of the circle served by the bank providing loans to the buyers, the transfer of deposits between buyers and sellers will just be a renaming of these bank’s liabilities. However, with an islandwide market, buyers can buy goods or servicess from any part of the circle. Because there is a continuum of locations in each island, under the Law of Large Numbers,
on average, a fraction $\delta$ of those purchases will live in the section of the circle served by the bank and the other fraction $(1 - \delta)$ will live on outside locations served by the other banks. This means, when transferring deposits to pay for expenditures, funds equal to $(1 - \delta)\delta L$ will leave the bank. This is, a fraction $(1 - \delta)$ of the total payments, $\delta L$. On the other hand, buyers in the locations not served by the bank will probably buy goods and services from sellers whose accounts are in that bank. Buyers in those outside locations will go to the other banks and ask for a loan of an average size $\hat{b}$ to pay for their purchases (so their total bill is $(1 - \delta)\hat{L}$). Then, a fraction $\delta$ of those transfers will end up in our segment of the circle. This means an inflow of funds to the bank equal to $\delta(1 - \delta)\hat{L}$.

Thus, when considering its liquidity needs, the bank foresees it will have a total outflow of funds equal to $(1 - \delta)\delta L$ and a total inflow of funds equal to $\delta(1 - \delta)\hat{L}$. The problem is that, apart from these deterministic payments, banks face uncertain fund flows, as described in Assumption 4. This assumption supposes that this uncertainty is governed by the random variable $\varepsilon$. With this in mind, the average change in liquidity for the bank would be

$$FF(\varepsilon) = \delta(1 - \delta) \left(\hat{L} - L\right) - \varepsilon \delta L.$$  \hspace{1cm} (2)

This change in liquidity is assumed to be permanent from the point of view of the bank. As mentioned above, banks deal with these changes in reserves obtaining $M$ units of reserves at the central bank’s OMO at the interest rate $i^0$. Thus, the ex post net worth of the bank (once both $x$ and $\varepsilon$ are realized) taking into account liquidity risks and reserve demand from the OMO would be

$$x \left(1 + \hat{i}^0\right) \delta L + \delta Q - \hat{i}^0 \hat{M} + FF(x) - \left(1 + \hat{i}^0\right) \left[\delta L + FF(x)\right] \geq 0.$$ \hspace{1cm} (3)

Furthermore, assume banks have to satisfy a reserve requirement of $\rho \delta L$, where $0 < \rho < 1$.

Then, the average reserve position of the bank would be

$$M - \rho \delta L + \delta(1 - \delta) \left(\hat{L} - L\right) - \varepsilon \delta L.$$ \hspace{1cm} (4)

It is this position what the central bank checks to see whether the bank has fulfilled its reserve obligation. By inspecting (4) one can see that, given the OMO allotment, $M$, the reserve requirement, $\rho \delta L$, and the planned loan and deposit productions, $L$ and $\hat{L}$, the larger the realization of $\varepsilon$ the more likely is the average reserve position to be negative. In particular, for

$$-\infty \leq \varepsilon \leq \varepsilon \equiv \frac{M + \delta(1 - \delta) \left(\hat{L} - L\right)}{\delta L} - \rho$$ \hspace{1cm} (5)

11 Notice in equilibrium $L$ and $\hat{L}$ will be equal as all banks are identical. However, our individual bank takes $\hat{L}$ as given when making decisions.

12 Notice I am assuming reserve requirements to be contemporaneous to deposit creation. On the contrary, typical central bank regulations define these requirements as lagged with respect to deposits. To include this feature, I would need a dynamic model. However, as discussed in section 4.2 this should not affect the main results of the paper.
the average reserve position will be positive while it will be negative for
\[ \mathcal{E}_0 < \varepsilon \leq \infty. \]  

(6)

Assume the central bank offers two permanent standing facilities. First, there is a marginal lending facility in which commercial banks can obtain as much liquidity as needed at a penalty rate \( i^{LF} > i^o \).\(^{13}\) Second, there is a deposit facility in which commercial banks can place any excess liquidity remunerated at the rate \( i^{DF} < i^o \).\(^{14}\) Now assume an economywide interbank market, connecting all islands, opens right after the uncertainty about \( \varepsilon \) is resolved but before the realization on \( x \) is known. In this market, banks with excess reserves can provide liquidity to banks with liquidity deficits. Let \( I(\varepsilon) \) be the lending of the bank in the interbank market (borrowing if negative) for a particular realization of \( \varepsilon \). Clearly, under no uncertainty about the liquidity position at the end of period 0, which is the case here once the value of \( \varepsilon \) is known, if the return on lending in the interbank market is larger than the rate of the deposit facility, \( i^{DF} \), and the cost of borrowing in the interbank market is smaller than the rate of the marginal lending facility, \( i^{LF} \), it must be the case that the supply of funds in the interbank market will equal (4), that is,

\[ I(\varepsilon) = M - \rho \delta L + \delta (1 - \delta) (\hat{L} - L) - \varepsilon \delta L. \]  

(7)

Notice a particular bank in a particular island will be a lender or a borrower in the interbank market depending on its particular realization of \( \varepsilon \) as compared with its threshold (5). With the continuum of banks producing different realizations for \( \varepsilon \), there will be a continuum of liquidity positions within the interbank market. This is the purpose of modeling the physical setup of the economy with a continuum of islands.

Because there are solvency risks in this economy and because loans in the interbank market are unsecured, a bank demanding reserves will have to pay a spread or risk premium, call it \( s > 0 \), to lenders. With this premium, lenders are sure to obtain on average a rate \( i \) on that interbank loan. This premium will be computed below once the solvency risks are determined. Thus, the condition for trade in the interbank market to happen is \( i^{DF} < i < i + s < i^{LF} \).

The implications for net worth of accessing the interbank market will depend on the reserve position of the bank which itself will depend upon the realization on \( \varepsilon \). For \( \varepsilon \in (-\infty, \mathcal{E}_0] \) the bank will have excess reserves that can be loaned out in the interbank market obtaining a rate \( i \) while for \( \varepsilon \in \mathcal{E}_0, \infty \) the bank will have a reserve deficit that will be borrowed from the interbank market at the

\(^{13}\)Actually, when in place, commercial banks can borrow from marginal lending facilities against the presentation of eligible collateral as they do with the OMO.

