Fiscal Rules, Bailouts, and Reputation in Federal Governments*

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Abstract

Expectations of bailouts by central governments incentivize over-borrowing by local governments. In this paper, we ask if fiscal rules can correct these incentives to over-borrow when central governments cannot commit and if they will arise in equilibrium. We address these questions in a reputation model in which the central government can either be a commitment or a no-commitment type and local governments learn about this type over time. Our first main result is that if the reputation of the central government is low enough, then fiscal rules can be welfare reducing as they can lead to even more debt accumulation relative the case with no rules. This is because the costs of enforcing the punishment associated with the fiscal rule worsens the payoffs of preserving reputation and incentivizes the no-commitment type to reveal its type earlier relative to an environment without rules. This early resolution of uncertainty makes over-borrowing more attractive for the local governments. Despite being welfare reducing, binding fiscal rules will arise in the equilibrium of a signaling game due to the incentives of the commitment type to reveal its type. The model can be used to shed light on the numerous examples throughout history where tight fiscal rules were instituted but were not enforced ex-post, such as the Stability and Growth Pact.

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1 Introduction

There are numerous examples throughout history in which excessive spending and debt accumulation by subnational governments have led to bailouts by central governments. Examples include provinces in Argentina, states in Brazil, länders in Germany and most recently Greece, Ireland and Portugal in the European Union.[1] One view of such events is that the lack of commitment on the part of central governments to not bail out leads to profligate fiscal policies ex-ante, which in turn justifies the bailouts ex-post. This idea has been formally studied by Chari and Kehoe (2007), Chari and Kehoe (2008) and Cooper et al. (2008) in the economics literature and Rodden (2002) in political science. See also Sargent (2012).

A commonly held view is that *fiscal rules* can correct these incentives to overborrow when central governments lack commitment. In practice, fiscal rules take the form of limits to debt-to-GDP or deficit-to-GDP ratios along with some penalty if these are violated. For example, the Stability and Growth Pact (SGP) calls for all member countries to keep budget deficits below 3% of GDP and public debt to below 60% of GDP. Member countries are liable to face financial penalties of up to 0.5% of GDP if they repeatedly fail to respect these limits.

A natural question that arises when thinking about the design of fiscal rules is why central governments can commit to enforcing these rules if they cannot commit to not bail out. In this paper, we ask if fiscal rules can be beneficial if central governments cannot commit and if they will arise in equilibrium. We address these questions in a reputation model in which the type of the central government is uncertain: it can be either a commitment type or a no-commitment type. The reputation of a central government is the probability that local governments assign to it being a commitment type.

Our first main result is that if the reputation of the central government is low enough then fiscal rules are welfare reducing and lead to even more debt accumulation relative the case with no rules. This is because the punishment associated with the fiscal rule enforcement makes it more attractive for the no-commitment type to reveal its type earlier relative to an environment without rules. This early resolution of uncertainty makes over borrowing more attractive for the local governments. Our second main result is that, despite being welfare reducing, binding fiscal rules arise in equilibrium because the commitment type wants to signal its type and it is optimal for the no-commitment type to initially mimic and then not enforce the rule once violated.

We show these results in a stylized three period environment with three strategic players: two local governments (North and South) and a benevolent central government. Local governments choose the provision of a local public good and have access to local tax

[1]See Rodden et al. (2003), Rodden (2006), and Bordo et al. (2013) for further documentation.

revenues. They can also borrow from the rest of the world at a given interest rate. We assume that the North has access to a larger period 0 tax revenue which leads to a non-degenerate distribution of debt holdings in period 1. The central government does not have tax revenues but it can impose transfers from one state to the other. We consider an institutional setup in which the constitution requires the central government to not impose such transfers (*no-bailout clause*) and local governments to keep their debt below some level or face an output cost if violated (*fiscal rule*).

The central government can either be a commitment type who enforces the fiscal constitution or a no-commitment type who chooses its policy sequentially. This type is initially unknown to the local governments who learn about it through the actions of the central government. In period 1 (the intermediate period), the benevolent no-commitment central government faces a trade-off between not enforcing the constitution and preserving its reputation which incentivizes local governments to keep future debt accumulation in check. An important feature of our environment is that conditional on knowing the type of the central government, the timing of bailouts is irrelevant. Therefore, the choice of the no-commitment central government is whether to reveal its type in period 1 or not (revelation of uncertainty).

We first consider the case in which the constitution only contains a no-bailout clause. We show that under some sufficient conditions, when initial reputation levels are low enough, there is a unique equilibrium in which the no-commitment central government does not bail out the local government in the intermediate periods and so there is no revelation of uncertainty until the terminal period. The reason why the central government prefers to delay the revelation of its type is that for low enough reputation levels, the costs of early information revelation are first order while the benefits of equalizing the provision of the local public good in the interim period via a bailout are second order. In fact, if the probability of facing the commitment type is close to zero, the provision of the local public good in the North and the South is almost identical even without a bailout in the interim period because the South borrows against the bailout transfer it anticipates in the final period. Moreover, we show that the local government with high spending needs does not have an incentive to undertake a large deviation in the first period to incentivize a bailout in the intermediate period.

We next consider a constitution with both a no-bailout clause and a fiscal rule. The first main result of the paper is that for low enough reputation levels there is a unique equilibrium with *early of resolution of uncertainty*, i.e. the central government reveals its type in period 1. Under some sufficient conditions, we show that this leads to even more debt accumulation relative to the case without rules. In other words, with the introduction of fiscal rules, the unique equilibrium switches from one in which the constitution is enforced in period 1 to one in which it is not.

The intuition behind this result is that with fiscal rules, the value of preserving reputation is lower since the enforcement of the constitution now requires the no-commitment type central government to impose costly penalties on local governments that violate the rule. The strategic local government with high spending needs (South) now has an incentive to overborrow and incentivize the central government to bail out (reveal its type) in period 1. We show that in the unique equilibrium both types of local governments overborrow in the first period relative to the case with no fiscal rules.

We next consider a Ramsey planner tasked with designing the optimal fiscal rule taking into account this lack of commitment. We show that if the prior of the central government being the commitment type is low enough, it is strictly optimal to not have fiscal rules.

The previous result raises the question of why we would ever see fiscal rules being instituted in practice if they were welfare reducing. We study a signaling game in which rules are chosen at the beginning of time by the central government. We show that in the equilibrium of this game, the commitment type chooses to announce a fiscal rule which is mimicked by the no-commitment type. However, in this equilibrium the rule is not enforced in period 1 by the no-commitment type leading to early resolution of uncertainty and even more debt accumulation.

This result sheds light on historical and contemporary episodes when fiscal rules were instituted but were not enforced ex-post. A leading example is the SGP in the Eurozone. The SGP was instituted for the newly formed monetary union, under the pressure of Germany, with the intent of constraining fiscal policy in member countries to insulate the ECB from the pressure to inflate or monetize the debt in member countries. The enforcement of the SGP has been very lax. For example, in 2003 both Germany and France violated it and sanctions were not imposed. Moreover, thesanctionary powers of the European commission was subsequently weakened. Through the lens of our theory, this corresponds to the case in which the central government reveals its type in the intermediate period. Consistent with our theory, after 2003, the power of the SGP in disciplining fiscal policy was arguably weakened. According to several commentators, this was a major factor in the current European debt crisis in which Greece, Ireland, and Portugal received bailout packages from the European Union and the ECB (the central government) as our theory predicts.

Arguably, after the bailouts to peripheral member countries, the reputation and credibility of the central European institutions were very low. Member countries and the European institutions agreed to impose tough fiscal rules by strengthening the SGP by introducing the so-called "Six-Pact" and "Fiscal Compact" consistent with the prediction of our signaling game. The provisions of the Six-Pact were soon violated by Spain and Portugal without any sanction being levied.[2] The governor of the Bundesbank, Jens Weidmann, has recently accused the Commission of not enforcing the fiscal rules: "My perception is that the European Commission has basically given up on enforcing the rules of the Stability and Growth Pact."[3]

Another leading example of federal governments with poor fiscal discipline among subnational governments is Brazil, the most decentralized state in the developing world. The fiscal behavior of the states and large municipal governments in Brazil were a major source of macroeconomic instability and resulted in sub-national debt crises in 1989, 1993, and 1997. "The federal government took a variety of measures to control state borrowing in the 1990s, and at a first glance it would appear to have had access to an impressive array of hierarchical control mechanisms through the constitution, additional federal legislation, and the central bank. Most of these mechanisms have been undermined however, by loopholes or bad incentives that discourage adequate enforcement. (Rodden et al. (2003) page 222)."

In 1997, the federal government assumed the debts of 25 of the 27 states that were unable to service their debt—an amount equivalent to about 13 percent of GDP. By September 2001, 84% of state debt was held by the national treasury (see Rodden et al. (2003), page 234). After the bailouts in 1997, the Cardoso administration approved the Fiscal Responsibility Law which instituted "a rule-based system of decentralized federalism that leaves little room for discretionary policymaking at the subnational level. It has been motivated by the recognition that market control over subnational finances should be replaced, or strengthened, by fiscal rules as well as appropriate legal constraints and sanctions for noncompliance, Afonso and De Mello (2000)." So also in this case, in a moment in which the the reputation of the central government was low because of the recent bailouts, the central government responded by imposing stringent fiscal rules.

We study various extensions of baseline model to see whether our results are robust to them. For example, we relax commitment on the part of local governments to repay their debts. We consider a short-term defaultable debt model similar to Eaton and Gersovitz where the costs of default include the inability to borrow and lend in the future as well as exogenous utility costs. This implies that local governments are subject to endogenous borrowing constraints and non-zero spreads. We show that versions of our main results go through in this case. In particular, with fiscal rules we have an equilibrium with more debt than in case without fiscal rules. The reason for this is that borrowing constraints are relaxed in the case in which there is early resolution of uncertainty. Conditional on a bailout, the value of repaying debt goes up relative to the case in which there is no bailout while the value of default is identical. This allows for more borrowing ex-ante.

[2]See https://www.ft.com/content/f66a5c1d-b023-3d0f-ad02-767a9656d4f9

[3]See https://www.ft.com/content/95e7ee7e-ad8e-11e6-ba7d-76378e4fef24.

Related Literature Our paper is related to several strands of the literature. First, it is related to a literature that studies the free rider problem in federal governments when the central government cannot commit. Examples of papers in this literature include Chari and Kehoe (2007), Chari and Kehoe (2008), Cooper et al. (2008), Aguiar et al. (2015), Chari et al. (2016), and Rodden (2002). The main result in this literature is that the inability of the central government (or monetary authority) to commit not to bail out ex-post leads to overborrowing ex-ante. In such settings, it is often argued that fiscal rules can improve outcomes by lowering the amount of debt issued. For such analysis see Beetsma and Uhlig (1999). The contribution of our paper is to analyze the effects of fiscal rules when the government also cannot commit to enforcing them.

Fiscal rules have been studied in several environments as the solution to time inconsistency problems. See for instance Amador et al. (2006) and Halac and Yared (2014) in the context of delegation and Hatchondo et al. (2015) and Alfaro and Kanczuk (2016) in the context of sovereign default. All these papers assume that the agents can commit to rules and do not analyze the enforcement problem which is the main focus of this paper.

The baseline model uses a reputational setup similar to Kreps et al. (1982) with uncertainty about the type of the central government. It also relates to papers that try and account for several features of policy outcomes by studying models in which a government with a hidden type interacts with a continuum of private agents. as in Phelan (2006) and D'Erasmo (2008). A key difference in our paper is the fact that the local governments are strategic and can incentivize the central government to reveal its type via its actions. In addition, we also study the optimal policy in this environment.