\(^{14}\)Nowadays many central banks offer explicitly such facilities with the aim of setting a corridor for overnight rates. Such corridors do also implicitly exist whenever these facilities are not provided. In such cases, the rate of the discount window (plus a valuation of any nonpecuniary costs associated with its use) can provide an upper bound (corresponding to the marginal lending rate) while zero provides the lower bound (corresponding to the deposit rate).
rate $i + s$. Then ex post net worth for a solvent bank will equal

$$x \left(1 + i^d\right) \delta L + \delta Q + FF(\varepsilon) - (1 + i^d) \left[\delta L + FF(\varepsilon)\right]$$

$$-i^o M + i^p \rho \delta L + i I(\varepsilon; \varepsilon \leq \varepsilon) + (i + s) I(\varepsilon; \varepsilon > \varepsilon) \geq 0.$$  

where $I(\varepsilon)$ is displayed in (7). Notice the expression includes the possibility that the central bank remunerates required reserves at the rate $i^o$.

For a given realization of the fraction of entrepreneurs paying back the loan, $x$, the expected net worth with respect to the liquidity shock, $\varepsilon$, would be

$$x \left(1 + i^d\right) \delta L + \delta Q - (1 + i^d) \delta L + \Pi(L)$$

where $\Pi(L)$ represents the optimized expected profits from liquidity management which only depend on the amount of loans produced by the bank, $L$, and equals

$$\Pi(L) = \max_M - i^d \delta (1 - \delta) \left(\hat{L} - L\right) - i^o M + i^p \rho \delta L$$

$$+ i \int_{-\infty}^{\varepsilon} I(\varepsilon) d\Psi(\varepsilon) + (i + s) \int_{\varepsilon}^{\infty} I(\varepsilon) d\Psi(\varepsilon),$$

where $I(\varepsilon)$ is described in (7).

This setup is convenient as it separates the liquidity problem from the solvency problem in the following way. Given a choice of loan provision by the bank, $L$, problem (8) determines the reserve demand at the central bank’s OMO together with the supply of funds in the interbank market. Once this problem is solved, $\Pi(L)$ summarizes the liquidity implications on the net worth of the bank streaming from its loan provision. The bank will then choose its loan supply in order to maximize expected net worth. These two steps are followed in the next two sections.

3.2.2 Solving the liquidity problem

Taking first order conditions with respect to $M$ in (8) implies the condition

$$i^o = i + s [1 - \Psi(\varepsilon)].$$

The intuition behind this expression is simple. The bank will demand reserves at the OMO up to the point in which the marginal cost of borrowing from the central bank ($i^o$) equals the expected marginal value from using these reserves later on in the period. This expected value is the interbank lending rate, $i$, if the bank has excess reserves, which happens with probability $\Psi(\varepsilon)$, and is the interbank borrowing rate, $i + s$, if the bank has a reserve deficit, which happens with probability $1 - \Psi(\varepsilon)$. Then, using (5) produces the following demand for reserves at the OMO

$$M = \rho \delta L - \delta (1 - \delta) \left(\hat{L} - L\right) + \Psi^{-1} \left(\frac{i + s - i^o}{s}\right) \delta L.$$  

(9)
where \( \Psi^{-1}(\cdot) \) represents the inverse of the distribution function of the liquidity shock. Notice this expression determines reserve demand to be decreasing in the OMO rate, \( i^o \), and increasing in the interbank rate, \( i \), and the risk premium, \( s \). Substituting this demand in (7) yields the supply of funds in the interbank market

\[
I(\varepsilon) = (\underline{\varepsilon} - \varepsilon) \delta L.
\]

(10)

Expressions (9) and (10) make evident the role of reserve requirements in the liquidity management problem of the bank. These requirements are just incorporated deterministically in the reserve demand at the OMO and do not directly affect the supply of funds in the interbank market. Substituting these expressions in the objective function (8) yields the expected profits from liquidity management which now is only a function of the amount of loans the bank has decided to extend

\[
\Pi(L) = (i^o - i^d)\delta(1 - \delta) \left( \hat{L} - L \right) - (i^o - i^r)\rho \delta L - s\delta L \int_{\underline{\varepsilon}}^{\infty} \varepsilon d\Psi(\varepsilon).
\]

(11)

Notice these liquidity management profits are a linear function of loan production, \( L \), which means marginal profits per additional loan produced

\[
\frac{\partial \Pi(L)}{\partial L} = -(i^o - i^d)\delta(1 - \delta) - (i^o - i^r)\rho \delta - s \int_{\underline{\varepsilon}}^{\infty} \varepsilon d\Psi(\varepsilon),
\]

are a constant. Also notice the role reserve requirements, \( \rho \), play in the determination of these profits. In particular, assuming central banks do not remunerate required reserves at a rate larger than their refinancing rate, \( i^r \leq i^o \), required reserves have a negative impact on marginal liquidity profits. There are two ways to drive this effect to zero, though, either by eliminating required reserves, \( \rho = 0 \), or by equating the remuneration of required reserves to the refinancing rate of the central bank, \( i^r = i^o \).

3.3 Closing the model

3.3.1 Solving the solvency problem

The previous section makes it clear that if required reserves are remunerated at the same rate as the refinancing operations of the central bank, so that \( i^o = i^r \), reserve requirements disappear from the problem and, therefore, cannot affect loan and deposit provision by commercial banks. Therefore, unless there is another channel in which reserve requirements affect allocations, one can argue, without modelling the rest of the economy, that the level of reserve requirements is completely innocuous. With this result, the main point of the paper is made. However, it may be the case that a central bank is not able or is unwilling to remunerate required reserves at that level. For that reason, this section solves the model for the general case and later a calibration will be presented in which the effects of changing reserve requirements are computed.
In this economy, the fraction of loans that are returned to the bank, \(x \in [0, 1]\), is a random variable with marginal distribution denoted by \(\Xi(x)\), corresponding density \(\xi(x)\) and mean \(\mu\). Since all buyers served by this very bank are exante identical, they will all ask for the same loan amount and the nonperforming loan (NPL) ratio of that bank will be \(1 - x\), which, of course, is also random. When providing loans, banks need to satisfy the capital requirement

\[
\delta Q \geq \kappa \delta L,
\]

where \(0 < \kappa < 1\) is a parameter determined by regulation.

As seen above, conditional on the realization of \(x\), the expected net worth of a solvent bank from the point of view of deciding the amount of loans to be produced, \(L\), would be

\[
x (1 + i^d) \delta L - (1 + i^d) \delta L \geq 0
\]

where \(i^d\) is the interest rate on loans, \(i^d\) is the interest rate on deposits and \(\Pi(L)\) is the expected profits from liquidity management determined by (11). Thus, the bank will be solvent as long as the loan recovery rate satisfies

\[
x \geq \frac{(1 + i^d) \delta L - \delta Q - \Pi(L)}{(1 + i^d) \delta L} \equiv \underline{x}.
\]  

Expression (12) defines the minimum loan recovery rate, \(\underline{x}\), to ensure solvency of the bank. As all banks start identical and face the same prices, the fraction of insolvent banks will be \(\Xi(\underline{x})\).