Uncertainty about the type of the central government plays a key role in the provision of incentives to local governments. Nosal and Ordoñez (2013) also consider an environment in which uncertainty can mitigate the time inconsistency problem when a central government cannot commit not to bailout banks. The mechanism is very different: here uncertainty about the type of the central government curbs debt issuances by local governments while in their paper it is the uncertainty about banks (local governments) that restraints the central government to not intervene ex-post.

2 Three period economy

Environment

Let $t = 0, 1, 2$. Consider a small open economy composed of two states or regions, the North and the South, $i \in \{N, S\}$. The representative citizen in state i has preferences over the local public good provision $\{G_{it}\}$

$$U = \sum_{t=0}^{2} \beta^t u\left(G_{it}\right).$$

The local public good provision is decided by a benevolent *local government* with local tax revenues $\{Y_{it}\}$. In particular, we let[4]

$$Y_{N0} > Y_{S0}, \quad Y_{Nt} = Y_{St} = Y \quad \text{for } t = 1, 2$$

So the North is "richer" at time 0 relative to the South. The local government can borrow from the rest of the world at a rate $1 + r^*$. We let $q = 1/\left(1 + r^*\right)$ be the price of a bond that promises to pay one unit of the consumption good next period. There is also a *central government*. The central government does not have tax revenues but it can impose transfers from one state to the other subject to a budget constraint

$$\sum_{i=N,S} T_{it} \leq 0$$

where T_{it} is the transfer to state i in period t.

Efficient allocation

As a benchmark, we consider the efficient allocation in this environment. An allocation is efficient if for some set of Pareto weights $\{\lambda_i\}$ it solves

$$\max \sum_i \lambda_i U\left(\{G_{it}\}\right)$$

subject to

$$\sum_{t=0}^{2} \sum_i q^t \left[G_{it} - Y_{it}\right] \leq 0 \tag{1}$$

Any efficient allocation must satisfy

$$q u'\left(G_{it}\right) = \beta u'\left(G_{it+1}\right) \tag{2}$$

and the consolidated budget constraint (1) with equality.

[4] Adding heterogeneity in tax revenues Y_{it} for $t > 0$ leaves the results unchanged.

Institutional setup and equilibrium

Consider an institutional setup in which it is written in the constitution that the central government should not bail the states out. We call such a provision the *no-bailout clause*. Moreover, the constitution requires local governments to keep their debt below a cap $\bar{b}_t$ for $t = 1, 2$. In case $b_{it} > \bar{b}_t$, the central government must impose a penalty $\psi_t Y$ on the state that violated the rule. We call this constitutional provision a *fiscal rule*. A fiscal rule is then fully described by $\left\{ \left(\bar{b}_t, \psi_t \right) \right\}_{t=1,2}$.

The central government can be of one of two types: a *commitment type*, which follows the prescriptions in the constitution, and a *no-commitment type* that is not bound to follow the prescriptions of the constitution as it chooses policy sequentially to maximize an equally weighted average of the utility of citizens in both countries:

$$W_r = \sum_{t \geq r}^{2} \sum_{i=N,S} \beta^t u \left(G_{it} \right).$$

The type of the central government is drawn at the beginning of period 0 and it is not known to the local governments. They have a common prior π $(1 - \pi)$ that the central government is the commitment type (no-commitment type).

The timing is as follows:

- At $t = 0$, local governments choose the local public good provision G_{i0} and debt b_{i1} subject to the budget constraint

$$G_{i0} \leq Y_{i0} + q b_{i1}.$$

- At $t = 1$, if the central government is the no-commitment type, it decides whether to make transfers $\{T_{i1}\}$ or not and whether to enforce the penalty if the fiscal rule is violated by a local government. After observing the central government actions, the local governments update their prior about the central government type and they decide the provision of the local public good G_{i1} and new debt issuance b_{i2} such that

$$G_{i1} + b_{i1} \leq Y + T_{i1} + q b_{i2} - \psi_1 Y \mathbb{I}_{\left\{ b_{i1} > \bar{b}_1 \text{ and central government enforces} \right\}}.$$

- At $t = 2$, if the central government is the no-commitment type, it decides whether to make a transfer $\{T_{i2}\}$ or not and fiscal rule enforcement. Next, the local governments choose G_{i2} subject to budget constraints

$$G_{i2} + b_{i2} \leq Y + T_{i2} - \psi_2 Y \mathbb{I}_{\left\{ b_{i2} > \bar{b}_2 \text{ and central government enforces} \right\}}.$$

We assume for now that the local government can commit to repaying its debt. This can be motivated by the existence of high default costs which makes repayment always optimal for the local government. In Section 7.2 we show that our results extend to the case in which the local government cannot commit to repay debt.

We now characterize the equilibrium by backward induction.

Period 2 The state in the last period is the distribution of debt among local governments, $b_2 = (b_{N2}, b_{S2})$. If the central government is the no-commitment type, it will choose transfers $T_{i2}(b_2)$ such that the consumption of the local public good is equalized between the two states: $T_{i2}(b_2) = b_{i2} - \frac{\sum_j b_{j2}}{2}$ so that

$$G_{it} = Y - \frac{\sum_j b_{j2}}{2}$$

and it will not impose the penalty if the fiscal rule is violated. We refer to this situation as *debt mutualization*. The value for the central government is

$$W_2(b_2) = \sum_i u\left(Y - \frac{\sum_j b_{j2}}{2}\right)$$

and the value for a local government is

$$V_{i2}(b_2) = u\left(Y - \frac{\sum_j b_{j2}}{2}\right).$$

If instead the central government is the commitment type, each state will consume $G_{i2} = Y_{i2} - b_{i2} - \psi_2 \mathbb{I}_{\{b_{i2} > \bar{b}\}} = Y - b_{i2} - \psi_2 \mathbb{I}_{\{b_{i2} > \bar{b}\}}$. The value for the local government is then

$$V_{i2}^c(b_2) = u\left(Y - b_{i2} - \psi_2 \mathbb{I}_{\{b_{i2} > \bar{b}\}}\right).$$

Period 1 The state in period 1 is the distribution of debt among local governments, $b_1 = (b_{N1}, b_{S1})$ and the prior on the type of the central government, π. Let σ be the equilibrium strategy of the central government in period 1. The central government can either enforce the fiscal constitution or not.[5] We consider equilibria where the law of motion for beliefs follows Bayes' Rule and is given by

$$\pi'(b_1, \zeta, \pi; \sigma) = \begin{cases} \frac{\pi}{\pi + (1-\pi)(1-\sigma(b_1,\pi))} & \text{if } \zeta = 0 \\ 0 & \text{if } \zeta = 1 \end{cases} \tag{3}$$

[5]To ease notation, we will not consider the case in which the central government enforces only one of the provisions of the fiscal constitution. This is without loss of generality since it will never be optimal to do so.

where $\zeta = 1$ if the central government does not enforce the fiscal constitution in period 1, and σ denotes the enforcement strategy for the central government and is defined by

$$\sigma\left(b_1, \pi, \psi; \pi'\right) = \begin{cases} 0 & W_1^{nb}\left(b_1, \pi'\left(b_1, 0, \pi; \sigma\right), \psi\right) > W_1^b\left(b_1\right) \\ 1 & W_1^{nb}\left(b_1, \pi'\left(b_1, 0, \pi; \sigma\right), \psi\right) < W_1^b\left(b_1\right) \\ 0 < \tilde{\sigma} < 1 & W_1^{nb}\left(b_1, \pi'\left(b_1, 0, \pi; \sigma\right), \psi\right) = W_1^b\left(b_1\right) \end{cases} \tag{4}$$

where $\sigma = 1$ means that the constitution is not enforced while $\sigma = 0$ denotes enforcement. W_1^{nb} is the value for the no-commitment type central government if it does enforce the fiscal constitution in period 1 and W_1^b is the value for the no-commitment type central government if it does not enforce the fiscal constitution in period 1. We will describe these value functions in detail in what follows.

We now analyze the decision of the local governments. Suppose first that there is enforcement and the posterior of the central government's type remains constant at π, $\pi'\left(b_1, 0, \pi; \sigma\right) = \pi$. In this case, local governments choose G_{i1}, b_{i2} to solve

$$V_{i1}^{nb}\left(b_1, \pi\right) = \max_{G_{i1}, b_{i2}} u\left(G_{i1}\right) + \beta\pi V_{i2}^c\left(b_{2i}\right) + \beta\left(1 - \pi\right) V_{i2}\left(b_{i2}, b_{-i2}\left(b_1, \pi\right)\right) \tag{5}$$

subject to

$$G_{i1} + b_{i1} \leq Y_{i1} + qb_{i2} - \psi Y \mathbb{I}_{\left\{b_{i1} > \bar{b}\right\}}$$

taking as given the strategy $b_{-i2}\left(b_1, \pi, \psi\right)$ followed by the other local government.

For later reference, the equilibrium outcome at this node will be given by $\mathbf{b}_2\left(b_1, \pi, \psi\right) = \left(\mathbf{b}_{N2}\left(b_1, \pi, \psi\right), \mathbf{b}_{S2}\left(b_1, \pi, \psi\right)\right)$ which solves for $i \in \{N, S\}$

$$qu'\left(Y - b_{i1} + qb_{i2} - \psi Y \mathbb{I}_{\left\{b_{i1} > \bar{b}\right\}}\right) = \beta\pi u'\left(Y - b_{i2}\right) + \beta\left(1 - \pi\right) \frac{u'\left(Y - \frac{\sum_j b_{j2}}{2}\right)}{2} \tag{6}$$

Notice that unless the probability of facing the commitment type is one, the optimality condition (6) differs from the Euler equation (2) that characterizes an efficient allocation. In particular, if $\pi < 1$, there is overborrowing because each local government internalizes only half the marginal cost of repaying its debt next period because it anticipates a bailout if the central government is the no-commitment type.

Next, suppose that the fiscal constitution is not enforced and $\pi'\left(b_1, 1, \pi; \sigma\right) = 0$ so that the central government reveals its type. In this case, the value for the local government given a set of transfers T_1 is

$$V_{i1}^b\left(b_1, 0, T_1\right) = \max_{G_{i1}, b_{i2}} u\left(G_{i1}\right) + \beta V_{i2}\left(b_{i2}, b_{-i2}\left(b_1, \pi\right)\right) \tag{7}$$

subject to

$$G_{i1} + b_{i1} \leq Y_{i1} + T_{i1} + qb_{i2}$$

taking as given the strategy $b_{-i2}(b_1, 0, T)$ followed by the other local government.

To simplify the exposition, note that the transfer T_1 is not welfare relevant. Moreover, if $\pi'(b_1, 1, \pi; \sigma) = 0$, whether there is debt mutualization in period 1 and 2 or only in period 2 is irrelevant in that the equilibrium consumption outcomes are identical.

Lemma 1. *For all $T_1 = (T_{N1}, T_{S1})$, the value of a violation of the no-bailout clause is independent of T*

$$V_{i1}^{b}(b_1, 0, T_1) = V_{i1}^{nb}(b_1, 0) = V_{i1}^{b}(B_1) \tag{8}$$

where $B_1 = \sum_i b_{i1}$.