Then, the expected net worth, per location, of a solvent bank will be

\[
\int_{\underline{x}}^{1} [x(1 + i^d) \delta L + \delta Q + \Pi(L) - (1 + i^d) \delta L] \, d\Xi(x).
\]

Using the definition of \(\underline{x}\) in (12), the expected net worth becomes

\[
(1 + i^d) \delta L \left( \int_{\underline{x}}^{1} x d\Xi(x) - [1 - \Xi(\underline{x})] \underline{x} \right) \equiv (1 + i^d) \delta L \left[ \mu - F(\underline{x}) \right],
\]

where \(\mu\) is the mean of the distribution \(\Xi(x)\),

\[
F(\underline{x}) = \underline{x} [1 - \Xi(\underline{x})] + G(\underline{x}),
\]

and

\[
G(x) = \int_{0}^{\underline{x}} xd\Xi(x).
\]

As in Bernanke et al. [4], \(0 \leq F(\underline{x}) \leq 1\). Furthermore,

\[
F'(x) = 1 - \Xi(x) > 0; F''(x) = -\xi(x) < 0
\]

\[
\lim_{\underline{x} \to 0} F(\underline{x}) = 0; \lim_{\underline{x} \to 1} F(\underline{x}) = \mu
\]
Depositors will get back their whole deposits only if the bank serving their location is solvent. Thus, they face default risk. As described before, because depositors are risk averse, a deposit insurance (DI) scheme is introduced in the economy. This DI insures the face value of deposits (not the interest) and is the one negotiating the terms of the deposit contracts with the banks. The details of the relation between the DI and the depositors will be spelled out below. When bargaining the terms of the deposit contract, I have assumed that the realization of $x$ is private information of investors (as equity holders) and banks and not known either by the DI or the depositors (workers). The DI can learn the true value of $x$ by hiring auditors. The nominal cost of the audit is born by the depositors and is equal to a proportion $0 < \phi < 1$ of the amount to be audited, that is, $\phi x (1 + i') L$. This cost is paid for with deposits. As in Carlstrom and Fuerst [7] or Bernanke et al. [4], this information asymmetry creates a moral hazard problem as banks have incentives to misreport the fraction of loans defaulting. The optimal financial contract should be structured so as to induce the bankers to truthfully report the realization of $x$. Carlstrom and Fuerst [7] and Bernanke et al. [4] use the findings of Gale and Hellwig [21], Townsend [39] and Williamson [44] to show that in this setup the optimal contract has the form of risky debt. Then, the expected revenues for the DI, per location, will be

$$
\int_0^x \left[ (1 - \phi)x(1 + i')L + \delta Q + \Pi(L) \right] d\Xi(x) + [1 - \Xi(x)] (1 + i')L.
$$

Again, using the definition of $x$, these revenues become

$$
\delta Q + \Pi(L) + (1 + i')L \left[ F(x) - \phi G(x) \right].
$$

In this case, notice that

$$
\lim_{x \to 0} [F(x) - \phi G(x)] = 0; \lim_{x \to 1} [F(x) - \phi G(x)] = \mu(1 - \phi)
$$

and

$$
F'(x) - \phi G'(x) = 1 - \Xi(x) - \phi g(x) = [1 - \Xi(x)][1 - \phi h(x)]
$$

with $h(x)$ defined as

$$
h(x) = \frac{xg(x)}{1 - \Xi(x)}.
$$

Thus, $\mu - F(x)$ represents the fraction of expected bank revenues going to equity holders, $F(x) - \phi G(x)$ represents the fraction of expected bank revenues going to depositors and $\phi G(x)$ represents the fraction of expected bank revenues remunerating auditors. Notice that

$$
[\mu - F(x)] + [F(x) - \phi G(x)] + \phi G(x) = \mu,
$$

i.e. the expected fraction of loans being repaid.
As locations are ex ante identical, we can define the optimization problem of each bank for a representative location among the ones it serves. Banks take \( \varphi, \vartheta, \varphi, \vartheta, \varbeta \) as given and choose loan supply, \( L \), and the threshold value, \( \underline{x} \), to maximize the return for shareholders (investors) over their opportunity cost

\[
\frac{(1 + \underline{i}) L [\mu - F(\underline{x})]}{(1 + \underline{i}) Q},
\]

subject to the participation constraint for depositors

\[
\delta Q + \Pi(L) + (1 + \underline{i})\delta L [F(\underline{x}) - \phi G(\underline{x})] \geq (1 + \underline{i})\delta L,
\]

and the capital requirement

\[
Q \geq \kappa L.
\]

Here \( \underline{i} \) is the nominal risk free rate which I take as exogenous and given from the point of view of both banks and the DI. Notice that I am assuming that rate to be the opportunity cost of both, equity deposits. Because investors are diversifying their equity holdings across all banks in the economy they will not be facing any risk from this investment.\(^{15}\)

When solving this maximization problem, I will not include the constraint (14). After solving the problem I will check whether this constraint binds or not. Let \( \lambda \) be the leverage of the bank defined as

\[
\lambda = \frac{L}{Q}.
\]

Recalling the expression for profits from liquidity management (11), these profits are

\[
\Pi(L) = (\varphi - \varphi)(1 - \delta) \left( \underline{L} - L \right) - (\varphi - \varphi)\rho \delta L - s \delta L \int_{-\infty}^{\infty} \varepsilon d\Psi(\varepsilon).
\]

With this in mind, the problem of the bank can be rewritten as choosing leverage \( \lambda \) and the threshold \( \underline{x} \) to maximize the equity premium

\[
\left( \frac{1 + \underline{i}}{1 + \underline{i}} \right) [\mu - F(\underline{x})] \lambda,
\]

subject to the participation constraint for the DI

\[
(1 + \underline{i})\lambda - (1 + \underline{i}) (F(\underline{x}) - \phi G(\underline{x})) \lambda = 1 + \pi(\lambda),
\]

where

\[
\pi(\lambda) = (\varphi - \varphi)(1 - \delta) \left( \frac{\underline{L}}{Q} - \lambda \right) - (\varphi - \varphi)\rho \lambda - s \lambda \int_{-\infty}^{\infty} \varepsilon d\Psi(\varepsilon),
\]

\(^{15}\) Although in the model the risk free rate \( \underline{i} \) and the official rate \( \varphi \) are independent they could be made to depend on each other.
are liquidity management profits per unit of bank equity. Notice these liquidity management profits are a linear function of leverage, $\lambda$, which means marginal profits per additional loan produced equal

$$\frac{d\pi(\lambda)}{d\lambda} = -(i^o - i^d)(1 - \delta) - (i^o - i^d)\rho - s \int_{-\infty}^{\mu} \varepsilon d\Psi(\varepsilon).$$ \hspace{1cm} (18)