Notice that $V_{i1}^{b}(B_1)$ means that the value for a local government in the event of non-enforcement depends only the aggregate level of debt rather than the distribution of debt.We can then use the Lemma to drop T_1 as a decision variable. The value for the non-commitment type central government is then

$$W_1(b_1, \pi, \psi) = [1 - \sigma(b_1, \pi, \psi)]\, W_1^{nb}\left(b_1, \pi'(b_1, 0, \pi), \psi\right) + \sigma(b_1, \pi, \psi)\, W_1^{b}(B_1) \tag{9}$$

where the value of not enforcing is

$$W^{b}(B_1) = \sum_i V_{i1}^{b}(B_1) \tag{10}$$

and the value of enforcing is[6]

$$W_1^{nb}(b_1, \pi, \psi) = \sum_i \left[u\left(Y - b_i + q\mathbf{b}_{i2}(b, \pi, \psi) - \psi Y \mathbb{I}_{\{b_{i1} > \bar{b}\}}\right) + \beta u\left(Y - \frac{\mathbf{b}_{i2}(b, \pi, \psi)}{2}\right)\right] \tag{11}$$

and the equilibrium enforcement strategy is given by (4).

Period 0 The state in period 0 is the prior on the type of the central government, π (the realization of Y_{i0} is incorporated by indexing the value functions by t and i). In period $t = 0$, a local government chooses the local public good provision and debt to solve

$$\begin{aligned} V_{i0}(\pi, \psi) = \max_{G_{i0}, b_{i1}} u(G_{i0}) &+ \beta[1 - \sigma(b_1, \pi, \psi)]\, V_{i1}^{nb}(b_{i1}, b_{-i1}, \pi, \psi) \\ &+ \beta\sigma(b_1, \pi, \psi)\left[\pi V_{i1}^{c}(b_{i1}) + \beta(1 - \pi)\, V_{i1}^{nb}(b_{i1}, b_{-i1}, 0)\right] \end{aligned} \tag{12}$$

[6]Note that $W_1^{nb} \neq \sum_i V_i^{nb}$ since the no-commitment type central government knows that it will mutualize debt in period 2 while local governments have uncertainty about the central government's type.

subject to the budget constraint

$$G_{i0} \leq Y_{i0} + qb_{i1}$$

taking as given the strategy $b_{-i1}(\pi, \psi)$ followed by the other local government.

For later reference, we also define the value for the no-commitment type central government

$$W_0(\pi,\psi) = \sum_i u(G_{i0}(\pi,\psi)) + \beta\left[1-\sigma(b_1(\pi,\psi),\pi,\psi)\right] W_{i1}^{nb}(b_1(\pi,\psi),\pi,\psi) \\ + \beta\sigma(b_1(\pi,\psi),\pi,\psi) W_1^{nb}(b_1(\pi,\psi),0,0) \tag{13}$$

where $G_{i0}(\pi,\psi)$ and $b_1(\pi,\psi)$ are the decision rules in (12). The value for the commitment type instead is

$$W_0^c(\pi,\psi) = \sum_i u(G_{i0}(\pi,\psi)) + \beta\left[1-\sigma(b_1(\pi,\psi),\pi,\psi)\right] W_1^c(b_1(\pi,\psi),\pi,\psi) \\ + \beta\sigma(b_1(\pi,\psi),\pi,\psi) W_1^c(b_1(\pi,\psi),\psi) \tag{14}$$

Equilibrium definition We can now define an equilibrium for this institutional setup.

Definition. An equilibrium is a set of strategies and beliefs for the local governments, $b_{i1}(\pi,\psi)$, $\pi'(b_1,\zeta,\pi)$, $b_{i2}(b_1,\pi,\psi)$, a strategy for the no-commitment type central government, $\sigma(b_1,\pi,\psi)$, and associated value functions, such that: i) Given $b_{-i1}(\pi,\psi)$ and $\sigma(b_1,\pi,\psi)$, $b_{i1}(\pi,\psi)$ solves (12); ii) Given $b_{-i2}(b_1,\pi,\psi)$, $b_{i2}(b_1,\pi,\psi)$ solves (5); iii) $\pi'(b_1,\zeta,\pi)$ satisfies (3); iii) $\sigma(b_1,\pi,\psi)$ satisfies (4).

3 Equilibrium outcomes without fiscal rules

We next turn to characterizing the equilibrium outcome for the policy game without fiscal rules or $\psi = 0$. The main result in this section is that without fiscal rules, for π close to zero, there exists a unique equilibrium outcome in which the central government does not bail the local governments out in period 1. For ease of notation we will drop the dependence on ψ from the strategies and value functions defined above. We start by establishing some properties of the equilibrium outcome in period 1 after the government decision of mutualizing debt or not.

Lemma 2. *The decision rule $\mathbf{b}_{i2}(b_1,\pi)$ defined in (6) is increasing in own debt and decreasing in debt of the other local government. Moreover, $\mathbf{b}_{S2}(b_1,\pi)$ is decreasing in π, and for π sufficiently small, $\mathbf{B}_2(b_1,\pi)$ is decreasing in π.*

We now turn to discussing properties of the value functions for the central and local governments.

Lemma 3. *i) For all π, $W_1^{nb}(\cdot,\pi)$ is continuous, differentiable, decreasing.*
ii) For all b, for π small enough, $W_1^{nb}(b,\cdot)$ is increasing in π.
iii) Consider $b_1=(b_{1N},b_{1S})$ with $b_{1N}\leq b_{1S}$ and $\Delta>0$. Then

$$W_1^{nb}(b_{1N},b_{1S},\pi)>W_1(b_{1N}+\Delta,b_{1S}-\Delta,\pi).$$

The figure below shows the main properties of W_1 and V_1. First, note that for any $\pi>0$ and inherited debt level for the North, b_{N1}, there is a maximal level of debt $\hat{b}_S(b_{N1},\pi)>b_{N1}$ after which it is optimal for the central government to violate the no-bailout clause. Enforcing the no-bailout clause has benefits and costs for the non-commitment type central government.

The benefits of enforcement are associated with a reduction of the distortions in the local governments' Euler equations (6) relative to the efficient one (2). By enforcing the no-bailout clause, the central government preserves its reputation. A higher π in turn promotes fiscal responsibility because the local government expects to repay its debt without a bailout from the central government with higher probability.

The costs of enforcing the no-bailout clause are associated with the high inequality in the provision of the local public good. A higher π will induce the South government to borrow less and cut the consumption of the local public good relative to the North. This creates dispersion in the consumption of the local public good across states (North and South) that it is costly from the perspective of the benevolent central government.

We can see these two effects by decomposing the welfare of the central government into two components: the fiscal responsibility benefits and the redistribution costs,

$$\begin{aligned}W_1^{nb}(b,\pi)-W_1^{nb}(b,0)&=\left[W_1^{nb}(\bar{b},\pi)-W_1^{nb}(b,0)\right]-\left[W_1^{nb}(\bar{b},\pi)-W_1^{nb}(b,\pi)\right]\\&=\left[W_1^{nb}(\bar{b},\pi)-W_1^{nb}(\bar{b},0)\right]-\left[W_1^{nb}(\bar{b},\pi)-W_1^{nb}(b,\pi)\right]\end{aligned}$$

where $\bar{b}\equiv\left\{\frac{\sum_i b_i}{2},\frac{\sum_i b_i}{2}\right\}$ and the first term captures the benefits associated with less distortions in the Euler equation relative to the efficient benchmark and the second captures the losses associated with more dispersion. Fixing the inherited debt of the North, as debt issued by the South increases, eventually the redistribution costs exceed the fiscal responsibility benefits and it is optimal for the central government to bail the states out. This is because as inherited debt issued by the South increases, the provision of the local public good in the South decreases relative to the North. This local public good inequality lowers the utility of the central government if it does not engage in a bailout. Eventually, if this inequality is sufficiently high, the central government prefers a bailout rather than enforcing the constitution.

Second, notice that while the value for the central government W_1 is continuous in the inherited debt b_1, the same is not true for the value of the local governments. Both V_{S1} and V_{N1} are discontinuous at $\hat{b}_{S1}$. In particular, the value function for the South has a positive jump at $\hat{b}_{S1}$ because for levels of inherited debt above the cutoff, the South receives a transfer from the central government. The opposite is true for the North.

The discontinuity of V_{i1} implies that a pure strategy equilibrium may not exist. Next, we are going to show that for π sufficiently close to zero a pure strategy equilibrium exists. To this end, note that if an equilibrium in pure strategies exists, the equilibrium outcome can take one of two forms. First, an outcome $\{b_{i1}, b_{i2}\}$ is an equilibrium outcome of the policy game *with no debt mutualization in period 1* or NB equilibrium for short if and only if it satisfies: i) optimality of debt issuances in period 0: local optimality for local governments

$$qu'\left(Y_{i0}+qb_{i1}\right)=\beta\frac{\partial V_{i1}\left(b_{i1},b_{-i1},\pi\right)}{\partial b_{i1}}\quad\text{for } i=N,S \tag{15}$$

and a global optimality condition for the South:

$$\begin{aligned} u\left(Y_{S0}+qb_{S1}\right)+\beta V_{S1}\left(b_{S1},b_{N1},\pi\right)\geq \max_{b\geq\hat{b}_S(b_{N1},\pi)} & u\left(Y_{S0}+qb\right)+ \\ & +\beta\pi V_{S1}\left(b,b_{N1},1\right)+\beta\left(1-\pi\right)V_{S1}\left(b,b_{N1},0\right) \end{aligned} \tag{16}$$

where $\hat{b}_S\left(b_{N1}\right)$ solves

$$W^{nb}\left(\hat{b}_S,b_{N1},1\right)-W^{b}\left(\hat{b}_S+b_{N1}\right)=0. \tag{17}$$

ii) optimality for the no-commitment type central government:

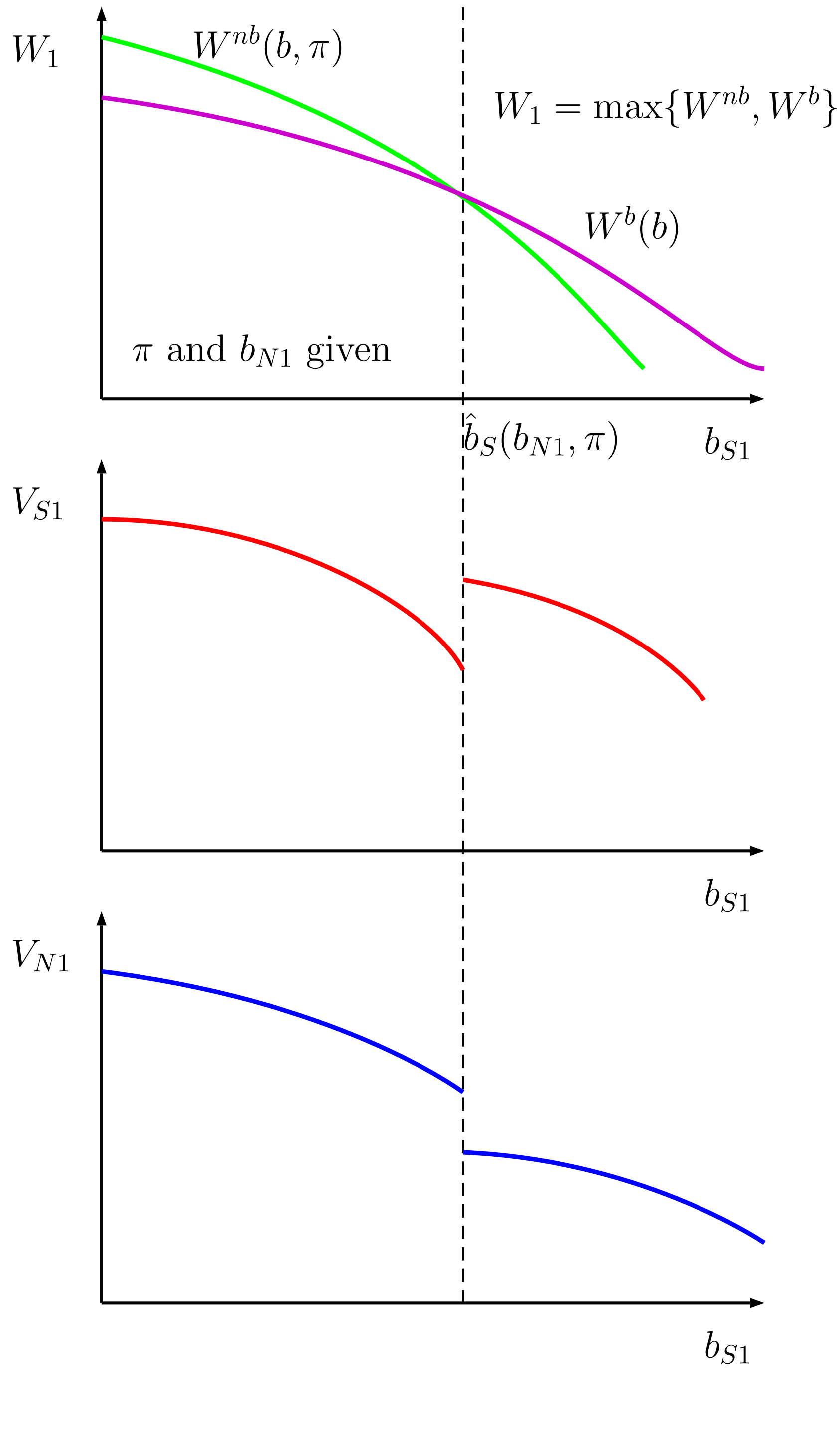
$$W^{nb}\left(b_i,\pi\right)\geq W^{b}\left(\sum_i b_{i1}\right) \tag{18}$$

and iii) optimality of debt issuances in period 2 in that $b_{i2}=\mathbf{b}_{i2}\left(b_1,\pi\right)$ from (6). We refer to this outcome as $\left\{b_{i1}^{nb}\left(\pi\right), b_{i2}^{nb}\left(\pi\right)\right\}$.

Second, an outcome $\left\{b_{i1}, b_{i2}, b_{i2}^{c}\right\}$ is an equilibrium outcome of the policy game *with debt mutualization in period 1* or B equilibrium for short, if and only if it satisfies: i) local optimality of debt issuances in period 0:

$$qu'\left(Y_{i0}+qb_{i1}\right)=\beta\pi\frac{\partial V_{i1}\left(b_{i1},b_{-i1},1\right)}{\partial b_{i1}}+\beta\left(1-\pi\right)\frac{\partial V_{i1}\left(b_{i1},b_{-i1},0\right)}{\partial b_{i1}}\quad\text{for } i=N,S \tag{19}$$

Figure 1: Value functions for central and local governments in period 1



and a global optimality condition for the North:

$$u\left(Y_{N0}+qb_{N1}\right)+\beta\pi V_{N1}\left(b_1,1\right)+\beta\left(1-\pi\right)V_{N1}\left(b_1,0\right)\geq\max_{b\geq\hat{b}_N\left(b_{S1},\pi\right)}u\left(Y_{N0}+qb\right)+\beta V_{N1}\left(b,b_{S1},\pi\right) \tag{20}$$

where $\hat{b}_N\left(b_{S1},\pi\right)$ solves

$$W^{nb}\left(\hat{b}_N,b_{S1},\pi\right)-W^b\left(\hat{b}_N+b_{S1}\right)=0. \tag{21}$$

ii) optimality for the no-commitment type central government

$$W^{nb}\left(b_1,1\right)<W^b\left(\sum_i b_{i1}\right) \tag{22}$$

and iii) optimality of debt issuances in period 2 in that $b_2=\mathbf{b}_2\left(b_1,0\right)$ and $b_2^c=\mathbf{b}_2\left(b_1,1\right)$. We refer to this outcome as $\left\{b_{i1}^b\left(\pi\right),b_{i2}^b\left(\pi\right),b_{i2}^c\left(\pi\right)\right\}$.

Optimal to delay type revelation The next proposition shows that when the central government has sufficiently low reputation, then under some sufficient conditions, there exists a unique equilibrium in which the no-commitment type enforces the constitution in period 1.

Proposition 1. *(No bailout in period 1 when credibility is low) At $\pi=0$ there are two equilibria with the same local public good provision that differ in the timing of debt mutualization. Suppose that $\frac{\beta}{q}\leq 1$, and $Y_{N0}-Y_{S0}$ and the coefficient of absolute risk aversion $\frac{-u''(c)}{u'(c)}$ are small. Then for $\pi>0$ but sufficiently small, there exists a unique equilibrium in which there is no debt mutualization in period 1 in that $\sigma\left(b_1\left(\pi\right),\pi\right)=0$.*

The proof of this and other propositions, except where noted, are provided in the Appendix .

The central insight from the Proposition is that when reputation is low, it is is optimal for the central government to delay the bailout. To gain intuition for this result, note that a bailout in period one will reveal that the central government is the no-commitment type. As we discussed earlier, this has costs associated with inducing more fiscal responsibility going forward (smaller distortions in the Euler equations relative to the efficient allocation) and benefits associated with lower inequality in the provision of the local public good across states. For π close to zero, if the central government enforces the constitution and does not bail out, there is essentially no inequality of the local public good consumption since the local governments expect a bailout with high probability in period 2 and so the redistribution benefits are second order. The benefits from inducing more fiscal discipline instead are first order since the Euler equation is distorted relative to the efficient allocation. Hence, it is optimal for the central government to not bail the states out (or to enforce the constitution) when its reputation is very low.

4 Equilibrium outcomes with fiscal rules

We now consider the case in which the no-commitment type central government cannot commit to enforce the no-bailout clause and the fiscal rule. Our main result is that when the reputation of the central government is low, fiscal rules promote *less* fiscal discipline when they are binding than a constitution without fiscal rules. This is because fiscal rules tighten the enforcement constraint for the central government in period 1: For a given distribution of debt that violates the rule, the government must punish the local government with more debt and doing so increases the dispersion in the consumption of the local public good and hence reducing the value of maintaining reputation. Anticipating this, the government in the South now has a stronger incentive to borrow more in order to induce the central government to bail it out and reveal its type in period 1. In particular, we show that if π is small enough and the debt limit implied by the fiscal rule is binding, there exists a unique equilibrium in which the South violates the rule by issuing more debt in period 0 and the no-commitment type central government does not enforce the rule and it reveals its type in period 1. This is in sharp contrast to the case without fiscal rules where in equilibrium, it was optimal for the central government to delay the revelation of its type.

Moreover, in the equilibrium with binding fiscal rules, the debt issued in period zero is larger than the level of debt issued in the equilibrium absent rules. This is because the anticipation of the early revelation of the type of the central government leads to more debt issuance in period 0. This is in sharp contrast to the case in which the central government can somehow commit to fiscal rules in the first period. In such case, the government can effectively implement any cap on government debt by appropriately choosing the punishment ψ.

Characterization We now turn to analyze how the outcome changes with fiscal rules. As in the case without rules, there can be two possible equilibrium outcomes. First, an outcome $\{b_{i1}, b_{i2}\}$ *with enforcement in period 1* or NB equilibrium for short if and only if it satisfies: i) optimality of debt issuances in period 0: local optimality

$$qu'\left(Y_{i0}+qb_{i1}\right)=\beta\frac{\partial V_{i1}\left(b_{i1},b_{-i1},\pi,\psi\right)}{\partial b_{i1}}\quad\text{for } i=N,S \tag{23}$$

and global optimality for the South:

$$u\left(Y_{S0}+qb_{S1}\right)+\beta V_{S1}\left(b_{S1},b_{N1},\pi,\psi\right)\geq\max_{b\geq\hat{b}_S(b_{N1},\pi,\psi)}u\left(Y_{S0}+qb\right)+ \\ +\beta\pi V_{S1}\left(b,b_{N1},1,\psi\right)+\beta\left(1-\pi\right)V_{S1}\left(b,b_{N1},0,\psi\right) \tag{24}$$

where $\hat{b}_S\left(b_{N1},\psi\right)$ solves

$$W^{nb}\left(\hat{b}_S,b_{N1},1,\psi\right)-W^b\left(\hat{b}_S+b_{N1}\right)=0. \tag{25}$$

ii) optimality for the no-commitment type central government:

$$W^{nb}\left(b_1,\pi,\psi\right)\geq W^b\left(\sum_i b_{i1}\right) \tag{26}$$

and iii) optimality of debt issuances in period 2 in that $b_{i2}=\mathbf{b}_{i2}\left(b_1,\pi,\psi\right)$. We refer to this outcome as $\left\{b_{i1}^{nb}\left(\pi,\psi\right),b_{i2}^{nb}\left(\pi,\psi\right)\right\}$.

Second, an outcome $\left\{b_{i1},b_{i2},b_{i2}^c\right\}$ is an equilibrium outcome of the policy game *without enforcement in period 1* or B equilibrium for short, if and only if it satisfies: i) optimality of debt issuances in period 0: local optimality

$$qu'\left(Y_{i0}+qb_{i1}\right)=\beta\pi\frac{\partial V_{i1}\left(b_{i1},b_{-i1},1,\psi\right)}{\partial b_{i1}}+\beta\left(1-\pi\right)\frac{\partial V_{i1}\left(b_{i1},b_{-i1},0,\psi\right)}{\partial b_{i1}}\quad\text{for } i=N,S \tag{27}$$

and global optimality for the North:

$$u\left(Y_{N0}+qb_{N1}\right)+\beta\pi V_{N1}\left(b_1,1,\psi\right)+\beta\left(1-\pi\right)V_{N1}\left(b_1,0,\psi\right)\geq \\ \geq\max_{b\geq\hat{b}_N(b_{S1},\pi,\psi)}u\left(Y_{N0}+qb\right)+\beta V_{N1}\left(b,b_{S1},\pi,\psi\right) \tag{28}$$

where $\hat{b}_N\left(b_{S1},\pi,\psi\right)$ solves

$$W^{nb}\left(\hat{b}_N,b_{S1},\pi,\psi\right)-W^b\left(\hat{b}_N+b_{S1}\right)=0. \tag{29}$$

ii) optimality for the no-commitment type central government:

$$W^{nb}\left(b_1,1,\psi\right)<W^b\left(\sum_i b_{i1}\right) \tag{30}$$

and iii) optimality of debt issuances in period 2 in that $b_2=\mathbf{b}_2\left(b_1,0,0\right)$ and $b_2^c=\mathbf{b}_2\left(b_1,1,\psi\right)$. We refer to this outcome as $\left\{b_{i1}^b\left(\pi,\psi\right),b_{i2}^b\left(\pi,\psi\right),b_{i2}^c\left(\pi,\psi\right)\right\}$.

Fiscal rules promote less fiscal discipline when reputation is low We now turn to the main result of the paper: when the central government's reputation is low and the fiscal rule is binding, there is a unique equilibrium with no enforcement in period 1. The debt levels in this equilibrium are higher than in the equilibrium without fiscal rules.