Let $\eta$ be the Lagrange multiplier associated with (16). The first order conditions determining bank leverage, $\lambda$, the bank solvency threshold level, $\mu$, and the Lagrange multiplier, $\eta$, are, respectively,

$$\left(1 + i^d \over 1 + i^f \right) [\mu - F(\mu)] = \eta \left[1 + i^f - \frac{d\pi(\lambda)}{d\lambda} - (1 + i^f) (F(\mu) - \phi G(\mu))\right]$$ \hspace{1cm} (19)

$$F'(\mu) = \eta (1 + i^f) [F'(\mu) - \phi G'(\mu)],$$ \hspace{1cm} (20)

and

$$\left(1 + i^f \right)\lambda = \frac{1 + \pi(\lambda)}{1 - \left(1 + i^f \over 1 + i^f \right) [F(\mu) - \phi G(\mu)]},$$ \hspace{1cm} (21)

As shown in the Appendix, this system defines a positive relationship between the cutoff value, $\mu$, bank leverage, $\lambda$, the lagrange multiplier, $\eta$, and the loan rate, $i^f$, for a given level of the official rates, $i^o$ and $i^o$. Furthermore, this relationship determines, for a given level of initial equity, $Q$, a positively sloped loan supply function

$$L^* = Q \times \lambda.$$

Larger loan rates $i^f$ induce banks to increase leverage through providing more loans to managers. Furthermore, it can be shown (see the Appendix) that the deposit rate is

$$i^d = i^f + \left(1 + i^f \right) \left[\mu - F(\mu) + \phi G(\mu)\right].$$

Also, the capital constraint (14) imposes an upper bound on leverage

$$\lambda \leq \frac{1}{\kappa},$$ \hspace{1cm} (22)

and, therefore, for a given level of equity $Q$, there is a limit on the amount of loans banks can provide, $L$. Notice this leverage bound is common to all banks. Because $\lambda$ is a choice of each bank, and because the agency problem between banks and the DI, limiting bank leverage, the constraint (14), or its equivalent (22), may not be binding.

Finally, given that the default probability of a bank is $\Xi(\mu)$, the spread paid by these banks in case they borrow in the interbank market should satisfy

$$1 + i = [1 - \Xi(\mu)] (1 + i + s)$$

or

$$s = \frac{\Xi(\mu)}{1 - \Xi(\mu)} (1 + i).$$ \hspace{1cm} (23)
3.3.2 Investors

As mentioned above, in the economy there is a measure 1 of identical, risk neutral investors. These investors are not assigned to any island in particular. They start period 0 with nominal assets \( A \), which are distributed between bank equity, \( Q \), earning a gross nominal return \( 1 + \vartheta \), or the risk free investment, \( B \), paying interest \( i^f \), i.e.

\[
Q + B \leq A.
\]

In general, total equity is distributed equally across all banks in the economy. Being risk neutral, investors make these portfolio decisions to maximize their net worth at period 1, \( A' \), that is,

\[
A' = (1 + \vartheta)Q + (1 + i^f)B.
\]

Because investors diversify across all banks and islands, the total return on bank equity is deterministic even though some of the banks are insolvent along the way.

3.3.3 The deposit insurance scheme

The DI stands between banks and households, negotiating the terms of the deposit contract with banks and providing insurance to depositors. The DI scheme is as follows. The DI insures the face value of deposits. It collects all deposit revenues from all banks and returns \( L \) to each depositor. Any excess of the deposit revenues from banks with respect to the insurance claims from depositors, after monitoring costs are payed, is transferred to all agents in the economy, independently of whether they are buyers or sellers, in a lump sum fashion. Let this nominal lump sum transfer be \( T \).

On the one hand, notice the DI as well as all agents know how much the auditors will cost in terms of deposits. As mentioned above, the total bill to pay auditors is

\[
\phi(1 + i^f)LG(x).
\]

On the other hand, all agents in any location receive from the DI an aggregate revenue of

\[
Q + \frac{\Pi(L)}{\delta} + (1 + i^f)L\left[F(x) - \phi G(\underline{x})\right],
\]

of which \( L \) is the deposit insurance, to be paid to sellers, and

\[
T = Q(\delta) + \frac{\Pi(L)}{\delta} + (1 + i^f)L\left[F(x) - \phi G(\underline{x})\right] - L
\]

is the lump sum interest transfer to be paid to all agents, sellers and buyers per location.
3.3.4 The central bank

The central bank is in charge of setting the policy rates, $i^o$, $i^{DF}$, $i^{LF}$, and $i^p$. It also determines regulation such as the capital to loan ratio, $\kappa$, and reserve requirement, $\rho$. I take these rates as parameters and look at how the equilibrium change when the rate of the OMO, $i^o$, and the remuneration of required reserves, $i^p$, move together with reserve requirements, $\rho$. As mentioned above, the rates of the deposit and lending facilities satisfy

$$i^{DF} \leq i < i + s \leq i^{LF}$$

where $i$ is the rate in the interbank market and $s$ is the borrower risk premium. Thus, I assume commercial banks do not use the standing facilities. I also assume the central bank is ready to supply all reserves banks demand at the official rates. Seigniorage by the central bank will then equal (per bank)

$$S(i^o, i^p) = i^o M - i^p \rho \delta L.$$

4 Equilibrium

4.1 Characterization

As mentioned above, on each island there are $\beta \geq 2$ identical banks in the economy with sizes $0 < \delta < 1/2$. To characterize the equilibrium of this economy, start with investors. These agents split their nominal assets, $\mathbb{F}$, between equity in all banks in the economy. As there is a measure 1 of locations served by these banks, $Q = A$.