Proposition 2. *(Fiscal rules promote less fiscal discipline when reputation is low.) For π small and $\psi > 0$, either*

1. The fiscal rule does not bind: the equilibrium debt holding $\left(b_{N1}^{nb}, b_{S1}^{nb}\right)$ is such that $b_{N1}^{nb} < \bar{b}$ and $b_{S1}^{nb} < \bar{b}$ and the no-commitment type central government enforces the fiscal constitution in period 1.

2. The fiscal rule binds: the equilibrium debt holding $\left(b_{N1}^{b}, b_{S1}^{b}\right)$ is such that $b_{N1}^{b} < \bar{b}$ and $b_{S1}^{b} > \bar{b}$ and the no-commitment type central government does not enforce the fiscal constitution in period 1.

If β is small enough, there exists an equilibrium of this form. Moreover, in case 2, there exists parameters such that the debt issued by the South is higher than in the case without rules in that $b_{S1}^{b}(\pi, \psi) \geq b_{S1}^{nb}(\pi, 0)$

We now provide some intuition for the Proposition. We first explain why with binding fiscal rules there cannot exist an equilibrium with enforcement in period 1. Suppose by way of contradiction we have an equilibrium with enforcement and local governments anticipate this. The state of the economy in period 1 is then $b_1^{nb}(\pi, \psi)$. Define

$$\begin{aligned} \mathcal{W}(\pi, \psi) &\equiv W^{nb}\left(b_1^{nb}(\pi, \psi), \pi, \psi\right) - W^{b}\left(b_1^{nb}(\pi)\right) \\ &= W^{nb}\left(b_1^{nb}(\pi), \pi, \psi\right) - W^{nb}\left(b_1^{nb}(\pi), 0, 0\right) \end{aligned}$$

If $\psi = 0$ we know that $\mathcal{W}(0,0) = 0$ and $\partial \mathcal{W}(0,0) / \partial \pi > 0$ so it is optimal for the central government to enforce if local governments expect it will do so. If $\psi > 0$ and the rules are binding then there is an additional cost of enforcing and so

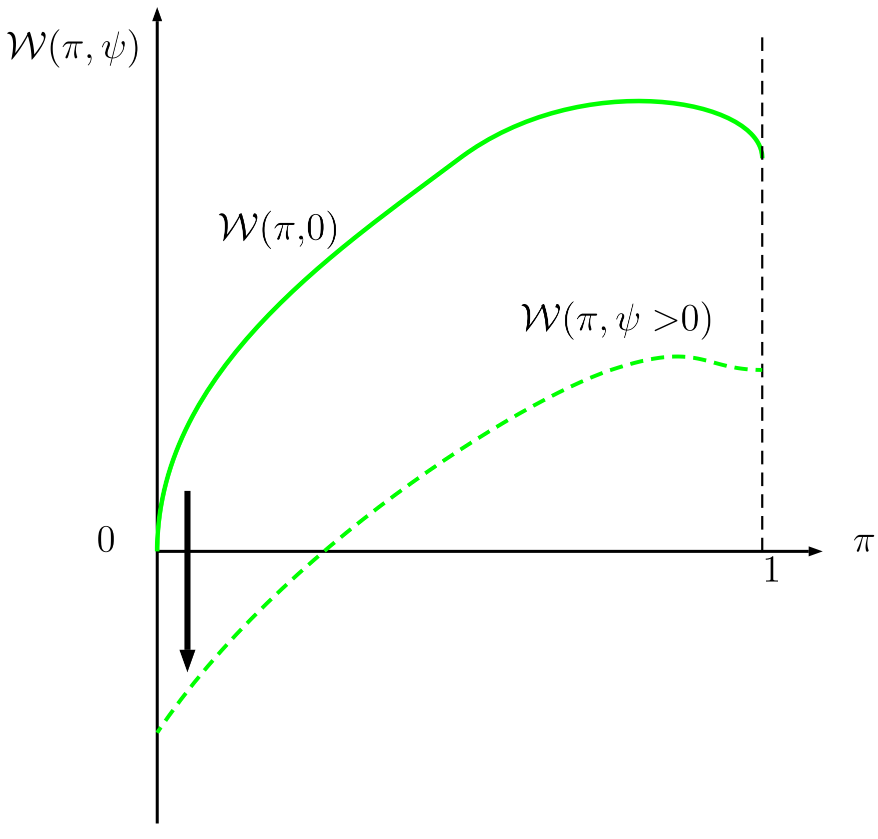
$$\mathcal{W}(0, \psi) < 0$$

Since $\mathcal{W}$ is continuous, if the central government's reputation is low enough, then if the local governments violate the fiscal rule, it is not optimal ex-post for the central government to enforce it and so we cannot have an equilibrium with binding rules and enforcement.

Therefore, either the rules are so lax that they are not binding and the outcomes with and without rules coincide (case 1) or they are binding and there is no enforcement by the central government in period 1 (case 2).

When the rules are binding, we show that if $\psi > 0$ there is more borrowing from period 0 to period 1 and also from period 1 to period 2 relative to case without fiscal rules

Figure 2: Value of enforcing for central government if local local governments anticipate enforcement in period 1



($\psi = 0$). This is due to the early resolution of uncertainty about the type of the central government.

Consider first debt issued in period zero. We can write the optimality condition for the South in period 0 if it expects a bailout in period 1 combined with optimality conditions in period 1:

$$\begin{aligned} u'\left(Y_{S0} + qb_{S1}^{b}\right) =& \frac{\beta^2}{q^2}\pi u'\left(Y - \mathbf{b}_{S2}\left(b_{S1}^{b}, 1, \psi\right)\right) \\ &+ \frac{\beta^2}{q^2}(1-\pi)\frac{u'\left(Y - \frac{\mathbf{B}_2\left(b_1^b,0,0\right)}{2}\right)}{2} \\ &+ \frac{\beta^2}{2q}(1-\pi)\,u'\left(Y - \frac{\mathbf{B}_2\left(b_1^b,0,0\right)}{2}\right)\frac{\partial \mathbf{b}_{N2}\left(b_1^b,0,0\right)}{\partial b_{S1}} \end{aligned} \tag{31}$$

We can obtain a similar optimality condition for the case in which the bailout is delayed to period 2:

$$\begin{aligned} u'\left(Y_{S0} + qb_{S1}^{nb}\right) =& \frac{\beta^2}{q^2}\pi u'\left(Y - \mathbf{b}_{S2}\left(b_{1}^{nb}, \pi, \psi\right)\right) \\ &+ \frac{\beta^2}{q^2}(1-\pi)\frac{u'\left(Y - \frac{\mathbf{B}_2\left(b_1^{nb},\pi,\psi\right)}{2}\right)}{2} \\ &+ \frac{\beta^2}{2q}(1-\pi)\,u'\left(Y - \frac{\mathbf{B}_2\left(b_1^{nb},\pi,\psi\right)}{2}\right)\frac{\partial \mathbf{b}_{N2}\left(b_1^{nb},\pi,\psi\right)}{\partial b_{S1}} \end{aligned} \tag{32}$$

There are two channels through which early revelation of the central government's type induces local governments to issue more debt: the *strategic* channel and the *prudence* channel. The strategic channel has to do with the third term on the right side of the two optimality conditions above while the prudence channel has to do with the first two terms.

Consider the strategic channel first. The strategic interaction in debt choices between the two local governments is captured by the third term on the right side of (31) and (32). Each local government understands that its choice of debt issuance in period 0 will affect the debt issuance decision of the other government in period 1 which in turn affects the utility of the local government in period 2 in case of debt mutualization (which happens with probability $1-\pi$). We can show that the sensitivity of the North's (South's) debt issuance in period 2 to the debt issued by the South (North) in period 1 is decreasing in π. Formally,

$$\frac{\partial \mathbf{b}_{i2}\left(b_1, \pi', \psi\right)}{\partial b_{-i1}} < \frac{\partial \mathbf{b}_{i2}\left(b_1, \pi, \psi\right)}{\partial b_{-i1}} < 0. \tag{33}$$

for $\pi' > \pi$. The intuition is straightforward. If $\pi = 0$ as is the case when there is revelation of the central government's type at $t = 1$, then a given local government has a high incentive to adjust its period 1 debt issuance in response to the inherited debt of the other local government. This is because at $\pi = 0$ local governments know there will be debt mutualization with probability one next period. If instead there is no early revelation and $\pi > 0$ then there is debt mutualization in period 2 only with probability $1 - \pi$ and so a local government's debt issuance will be less sensitive to debt issued the previous period by the other local government.

Everything else equal, using condition (33) in (31) and (32) implies that the expected marginal cost of issuing debt in period zero is lower when there is early revelation about the central government's type because the other local government will cut its newly issued debt in period 1 by more when it knows that it is facing the no-commitment type. Hence the South (and the North) will issue more debt in period 0 because of the lower expected marginal cost.

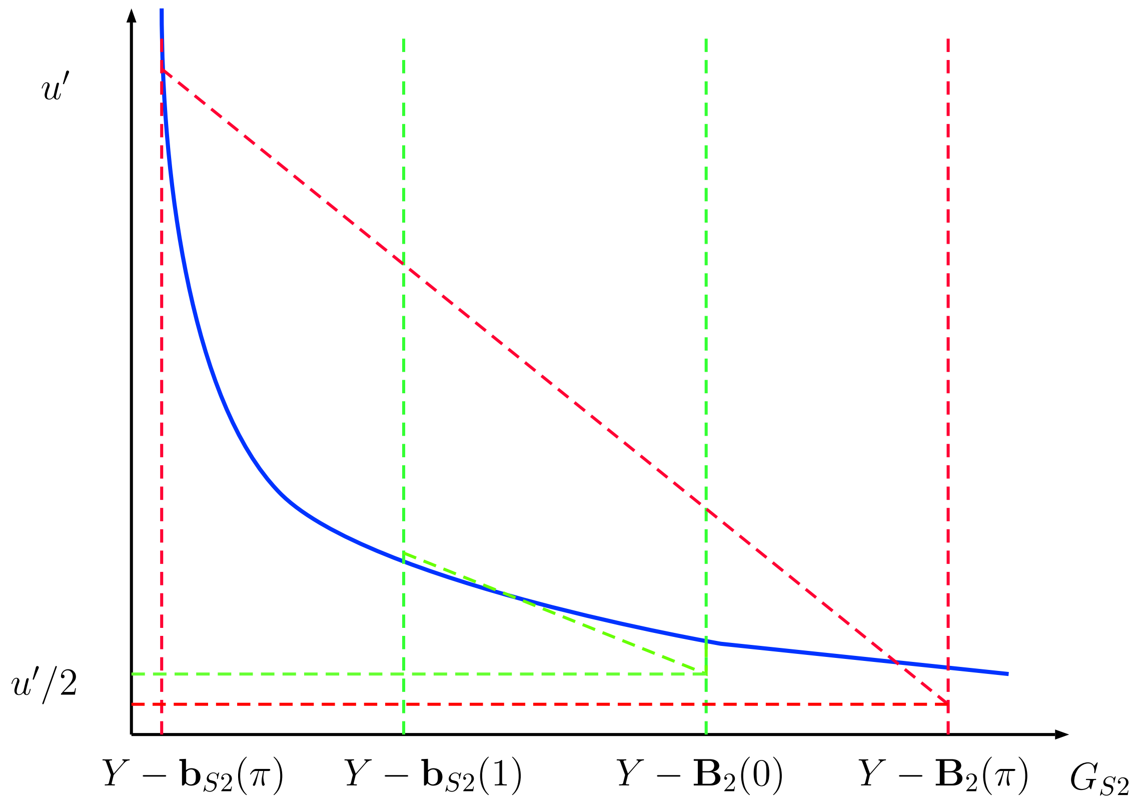
Consider the prudence channel now. This channel operates only if the utility function u displays prudence i.e. $u''' > 0$. Consider the first two terms on the right side of (31) and (32). With convex marginal utility, for an arbitrary b_1 and ψ small enough we have that

$$\pi u' \left(Y - \mathbf{b}_{S2}\left(b_1, 1, 0\right)\right) + (1-\pi) \frac{u'\left(Y - \frac{\mathbf{B}_2(b_1,0,0)}{2}\right)}{2} < \pi u' \left(Y - \mathbf{b}_{S2}\left(b_1, \pi, 0\right)\right) + (1-\pi) \frac{u'\left(Y - \frac{\mathbf{B}_2(b_1,\pi,0)}{2}\right)}{2}$$

This is illustrated in Figure 3 below. Intuitively, if the central government reveals its type in period 2 only, even if the South is confident it will receive a bailout in period 2 it does not borrow a lot in period 0 since it knows that if the central government is the commitment type, consumption in period 2 will be very low. If instead the central government reveals its type in period 1, the South will borrow more because, in the unlikely event that the central government is the commitment type, it can spread the losses associated with not having a bailout over period 1 and period 2. Because of prudence, this is preferable and so the government has a higher incentive to borrow more in period 0 because it can better insure the risk of facing the commitment type.

Consider now debt issuances in period 1 if the central government is the no-commitment type. In this case, debt issued into period 2 is higher with rules than without for two reasons: first, the inherited debt is larger; second, the local governments face no uncertainty about the type of central government and therefore now internalize only one half of the cost of issuing debt while with rules they internalize the full cost with probability π and one half the cost with probability $1 - \pi$. To see this, notice that $B_2^{nb} = \sum_i \mathbf{b}_2\left(b_1^b, \pi, 0\right)$ and

Figure 3: State contingent debt lowers marginal cost of debt issued in period 1



$B_2^b = \sum_i \mathbf{b}_2\left(b_1^b, 0, 0\right)$ and

$$B_2^b = \sum_i \mathbf{b}_2\left(b_1^b, 0, 0\right) > \sum_i \mathbf{b}_2\left(b_1^{nb}, 0, 0\right) > \sum_i \mathbf{b}_2\left(b_1^{nb}, \pi, 0\right) = B_2^{nb}$$

where the first inequality follows from the fact that debt inherited in period 1 is higher with rules, $b_1^b > b_1^{nb}$, as argued above; the second inequality follows from the fact that if we differentiate the first order condition (6) with respect to π holding fixed b_1 and evaluate it at $\pi = 0$ we obtain

$$\begin{aligned}
\frac{\partial \sum_i \mathbf{b}_{i2}\left(b_1^{nb}, 0, 0\right)}{\partial \pi} &= \frac{\beta\left[u'\left(Y - \mathbf{b}_{S2}\right) + u'\left(Y - \mathbf{b}_{N2}\right) - u'\left(Y - \frac{\mathbf{B}_2}{2}\right)\right]}{\left[u''\left(Y - b_{i1}^{nb} + q\mathbf{b}_{i2}\right)q^2 + \beta\frac{u''\left(Y - \frac{\mathbf{B}_2}{2}\right)}{4}\right]} \\
&\leq \frac{\beta\left[u'\left(Y - \mathbf{b}_{S2}\right) - \frac{1}{2}u'\left(Y - \mathbf{b}_{S2}\right) + u'\left(Y - \mathbf{b}_{N2}\right) - \frac{1}{2}u'\left(Y - \mathbf{b}_{N2}\right)\right]}{\left[u''\left(Y - b_{i1}^{nb} + q\mathbf{b}_{i2}\right)q^2 + \beta\frac{u''\left(Y - \frac{\mathbf{B}_2}{2}\right)}{4}\right]} \\
&< 0
\end{aligned}$$

so for π close to zero we have that $\sum_i \mathbf{b}_2 \left(b_1^{nb}, 0, 0\right) > \sum_i \mathbf{b}_2 \left(b_1^{nb}, \pi, 0\right)$.

This behavior is consistent with the experience of several federal governments in which after a bailout subnational governments kept on borrowing excessively. Arguably this is what happened in the EMU after the violation of Maastricht treaty in 2005 and the subsequent relaxation of the rules and penalties. This is also consistent with the experience in Brazil where "Debt burden continued to grow in the 1990s. Despite the previous crises and bailouts - or perhaps because of them - the states continued to increase spending." (Rodden et al. (2003)).

General case We illustrate Proposition 2 with a numerical example. The four panels of Figure 4 below display the debt issued by the South and North along the equilibrium path without rules (blue line) and with rules (red line) as a function of the prior in period 0 that the central government is the commitment type. As showed in Proposition 2, for π close to zero the debt issued in period 0 by all local governments is higher with rules than without. In the period 1, the debt issued by the South is lower with rules than without while the opposite is true for the North but total debt is larger with rules as shown in Proposition 2 and illustrated in Figure 5. The South issues less debt with rules in response to the large increase in debt issued by the North.

Figure 4 and 5 also illustrate the equilibrium dynamics when the initial prior π is not close to zero where we are not able to characterize the equilibrium analytically. Numerically, we show that without rules there exists an equilibrium with delayed revelation of uncertainty as is the case for π close to zero. With rules, when π is close enough to zero, there is early resolution of uncertainty where rules are not followed and total indebtedness is higher than the case without rules. When instead π is above a threshold, there exists an equilibrium where rules are followed by both countries, rules are binding for the South that borrows up to the limit, and the central government does not reveal its type in period one. In this case, total indebtedness is lower than in the case without rules. Note that the North still borrows more with rules because it now anticipates that the South will borrow less which implies that it will have to transfer less in the event of a bailout and so its expected marginal utility of consumption is lower. We can then conclude that fiscal rules may be effective in reducing debt when the central government's reputation is sufficiently high.

Figure 4: Equilibrium outcomes: Debt issued in period 1 and 2 by North and South

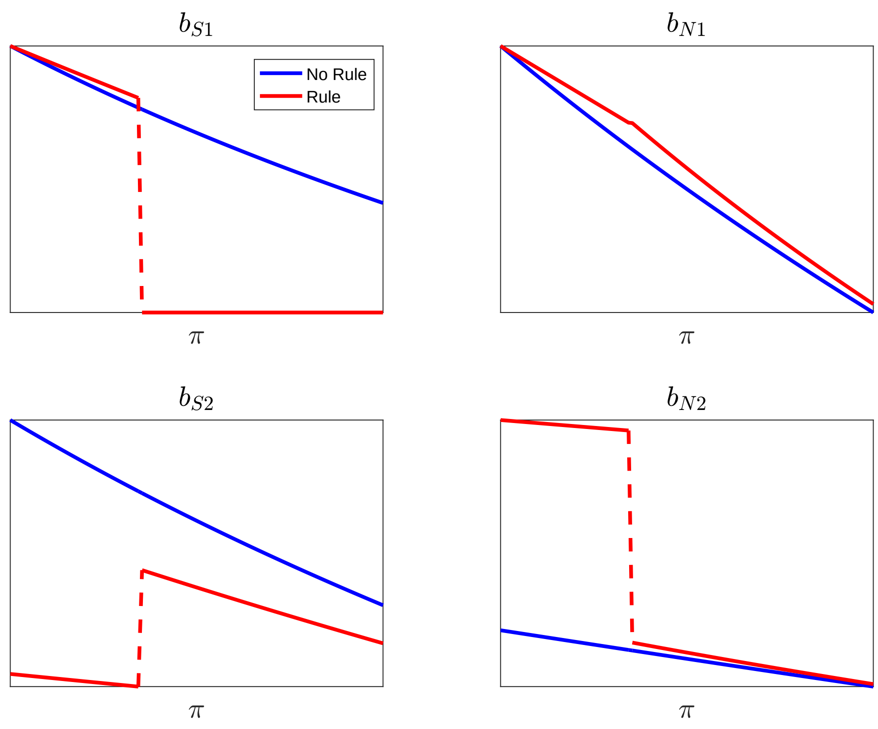
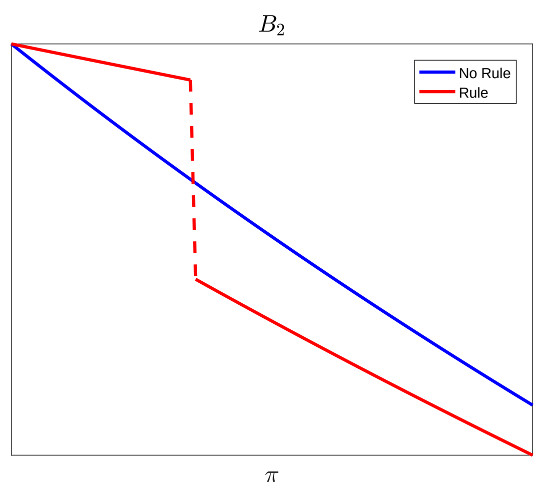


Figure 5: Equilibrium outcomes: Total debt issued



5 Ramsey problem

Optimality of no rules when credibility is low

We now turn to the optimal design of the fiscal rule. In particular, we analyze whether the optimal fiscal constitution should have fiscal rules. By optimal we mean the fiscal constitution that induces the maximal average welfare for the citizens knowing that the central government is the commitment type with probability π and the no-commitment type with probability $1-\pi$. We show that if the reputation of the central government is low, it is optimal to have no fiscal rules. This follows as a corollary of the previous result. Fiscal rules can be welfare improving only if they restrain local governments - in particular the South - from overborrowing. But we showed that the rules actually induce more borrowing for low π. Hence rules only have costs relative to the outcome without rules. In particular: i) rules promote more borrowing in period 0 when the local governments are already overborrowing relative to the efficient benchmark; ii) if the central government is the commitment type, there are output costs associated with the enforcement of rules ; iii) if the central government is the no-commitment type there is also more borrowing from period 1 to period 2 which is also detrimental for welfare.

As an illustration, we consider a numerical example with a given fiscal rule. This fiscal rule has the property that if the central government can somehow commit to enforce the rule in period 1,[7] aggregate welfare is higher with the rule. However, without commitment, we show that for π small enough, welfare is lower in the unique equilibrium with rules than in the one without rules. The figure also shows that rules can be beneficial when reputation is high enough. In particular, π must be large enough so that it is optimal ex-post for the central government to enforce the penalty associated with the fiscal rule. In this case, as illustrated in Figure 4, the South's debt issuance in equilibrium is constrained by the rule and so fiscal rules are effective in curbing debt issuances which leads to an increase in welfare.