For all banks to be financed through equity, and therefore function, it must be the case that investors should receive the same expected return on all types of equity. Furthermore, competition in the market for bank equity will drive down the return on equity to its opportunity cost, $i^f$. Then,

$$\left( \frac{1 + i^f}{1 + i^f} \right) [\mu - F(z)] \lambda = 1. \quad (26)$$

This condition, together with the FOCs of the banks’ solvency problem (19) through (21) provide with the equilibrium loan recovery threshold, $z$, leverage $\lambda$, loan rate $i^f$ and lagrange multiplier, $\eta$, for each bank

$$\left( \frac{1 + i^f}{1 + i^f} \right) [\mu - F(z)] = \eta \left[ 1 + i^f - \frac{d\pi(\lambda)}{d\lambda} - (1 + i^f) (F(z) - \phi G(z)) \right] = 0 \quad \text{(27)}$$

and

$$F'(z) = \eta (1 + i^f) [F'(z) - \phi G'(z)], \quad \text{(28)}$$

and

$$(1 + i^f) \lambda = \frac{1 + \pi(\lambda)}{1 - \left( \frac{1 + i^f}{1 + i^f} \right) [F(z) - \phi G(z)]}. \quad \text{(29)}$$
Leverage $\lambda$ together with bank equity $Q$ generate the supply of loans to be produced in each of the locations served by each bank

$$L^* = \lambda \times Q.$$ 

Additionally, deposit rates $i^d$ are computed from

$$i^d = i^f + (1 + \delta) \left[ \bar{\delta} - F(\bar{\delta}) + \phi G(\bar{\delta}) \right].$$

Because there is a continuum of islands, and because the liquidity shock $\varepsilon$ is idiosyncratic to each bank and island, equilibrium in the interbank market implies

$$\int_{-\infty}^{\infty} I(\varepsilon) d\Psi(\varepsilon) = 0,$$

where, from (10)

$$I(\varepsilon) = (\bar{\varepsilon} - \varepsilon) \delta L.$$

Market clearing implies

$$\bar{\varepsilon} = 0,$$

which determines the equilibrium interbank rate

$$i = i^o - s \Psi(0) = i^o - \frac{s}{2}. \quad (30)$$

Using (23), the risk spread equals

$$s = \frac{2\Xi(x)}{2 - \Xi(x)} (1 + i^o). \quad (31)$$

Substituting back this rate in (10) produces the net positions in the interbank market:

$$I(\varepsilon) = -\varepsilon \delta L. \quad (32)$$

Notice that although the interbank rate is below the official rate, $i < i^o$, see (30), the rate paid by borrowers, which includes the risk premium, exceeds the official rate, i.e.,

$$i + s = i^o + s [1 - \Psi(0)] > i^o.$$

Furthermore, substituting the equilibrium rates in (9) and the fact that all banks are identical ($\tilde{L} = L$), produces the equilibrium bids at the OMO by banks:

$$M = \rho \delta L, \quad (33)$$

that is, the reserve requirement. At equilibrium, average nominal profits from liquidity management become (see (11) evaluated at the equilibrium interbank rate (30) and risk premium (31), with $\tilde{L} = L$),

$$\Pi(L) = -(i^o - i^o) \rho L - s \delta L \int_{0}^{\infty} \varepsilon d\Psi(\varepsilon).$$

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The first term corresponds to seigniorage by the central bank and the second to the losses associated with banks being bankrupt. Thus, of all nominal revenue generated in this economy, 
\[(1+i)\left[\mu - F(\xi)\right] L = (1+i)Q\] 
remunerates investors, \[Q + \Pi(L) + (1+i)L[F(\xi) - \phi G(\xi)]\] is obtained by risk averse agents, \((1+i)L\phi G(\xi)\) is earned by workers working as auditors, 
\[-(i^o - i^o)\rho L\] is collected by the central bank as seigniorage, and 
\[\delta(1 - \delta)\frac{\Xi(\xi)}{2 - \Xi(\xi)}(1+i)\] 
is lost from either the loan or interbank market due to failures by entrepreneurs.

Equilibrium allocations and prices depend on the givens of the problem, namely, initial assets from investors, \(A\), the monetary policy parameters, \(\delta, \lambda,\) and \(\rho\), and the risk free rate, \(i^f\). Notice changes in investors’ initial assets, \(A\), only translate into the amount of liquidity provision \(L\) without affecting loan rates or any other variable in the model. On the contrary, in general, monetary policy parameters will affect general monetary conditions such as rates as they influence the solvency and/or the liquidity problem of banks.

4.2 Discussion of extensions

The characterization of the equilibrium shows how reserve requirements affect the provision of liquidity at the two layers described in section 2, namely, reserves, used by monetary financial institutions, and deposits, used by the non-financial sector. Regarding reserves, reserve requirements are translated directly into the bid at the OMO by all banks, \(M\), and do not have a direct impact on the interbank positions of banks, \(I(\xi)\). These interbank positions are only affected indirectly by reserve requirements to the extent that these requirements modify deposit and loan creation, \(L\), which by themselves influence reserve demand. Clearly, if excess reserves were to be remunerated instead, the central bank could change the liquidity management of the bank and will be able to move the overall demand for reserves. It is in this sense that excess reserve remuneration helps with the separation of prices and quantities in monetary policy but only remunerating required reserves does no do the job.

Regarding the second liquidity layer, deposit and loan creation, \(L\), reserve requirements only affect it through its impact on profits from liquidity management, \(\pi(\lambda)\). These profits feed into the FOCs of the solvency problem faced by banks (expressions (27) through (29) above) and affect leverage, \(\lambda\), the solvency ratio of banks, \(\Xi(\xi)\), and the lending rate, \(i^f\). This effect is cancelled, though, either if there are no reserve requirements, \(\rho = 0\), or else, if required reserves are remunerated at the rate of the OMO, \(i^o = i^f\). These two situations are different in terms of equilibrium allocations though. Not imposing reserve requirements, \(\rho = 0\), supports a bank run equilibrium in which, fearing bank insolvency, agents rush to the bank to withdraw their deposits. This situation could be resolved by imposing a deposit insurance scheme as it was done in the model. Notice this DI scheme could be financed by taxes on deposit rates. On the contrary, if
required reserves are remunerated at the OMO rate, \( i^p = i^o \), there will never be a run on a bank, independently of its solvency and, therefore, a DI scheme will no longer be needed.

Do these conclusion survive extensions of the model? First it is easy to see that the initial assumption of separating the islands in terms of the payment streams is completely innocuous. Even if agents are making payments across islands, banks will always demand at the OMO the deterministic part of these flows which includes reserve requirements. Thus, if these requirements are remunerated at the cost of the OMO, the result will still go through. This will also be true if we assumed banks to be risk averse instead of risk neutral. With risk aversion, banks will demand excess reserves for precautionary reasons associated with payment uncertainty but, again, still will demand the deterministic reserve requirements at the OMO. We could also dispose of the assumed market segmentation in banking payment services by allowing agents to deal with several banks as long as individual banks still faced solvency and liquidity risks associated with their loan and deposit provision.