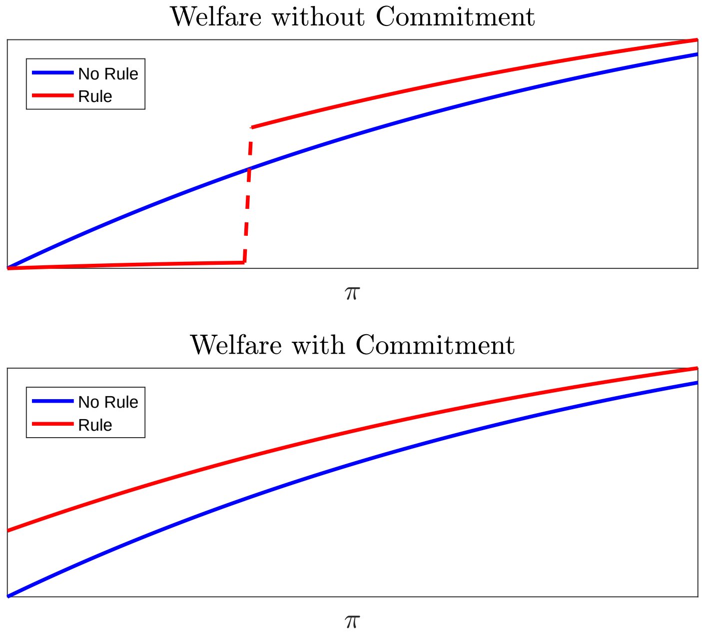
The next proposition establishes our second main result: if government's credibility is low, then it is optimal to have no fiscal rules.

Proposition 3. *Suppose that $u(c) = c - \frac{1}{2}\alpha c^2$ and the set of feasible punishments is bounded above by $\bar{\psi} < \infty$. For all such $\bar{\psi}$, if α and π are sufficiently small, the optimal fiscal rule without commitment has $\psi = 0$.*

When the central government has low reputation (π close to zero), having a fiscal rule

[7] More precisely, the central government can commit to enforce the constitution in period 1 but not in period 2. We do not think that this is a reasonable assumption. However, this assumption seems to be in line with a literature which assumes that the central government cannot commit to not bail out but can commit to enforcing fiscal rules. The example illustrates that if this was indeed the case, then fiscal rules can raise welfare.

Figure 6: Welfare with and without fiscal rules



lowers welfare. There is no $\psi < \infty$ and $\bar{b}$ that can improve welfare relative to the case without rules. The reason is that when reputation is low, the central government does not have the incentive to enforce the rule ex-post and there is early resolution of uncertainty that leads to more borrowing.

6 Equilibrium fiscal constitution

So far we have assumed that fiscal rules are given in period 0. In this section we study the *equilibrium fiscal constitution*, that is, the fiscal constitution that arises as the outcome of the signaling game between the two types of government in period 0. We show that if the commitment type is sufficiently patient, it is optimal for the commitment type to impose fiscal rules which will promote early resolution of uncertainty in period 1 and the no-commitment type will choose to mimic the commitment type in period 0 and also impose such rules (and violate them in period 1). This outcome can arise despite the fact that ex-ante it is efficient to impose no rules as shown in the previous section.

More formally, we add an additional stage to the policy game described in Section 2. In the initial stage, given the prior π about the type of central government, the central government chooses to write a fiscal constitution. A fiscal constitution has a no-bailout clause and a fiscal rule $(\psi, \bar{b})$ with $\psi \leq \bar{\psi}$. After observing the chosen fiscal constitution, the local governments update their prior about the type of the central government and the subsequent equilibrium outcome is an equilibrium outcome of the policy game described in the previous sections.

Definition. (Equilibrium fiscal constitution) An equilibrium fiscal constitution is the equilibrium outcome of the signaling game between the two type of the central government. Given a prior π, an equilibrium of the signaling game is a strategy for the commitment type central government ψ^c, a strategy by the non-commitment type ψ^{nc}, and beliefs π_0' such that: i) beliefs evolve according to

$$\pi_0'\left(\psi,\pi;\psi^c,\psi^{nc}\right)=\begin{cases}\pi & \text{if } \psi=\psi^{nc}=\psi^c\\ 0 & \text{if } \psi\text{=}\psi^{nc}\neq\psi^c\\ 1 & \text{if } \psi=\psi^c\neq\psi^{nc}\\ 0 & \text{if } \psi\notin\{\psi^c,\psi^{nc}\}\end{cases} \tag{34}$$

ii) given ψ^{nc}, the strategy for the commitment type ψ^c is optimal, in that for all ψ

$$W_0^c\left(\pi_0'\left(\psi^c,\pi;\psi^c,\psi^{nc}\right),\psi^c\right)\geq W_0^c\left(\pi_0'\left(\psi,\pi;\psi^c,\psi^{nc}\right),\psi\right)$$

where W_0^c is defined in (14); iii) given ψ^c, the strategy ψ^{nc} for the no-commitment type is optimal, in that for all ψ

$$W_0\left(\pi_0'\left(\psi^{nc},\pi;\psi^c,\psi^{nc}\right),\psi^{nc}\right)\geq W_0\left(\pi_0'\left(\psi,\pi;\psi^c,\psi^{nc}\right),\psi\right)$$

where W_0 is defined in (13).

We can characterize an equilibrium of this game by considering the fiscal rule chosen by the commitment type given the prior π. We can think of the problem for the commitment type in period 0 to be:

$$W_0^c=\max\left\{W_0^{c,\text{sep}},W_0^{c,\text{pool}}\right\}$$

where $W_0^{c,\text{sep}}$ is the value for the commitment type if it chooses a fiscal rule that ensures separation in period 1:

$$\begin{aligned}W_0^{c,\text{sep}}=&\max_{\psi,\bar{b}}\sum_i u\left(Y_{i0}+qb_i^b\left(\pi,\psi\right)\right)+\\&+\beta\sum_i\left[\begin{array}{c}u\left(Y-\psi Y\mathbb{I}_{b_{i1}>\bar{b}}-b_{i1}^b\left(\pi,\psi\right)+q\mathbf{b}_{i2}\left(b_{i1}^b\left(\pi,\psi\right),1,\psi\right)\right)\\+\beta u\left(Y-\mathbf{b}_{i2}\left(b_{i1}^b\left(\pi,\psi\right),1,\psi\right)\right)\end{array}\right]\end{aligned}$$

subject to

$$W_1^b\left(b_{i1}^b(\pi,\psi)\right) = W_1\left(b_{i1}^b(\pi,\psi),0,0\right) \geq W_1\left(b_{i1}^b(\pi,\psi),1,\psi\right)$$

and global optimality for the north in period 0. Conversely, $W_0^{c,\text{pool}}$is the value for the commitment type if the fiscal constitution it chooses is such that the no-commitment type will have an incentive to enforce the rule in period 1:

$$\begin{aligned} W_0^{c,\text{pool}} &= \max_{\psi,\bar{b}} \sum_i u\left(Y_{i0} + q b_i^{nb}(\pi,\psi)\right) + \\ &+ \beta \sum_i \left[\begin{array}{c} u\left(Y - \psi Y \mathbb{I}_{b_{i1} > \bar{b}} - b_{i1}^{nb}(\pi,\psi) + q \mathbf{b}_{i2}\left(b_{i1}^{nb}(\pi,\psi),\pi,\psi\right)\right) \\ +\beta u\left(Y - \mathbf{b}_{i2}\left(b_{i1}^{nb}(\pi,\psi),\pi,\psi\right)\right) \end{array}\right] \end{aligned}$$

subject to

$$W_1\left(b_{i1}^{nb}(\pi,\psi),\pi,\psi\right) \geq W_1^b\left(b_{i1}^{nb}(\pi,\psi)\right) = W_1\left(b_{i1}^{nb}(\pi,\psi),0,0\right)$$

and global optimality for the South in period 0. In setting up the problem we assumed that it was optimal for the no-commitment type to mimic the strategy of the commitment type in period zero. In the next Proposition we provide sufficient conditions for this to be the case.

Suppose first that the commitment type can only choose between two levels of $\psi \in \{0, \bar{\psi}\}$. The next proposition shows that if the commitment type central government is sufficiently patient then there exists a unique equilibrium fiscal constitution that has fiscal rules. Moreover, the no-commitment type central government prefers to mimic the strategy of the commitment type in period zero and chooses a constitution with fiscal rules despite the fact that it knows that it will not enforce the constitution in period 1.

Proposition 4. *For π close to 0, there exists $\underline{\beta}$ such that for $\beta < \underline{\beta}$, there exists a unique constitution with no fiscal rules and $\psi = 0$. For π close to 0 and Y small enough, there exists $\bar{\beta}$ such that if $\beta \in \left[\underline{\beta}, \bar{\beta}\right]$, there exists a unique constitution with fiscal rules which are violated by local governments and there is early resolution of uncertainty in period 1.*

Despite the optimality of no fiscal rules under the veil of ignorance, when the reputation of the central government is sufficiently close to zero, fiscal rules arise in equilibrium. This is because the commitment type wants to set up a fiscal constitution in which its type is revealed in period 1. This has benefits because in period 1 the reputation of the central government will jump from almost zero to one and so promoting fiscal discipline. In particular, the local government's decision will satisfy the Euler equation and so is efficient from period 1 onward.[8] But it also has costs. As we have shown in Proposition 2, instituting fiscal rules promotes overborrowing and fiscal indiscipline in period 0. When β is sufficiently high the benefits outweigh the costs.

Next, suppose that the commitment type can choose ψ in an interval $[0, \bar{\psi}]$. The proof for the above proposition is identical except that the objects $\underline{\beta}, \bar{\beta}$ now depend on the optimal choice ψ and thus are no longer defined in the terms of fundamentals. However, if the equations defining these bounds are well defined, then the proposition holds in this case as well.

Proposition 4 rationalizes why we often observe central governments with low reputation setting up tough fiscal rules. See for instance the case of Eurozone after the European debt crisis and the bailouts in Greece, Portugal, Ireland and Spain with the institution of the "Six-Pact" and the case of Brazil after the bailouts in 1997 and the Fiscal Responsibility Law approved by the Cardoso administration. In both cases, the reputation of the central government was low because of the recent bailouts to local governments.

As a final point in this section, we argue that equilibrium fiscal constitution is *unique*. There are three types of outcomes than can be equilibria. The first is one in which there is no fiscal rule instituted by any of the central government types. The second and third are ones in which the commitment type announces a rule and the no-commitment type mimics and does not mimic respectively. Notice that an outcome in which the no-commitment type announces a fiscal rule and the commitment type does not can never be an equilibrium. The first step of the argument is to show that conditional on the commitment type announcing a fiscal rule, for π close to zero, the no-commitment type always prefers to mimic. This is established in the proof of Proposition 4, where we show that the the reputation cost of not mimicking is of first order while the benefit of equalizing consumption is of second order when π is close to zero. The final step is show that an outcome with no fiscal rules cannot co-exist with an outcome with fiscal rules and mimicking. But again, as the previous proposition shows, these two equilbrium outcomes exist at disjoint regions of the parameter space and so cannot co-exist. We summarize this in the lemma below.

Proposition 5. *The equilibrium fiscal constitution is unique.*

The proof follows from proposition 4 and the discussion above.

[8] Of course, the commitment type central government would like to redistribute resources from the North to the South, but in our setup it has no instruments to do so.

7 Extensions

7.1 Finite Horizon Economy

We argue that our main results extend to a finite horizon economy. In particular, consider a $T < \infty$ environment with heterogeneity in period 0 tax revenues. Consider first the case without rules and the extension of Proposition 1 to this setup. As before, at $\pi = 0$, at each t, $W_t^{nb}(b_t, 0) = W_t^b(b_t)$. Therefore, at any t, for π sufficiently close to zero, the gains of redistribution are second order while the costs of losing reputation are first order. Hence it always optimal for the no-commitment type to delay the bailout until the very last period. Moreover, our sufficient conditions on the coefficient of absolute risk-aversion will continue to ensure that the South does not want to deviate and induce a bailout for π close to zero. Note that the assumption that T is finite is important for this result. It would be interesting to study the infinite horizon economy.