Two possible caveats to the previous argument are related to the central bank. First, one problem of remunerating required reserves could be the elimination of one source of revenues for central banks. However, the fact that many central banks function nowadays without reserve requirements, and, therefore, without income of this sort, suggests that this does not seem a major concern for their operations. In general, that income could be brought about through demand for excess reserves which have nothing to do with reserve requirements. As a second qualification, although by remunerating required reserves central banks could reduce to zero the implicit tax associated with reserve requirements, there is a channel by which these requirements can affect loan provision by depository institutions. As a whole, the banking system needs collateral to borrow reserves from the central bank. Imposing a 100 percent reserve requirement would call for the banking system to maintain eligible assets in an amount at least equal to their loan and deposit production. Because the overall supply of assets accepted as collateral is exogenous to the banking system, current collateral regulations could effectively constrain the implementation of narrow banking. The obvious way out from this situation would be to reform these collateral regulations as it has been done by the Eurosystem, the Fed and other central banks during the crisis. One possibility is to securitize bank loans after checking on compliance with some minimum standards and to use these ABS as collateral for the central bank’s refinancing operations.

The model is intentionally fuzzy about what the payment streams among agents in each island were so it seems hard to grasp what the consequences different reserve requirements could have on real outcomes and welfare. We could think these payments to be wage bills to be payed in advance of workers providing labor services, or payments for consumption durables or for schooling. At the end of the day, if there is a connection between the banking and the real sectors this has to go either through the amount, \( L \), and/or the price, \( i^l \) and \( i^d \), of the financial intermediation. To the extent that an institution such as reserve requirements does not change the terms of this financing it will be very hard
to argue that it will have an impact on real outcomes even without specifying what these banks finance.

Finally, the model is static. Arguably, this may not seem an adequate assumption as dynamics plague the financial problem of banks. On the one hand, reserve requirements are usually lagged and not contemporaneous as is assumed here. This should not be a problem, since banks will anticipate the costs from future reserve requirements associated with current loan and deposit provision. Because these requirements are weekly or monthly at most, these future requirements will barely be discounted and expressions in the current static model will be very close to those of the dynamic model. Another issue has to do with the life of a loan and the deposits it creates. As banks are making and receiving payments, the deposits they will hold and the reserves they will have to demand to meet reserve requirements will be random during the whole maturity of the loan that created the original deposits. However, with 100 percent reserve requirements, deposits will always travel across banks with their own reserves so that these requirements will be always fulfilled at zero cost. The only significant cost is the one born by the bank originating the initial loan and deposits. As discussed this cost could be driven to zero if $i_p = i^o$.

5 Simulation

5.1 Calibration

The previous sections have discussed the extent to which reserve requirements affect bank liquidity provision to the nonfinancial sector. In particular, it has been shown that reserve requirements should not have any effect on real outcomes if they are remunerated at the refinancing rate of the central bank. However, with the exception of the Eurosystem, the Federal Reserve and a few other cases, the majority of central banks do not remunerate required reserves (see Gray [25]). In this section I calibrate the model to assess quantitatively the real effects of imposing unremented required reserves.

The model includes two parameters, the size of the average bank, $\delta$, and the cost of bank monitoring, $\phi$. Furthermore, there are two probability distributions, namely, the distribution of the performing loan ratio for the average bank, $\Xi(x)$, and the distribution of liquidity shocks, $\Psi(\varepsilon)$. Finally, there are 3 policy rates (the refinancing rate of the central bank, $i^r$, the remuneration of required reserves, $i^p$, and the risk-free rate, $i^f$) together with two regulation parameters (the reserve ratio, $\rho$, and the capital requirement, $\kappa$).

To calibrate the size of the average bank and the distribution of the performing loan ratio, I use the database Statistics on Depository Institutions provided by the Federal Deposit Insurance Corporation (FDIC) and available at www.fdic.gov. This data is obtained from the Federal Financial Institution Examination Council (FFIEC) Consolidated Report of Condition and Income (also known as Call Reports) and the Office of Thrift Supervision (OTS) Thrift Financial Reports submitted by all FDIC-insured depository institutions. The
data set spans from the last quarter of 1992 until the second quarter of 2016 which represent 95 periods of data. This dataset includes information of the amount of loans and leases produced by each depository institution in the US together with their noncurrent loans and leases, defined as the fraction of loans and leases 90 days or more past due plus loans in nonaccrual status. The size of the average bank, $\delta$, is calibrated by the time average of the share of loans and leases by each bank. The resulting value is $\delta = 0.0001$ or 0.01 percent. To set the value of the cost of bank monitoring, $\phi$, I look at the value of losses from bank failures between 1992 and 2016 as a fraction of deposits. The data is provided by the FDIC. For failed banks in this period, the losses represented approximately 22 percent of deposits. Thus, I take $\phi = 0.22$.

On the other hand, data on noncurrent loans and leases is pooled for all banks and all periods to produce the empirical counterpart for the distribution of $x$, $\Xi(x)$. This distribution is then approximated by a lognormal truncated at $x = 1$. The approximation finds the mean and standard deviation of the lognormal to match the mean and standard deviation of nonperforming loan ratios in the sample. The resulting mean and standard deviation are $\mu_x = 5.860$, and $\sigma_x = 0.36$. To test that this theoretical distribution fits the empirical one, I run a Kolmogorof-Smirnof test. The test cannot reject the null hypothesis of equality of distributions at the 10 percent significant level.

The capital requirement is set to $\kappa = 0.08$ as it was required by the Basel I accord which was in place for most of the periods in the sample. In any case, in the computations done below this requirement is never binding as banks end up having larger capital ratios. The refinancing rate is approximated as the time average of the federal funds rate during the sample covered by the FDIC Call Reports. This average equals $i^r = 0.0278$. The risk-free rate is computed equivalently by using the 3-month T-bill rate which produces a value of $i^f = 0.0257$. Because the exercise is to compute the effects of unremunerated required reserves, this rate is set to $\rho = 0$ and the reserve requirement $\rho$ is set to vary between 0 and 100 percent.

Finally, the liquidity shock, $\varepsilon$, is assumed to follow a normal distribution, $\Psi(\varepsilon)$, with mean $\mu_\varepsilon = 0$. Its standard deviation is calibrated to $\sigma_\varepsilon = 0.22$ to match the average of federal funds trade as a fraction of total loans by banks. The FDIC Call Reports also include information about the net Federal Funds market position for each bank. I then compute the average of that position as a fraction of loan provision both for banks selling as well as borrowing funds in the market. The average is between 8.13 percent (for banks selling funds) and 8.64 percent (for banks borrowing funds).

### 5.2 Results

Figure 1 and 2 present information about leverage and interest rates produced by the model for different values for the reserve requirement ranging from $\rho = 0$ to $\rho = 1$. Figure 1 includes leverage. Moving from no reserve requirement

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16 The Appendix provides information about the data used in the calibration.
(which computationally is equivalent to remunerating required reserves at the refinancing rate of the central bank) to 100 percent reserve requirements reduces leverage by roughly 2 percent (from 8.70 to 8.54). Interestingly, these numbers are in line with data for the US where average leverage for the period considered equals 7.84 with a standard deviation of 0.54. As Figure 2 shows, this reduction in leverage is associated with a significant increase of lending rates, from 5.67 percent with $\rho = 0$ to 8.54 percent with $\rho = 1$. Because deposit rates are basically constant at around 2.59 percent, raising reserve requirements also mean increasing the spread between the two rates.

6 Concluding comments

This paper develops a new model of banking and payment systems to evaluate the benefits and costs of imposing a 100 percent reserve requirement to depository institutions. When developing the model, care has been taken in reproducing actual institutions present in our monetary systems. In particular, broad money (deposits) is created by commercial banks and used by the non-financial sector of the economy to finance real activity. At the same time, narrow money (reserves) is created by the central bank, used by commercial banks to face the net payments derived from the creation of broad money and exchanged between these banks through the interbank market. The connection between these two layers of liquidity (narrow and broad) is provided by the endogenous loan and deposit creation of commercial banks.

With this model in hand, the paper shows that, contrary to traditional banking models existing in the literature, a fully reserve-backed monetary system does not necessarily have to reduce the amount of liquidity produced by depository institutions. In particular, required reserves do not force banks to maintain deposited assets idle in an inferior investment. Required reserves only affect loan and deposit creation indirectly through it effect on the costs of liquidity provision. These costs could be driven to zero, and, therefore, isolate liquidity provision from the level of reserve requirements if required reserves were to be remunerated at the rate the central banks supply liquidity at its OMOs.

The model has been calibrated to assess the quantitative impact of changing reserve requirements whenever central banks do not remunerate required reserves. Moving from no reserve requirements to a fully backed depository system has significant effects on lending rates and, to some extent, to bank leverage.

The model has immediate implications regarding banking regulations and the role of the central bank as a lender of last resort. The first important implication has to do with the role of collateral. Because banks have to pledge collateral when demanding reserves at the central bank open market operation, for the narrow banking proposal to be implementable, depository institutions should maintain a stock of eligible assets at least equal to its deposit liabilities. Although this point was obviated in the model above by assuming refinancing
operations of the central bank to be unsecured, clearly, this could prove to be a binding constraint in deposit and liquidity production. This is because the stock of available collateral is beyond the control of the banking system and, therefore, could fall short of the amount needed to fully back deposits with reserves. One possible solution to this problem could be to expand the list of eligible collateral but that would mean making central banks take on more risk which they may not be willing to do even after applying the appropriate haircut. Another possibility is to use some government-sponsored enterprise to securitize bank loans after checking on compliance with some minimum standards and used these ABS as collateral for the open market operations.

If the collateral constraints described in the previous paragraph cannot be overcome in the aggregate, the economy must then rely on a fractional banking system. A second implication of the model is that, in such a case, as long as the central bank provides liquidity to needed banks, depositors do not have any incentive to coordinate in a bank run on solvent banks. However, liquidity provision by the central bank does not solve insolvency problems as long as the monetary authority is reluctant to take on losses of the assisted commercial banks. This means that whenever depositors cannot distinguish between solvent and insolvent banks, there exists the possibility of a bank run. Thus, with fractional reserves, a combination of a deposit insurance scheme, insuring deposits at face value and financed through taxing interests on deposits, to engage in the resolution of insolvent banks, and a central bank as a lender of last resort is needed to prevent bank runs in case depositors are not sure about which banks are solvent and which ones are not.

Third, it is not clear that the existence of a deposit insurance scheme introduces a moral hazard problem with respect to the situation without it, as Kareken and Wallace [28] or Freeman [18] have pointed out. The reason is that the DI acts as an intermediary between banks and depositors. However, from the point of view of the bank, its deposit liabilities have always the form of unsecured short term debt. The DI does not change that. It only changes the agent holding that debt. The issue here is whether the existence of this type of liabilities introduces a moral hazard problem as compared with other types of liabilities such as equity.

References


A The data used in the calibration

I have used several databases for the calibration and simulation exercises. First, I use the database Statistics on Depository Institutions provided by the Federal Deposit Insurance Corporation (FDIC) and available at www.fdic.gov. This data is obtained from the Federal Financial Institution Examination Council (FFIEC) Consolidated Report of Condition and Income (also known as Call Reports) and the Office of Thrift Supervision (OTS) Thrift Financial Reports submitted by all FDIC-insured depository institutions. The data set spans from the last quarter of 1992 until the second quarter of 2016 which represent 95 periods of data. From this dataset I have collected the following variables (acronyms in parenthesis):

- Total assets (asset). The sum of all assets owned by the institution including cash, loans, securities, bank premises and other assets. This total does not include off-balance-sheet accounts.
- Net loans and leases (lnlsnet). Total loans and lease financing receivables minus unearned income and loan loss allowances.
- Noncurrent loans and leases (nclnls). Assets past due 90 days or more, plus assets placed in nonaccrual status.
- Federal funds sold and reverse repurchase (frepo). Total federal funds sold and securities purchased under agreements to resell in domestic offices. Includes only federal Funds sold for TRF Reporters before March 1998.
- Federal funds purchased and repurchase agreements (frepp). Total federal funds purchased and securities sold under agreements to repurchase in domestic offices. Thrift Financial Reports include only federal funds purchased.

From the FDIC I also used bank failure data produced by the corresponding report in their webpage.

From the St. Louis Dataset FRED, I used the Effective Federal Funds Rate (FEDFUNDS) to approximate $i^n$ and the 3-month T-bill rate (TB3MS) to compute $i^t$.

B The Loan Supply Schedule

Here I show how to derive the properties of the loan supply schedule from expressions (19)-(21) in the text. For convenience, these expressions are reproduced here:

\[
\left( \frac{1 + i^t}{1 + i^f} \right) \left[ \mu - F(x) \right] = \eta \left[ 1 + i^f - \frac{d\pi(\lambda)}{d\lambda} - (1 + i^f) (F(x) - \phi G(x)) \right], \quad (34)
\]

\[
F'(x) = \eta (1 + i^f) [F'(x) - \phi G'(x)], \quad (35)
\]
(1 + i^f)\lambda = \frac{1 + \pi(\lambda)}{1 - \left(1 + \frac{i^n}{1 + i^f}\right)[F(x) - \phi G(x)]}.

The argument closely follows the derivations in Bernanke et al. [4]. First, notice that, for given rates \(i^n, i^f\) and \(i^l\),

\[ F'(x) - \phi G'(x) = 1 - \Xi(x) - \phi g\xi(x) = [1 - \Xi(x)][1 - \phi h(x)] \]

with

\[ h(x) = \frac{x\xi(x)}{1 - \Xi(x)} \]

being the hazard rate. By assumption (1.iv), \(h(x)\) is a strictly increasing function of \(x\). This means there must be a level of \(\bar{x}\), call it \(\bar{x}^*\), such that \(F(x) - \phi G(x)\) is maximum which implies that leverage, \(\lambda\), defined by (21) is also maximum. Thus, the relevant range to choose \(x\) from should be \(\bar{x} \in [0, \bar{x}^*]\). No bank will choose a higher cutoff level \(\bar{x}\) if, at the same leverage there is a lower level for \(\bar{x}\) such as depositors are indifferent between the two. This is because the lower level for \(\bar{x}\) saves on monitoring costs.

Use (20) to define the Lagrange multiplier \(\eta\) as a function of the cutoff level \(\bar{x}\):

\[ \eta(x) = \frac{F'(x)}{(1 + i^f)[F'(x) - \phi G'(x)]}. \]

Taking derivatives we have

\[ \frac{d\eta(x)}{dx} = \frac{\phi [F'(x)G''(x) - F''(x)G'(x)]}{(1 + i^f)[F'(x) - \phi G'(x)]} > 0. \]

for all \(\bar{x} \in [0, \bar{x}^*]\). Furthermore, notice

\[ \lim_{\bar{x} \to 0} \eta(x) = \frac{1}{1 + i^f}; \lim_{\bar{x} \to \bar{x}^*} \eta(x) = \infty. \]

Thus, expression (20) defines a one-to-one increasing mapping between the cutoff level \(\bar{x}\) and the Lagrange multiplier \(\eta\).

Next, from (19) define the function

\[ 1 + i^l = R^l(x) \equiv \frac{\eta(x)\left(1 + i^f - \frac{\pi(\lambda)}{d\lambda}\right)}{\mu - F(x) + \eta(x)[F(x) - \phi G(x)]}. \]

Again, it can be shown that

\[ \frac{dR^l(x)}{dx} \equiv \left[\frac{R^l(x)\eta'(x)}{(1 + i^f)\eta(x)}\right] \frac{\mu - F(x)}{1 + i^f + \eta(x)[F(x) - \phi G(x)]} > 0. \]
for all $\varepsilon \in [0, \varepsilon^*]$. Also, notice

$$\lim_{\varepsilon \to 0} R^l(\varepsilon) = \frac{1 + i^l}{\mu}; \quad \lim_{\varepsilon \to \varepsilon^*} R^l(\varepsilon) = \frac{1 + i^l - \frac{d\pi(\lambda)}{d\lambda}}{F(\varepsilon^*) - \phi G(\varepsilon^*)}.$$ 

Thus, expression (19), together with (20), define a one-to-one increasing mapping between the cutoff level $\varepsilon$ and the loan rate $i^l$. Notice the previous expression bounds the loan rate from below and above.

Furthermore, use (21) to define the function

$$\Lambda(\varepsilon) \equiv \frac{\lambda(\varepsilon)}{1 + \pi(\lambda(\varepsilon))} = \frac{1}{1 + i^l} \left[ 1 + \eta(\varepsilon) \left( 1 + i^l - \frac{d\pi(\lambda)}{d\lambda} \right) \frac{\mu - F(\varepsilon) + \eta(\varepsilon) \frac{d\pi(\lambda)}{d\lambda}}{(F(\varepsilon) - \phi G(\varepsilon))} \right]$$

$$= \frac{\left( 1 + i^l \frac{d\pi(\lambda)}{d\lambda} \right)}{(1 + i^l) \frac{d\pi(\lambda)}{d\lambda}} \left[ 1 + \frac{\mu - F(\varepsilon) + \eta(\varepsilon) \frac{d\pi(\lambda)}{d\lambda}}{\mu - F(\varepsilon) + \eta(\varepsilon) \frac{d\pi(\lambda)}{d\lambda}} (F(\varepsilon) - \phi G(\varepsilon)) \right]$$

$$+ \frac{1}{1 + i^l}.$$ 

Notice the left hand side is an increasing function of $\lambda$.

Taking derivatives one can be shown that

$$\Lambda'(\varepsilon) = \frac{\left( 1 + i^l - \frac{d\pi(\lambda)}{d\lambda} \right)}{\left( 1 + i^l \frac{d\pi(\lambda)}{d\lambda} \right) \left[ \mu - F(\varepsilon) + \eta(\varepsilon) \frac{d\pi(\lambda)}{d\lambda} (F(\varepsilon) - \phi G(\varepsilon)) \right]}$$

$$\times \left[ F'(\varepsilon) \eta(\varepsilon) \frac{d\pi(\lambda)}{d\lambda} (F(\varepsilon) - \phi G(\varepsilon)) \right]$$

$$+ (\mu - F(\varepsilon)) \eta(\varepsilon) \frac{d\pi(\lambda)}{d\lambda} (F(\varepsilon) - \phi G(\varepsilon))$$

$$+ (\mu - F(\varepsilon)) \eta(\varepsilon) \frac{d\pi(\lambda)}{d\lambda} (F(\varepsilon) - \phi G'(\varepsilon))$$

$$> 0$$

for all $\varepsilon \in [0, \varepsilon^*]$ with

$$\lim_{\varepsilon \to 0} \Lambda(\varepsilon) = \frac{1}{1 + i^l}; \quad \lim_{\varepsilon \to \varepsilon^*} \Lambda(\varepsilon) = \infty.$$ 

Thus, expression (21), together with (19) and (20), define a one-to-one increasing mapping between the cutoff level $\varepsilon$ and the leverage $\lambda$. 

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Also, using (34) and (36), the excess return of bank equity over its opportunity cost can be written as

\[
\left( \frac{1 + \delta^l}{1 + \delta^f} \right) [\mu - F(x, \delta)] \lambda = \eta \left( 1 - \frac{d\pi(\lambda)}{d\lambda} \right) \left( 1 + \delta^f - (1 + \delta^l) (F(x) - \phi G(x)) \right) [1 + \pi(\lambda)],
\]

Finally, using (12),

\[
\bar{x} = \frac{(1 + \delta^d) \delta Q - \Pi(L)}{(1 + \delta^d) \delta L},
\]

1 + \delta^d = \bar{x} (1 + \delta^l) + \frac{Q}{L} + \frac{\Pi(L)}{\delta L}

= 1 + \delta^f + (1 + \delta^l) [\bar{x} - F(x) + \phi G(x)]

so that the deposit rate is

\[
i^d = i^f + (1 + \delta^l) [\bar{x} - F(x) + \phi G(x)].
\]
FIGURE 1

Leverage

FIGURE 2

Lending rate ($i^l$) and deposit rate ($i^d$)