Next, consider the setup with fiscal rules. As before, if we assume that β is small enough so that $W_1^{nb}\left(b_1^b(0), 0, 0\right) > W_1^{nb}\left(b_1^b(0), 1, \psi\right)$, then for π close to and including zero, we have a unique equilibrium in which all uncertainty is resolved in period 1.

7.2 Lack of commitment on the part of local governments

We now relax the assumption that the local governments can commit to repaying their debt to foreign lenders. We will show that our main results are unchanged in this case. In particular, we show that debt issuance with rules is larger than without rules.

As is standard in these environments, in order to incentivize governments to pay their debts back default must be costly. Suppose that default in period 2 entails a utility cost κ while default in period 1 entails a utility cost κ as well as exclusion from credit markets. First consider the case without fiscal rules. In period 2, if there is no debt-mutualization, then the local authority repays its debt if

$$u(Y - b_{i2}) \geq u(Y) - \kappa$$

which implies a debt constraint: $b_{i2} \leq \phi_2$. If there debt-mutualization, then the local authority repays its debt if

$$u\left(Y - \frac{B_2}{2}\right) \geq u(Y) - \kappa$$

which implies a constraint on total debt: $B_2 \leq 2\phi_2$. Therefore, in period 1 given beliefs π, the price of debt is given by

$$Q_2(b_{i2}, b_{-i2}, \pi) = \begin{cases} q & b_{i2} \leq \phi_2,\ b_{-i2} \leq \phi_2 \\ q(1-\pi) & b_{i2} > \phi_2,\ B_2 \leq 2\phi_2 \\ 0 & b_{i2} > \phi_2,\ B_2 > 2\phi_2 \end{cases}$$

In particular, notice that lower π implies lower spreads on debt. So the problem for the local authority in period 1 (conditional on repayment) is

$$V_{i1}(b_1, \pi) = \max_{b_{i2}} u(Y - b_{i1} + Q_2(\pi, b_{i2}, b_{-i2}) b_{i2}) + \beta\pi u(Y - b_{i2}) + \beta(1-\pi) u\left(Y - \frac{B_2}{2}\right)$$

Now consider the incentives to default in period 1. Default is associated with a utility cost as well as exclusion from borrowing and lending. Suppose further that transfers cannot be made to or from a government that has defaulted.[9] This implies that the value of default is is $V_1^d = (1+\beta) u(Y) - \kappa$. Therefore, the local authority will repay its debt in period 1 if $V_{S1}(b_1, \pi) \geq V_1^d$. Suppose that there is no revelation of uncertainty in period 1. Then the above constraint implies a borrowing constraint $\phi_{i1}(b_{-i}, \pi)$ on country i and debt price

$$Q_1^{nb}(b_{-i1}, \pi) = \begin{cases} q & b_{i1} \leq \phi_{i1}(b_{-i}, \pi) \\ 0 & o.w \end{cases}$$

On the other hand, if there is revelation of uncertainty in period 1 we have a constraint on total debt, $B_1 \leq 2\phi_1(0)$, since $V_{i1}(b_1, 0) = V_{i1}\left(\frac{B_1}{2}, \frac{B_1}{2}, 0\right)$. Let the constraint associated with the local authority facing the commitment type in period 1 be $\phi_1(1)$. We will consider the case in which these constraints are potentially binding for the South and since, $V_{S1}\left(\frac{B_1}{2}, \frac{B_1}{2}, 0\right) > V_{S1}(b_1, 1)$, we have that $\phi(0) > \phi(1)$. Therefore, the price of debt is given by

$$Q_1^b(b_{i1}, b_{-i1}, \pi) = \begin{cases} q & b_{i1} \leq \phi(1) \\ q(1-\pi) & b_{i1} > \phi(1),\ B_1 \leq 2\phi(0) \\ 0 & b_{i1} > \phi(1),\ B_1 > 2\phi(0) \end{cases}$$

So as before we can define the NB equilibrium outcomes which solve

$$\max_{b_{i1}} u\left(Y_{i0} + Q_1^{nb}\left(b_1^{nb}, \pi\right) b_{i1}\right) + \beta V_{i1}\left(b_1^{nb}, \pi\right)$$

[9] This assumption ensures that the value of default is independent of whether there is revelation of uncertainty in period 1 or not.

and the B outcomes which solve

$$\max_{b_{i1}} u\left(Y_{i0} + Q_1^b\left(b_1^b, \pi\right) b_{i1}\right) + \beta\pi V_{i1}\left(b_1^b, 1\right) + \beta\left(1-\pi\right) V_{i1}\left(b_1^b, 0\right)$$

Our next result shows that the analog of Proposition 1 holds in this environment namely that under sufficient conditions, the equilibrium without rules in one with delayed revelation of uncertainty. The interesting case in the one in which these borrowing constraints bind in period 1 and so we consider the case in which for a neighborhood around $\pi = 0$, $b_{S1}^{nb,c}(\pi) > \phi_{S1}\left(b_{N1}^{nb,c}, \pi\right)$ where $b_1^{nb,c}(\pi)$ denotes the NB equilibrium outcome if the local authorities can commit to repayment of debt. Therefore, the definition of an equilibrium outcome with no debt mutualization is identical to the one previously, with the additional constraint $b_{S1} \leq \phi_{S1}(b_{N1}, \phi)$. For the relevant region, this will hold with an equality.

Proposition 6. *(No bailout in period 1 when credibility is low) At $\pi = 0$ there are two equilibria with the same local public good provision that differ in the timing of debt mutualization . Suppose that the coefficient of absolute risk aversion $\frac{-u''(c)}{u'(c)}$ and π are sufficiently small. Then, there exists an equilibrium where there is no debt mutualization in period 1 in that $\sigma(b_1(\pi), \pi) = 0$.*

Next, lets consider the environment with rules. As before, we assume β small enough so that $W_1^b\left(b_1^b(0)\right) > W_1^{nb}\left(b_1^b(0), 1\right)$. We know that at the bailout allocation for π close to zero the central authority will want to bail out and the North cannot deviate to prevent it. Therefore it is straightforward to show (as in Proposition 2) that there exists an equilibrium with early resolution of uncertainty.

We will establish that total debt is larger in the equilibrium with rules. Since $V_{S1}\left(\frac{B_1}{2}, \frac{B_1}{2}, 0\right) > V_{S1}(b_1, \pi) > V_{S1}(b_1, 1)$, we have that $\phi(0) > \phi_{i1}(b_{-i}, \pi) > \phi_1(1)$. Suppose by way of contradiction that for π close to zero we have $B_1^{nb}(\pi, 0) > B_1^b(\pi, \psi)$. Then it must be that $V_{S1}\left(\frac{B_1^b}{2}, \frac{B_1^b}{2}, 0\right) > V_{S1}\left(\frac{B_1^{nb}}{2}, \frac{B_1^{nb}}{2}, 0\right) > V_{S1}\left(b_1^{nb}, \pi\right) = V_1^d$. But since the South was borrowing constrained at the debt level in the NB equilibrium at $\pi = 0$ it must be that it also constrained at the debt level associated with the bailout equilibrium at $\pi = 0$ i.e.

$$u'\left(Y_{S0} + qb_{S1}^b(0)\right) > \beta u'\left(Y - b_{S1}^b(0) + qb_{S2}^b(0)\right) + \frac{\beta^2}{2} u'\left(Y - \frac{B_2^m}{2}\right) \frac{\partial b_{N2}^b}{\partial b_{S1}}$$

Therefore, for π close to zero it will be borrowing constrained at the bailout equilibrium debt level and since $V_{S1}\left(\frac{B_1^b}{2}, \frac{B_1^b}{2}, 0\right) > V_1^d$ implies that it can borrow more we have a contradiction. Therefore it must be that $B_1^b(\pi, \psi) \geq B_1^{nb}(\pi, 0)$. The next proposition summarizes the argument above:

Proposition 7. *Total debt inherited in period 1 is larger in the equilibrium with fiscal rules.*

7.3 Non strategic local governments

We now show that having large and strategic local governments is crucial for the lack of desirability of fiscal rules in our environment. To see this, we consider the case in which local governments are not strategic. In particular, suppose there is a continuum of local governments. Each local government can be one of two types: North and South. It is immediate to show that in this case, without a no-bailout clause, even if the central government cannot commit, the efficient allocation is an equilibrium outcome of the policy game. The idea is that it is always ex-post optimal for the central government not to bailout a measure zero local government that issues more debt than the efficient level.[10] Hence the equilibrium transfer rules will depend only on the aggregate debt level in the North and South and not on the individual level of debt. Hence the bailout transfers do not distort the decisions of the local governments.

This result is specific to our environment where transfers by the central government are lump-sum and non-distortionary. Chari et al. (2016) and Aguiar et al. (2015) reach different conclusions due to the assumption of distortionary transfers. We consider this extension next in the context of a monetary economy. Of course, we could reach similar conclusions simply by adding a utility cost $\tau(T)$ of implementing a transfer of size T.

7.3.1 Extension to a monetary economy

We now consider an extension of the baseline model in which bailouts are distortionary. We show that when local governments are non-atomistic, there exits an equilibrium outcome with fiscal rules that can improve upon the equilibrium outcome without fiscal rules. The improvement is fragile because therealso exists an equilibrium in which rules are violated and not enforced ex-post. When local governments are large instead, then the logic of Proposition 2 and Proposition 3 holds and we can have a unique equilibrium with overborrowing relative to the case without rules.

Consider a monetary version of the baseline model in which the monetary authority (central government) chooses the inflation rate and the enforcement of the fiscal rule. The preferences of the local governments are

$$\sum_{t=0}^{3} \beta^t \left[u\left(G_{it}\right) - \tau_t\left(\Pi_t\right)\right]$$

where u is strictly increasing, strictly concave and satisfies inada and τ_t is weakly increas-

[10] It is worthwhile to note that a fiscal constitution with a no-bailout clause is welfare reducing with non-strategic local governments as the efficient allocation from an utilitarian perspective can never be an equilibrium of the game as it prevents the optimal level of redistribution (from the perspective of a utilitarian welfare criterion).

ing and convex. The budget constraint for the local government is

$$G_{it} \leq Y - \psi Y \mathbb{I}_{b_{it} > \bar{b}} + Q_{t+1} b_{it+1} - \frac{b_{it}}{\Pi_t}$$

given some initial $b_{i0} = b_0$. The monetary authority chooses Π_t and the enforcement of the fiscal rule to maximize an equally weighted sum of the local governments' utility. Instead of a no-bailout clause, we assume that the constitution contains a strict inflation target, which we assume to be zero.[11] As before, local governments believe that the monetary authority is a commitment type with probability π. We also assume that the local governments can commit to repaying their debts and that there a continuum of risk-neutral lenders who discount the future at a rate q. The no-arbitrage condition for the lenders requires that

$$Q_t(b_t, \pi) = q \left[\pi + (1 - \pi) \frac{1}{\Pi_t(b_t, \pi)} \right]$$

where as before $b_t = \{b_{it}\}_{i \in I}$ and Π_t is the equilibrium decision rule of the no-commitment type monetary authority.

Notice that this environment features a free-rider problem that is different from our baseline model. Each local government does not internalize the costs of debt accumulation on other local governments through the price of debt. This is true independently of whether local governments are non-atomistic or not. We first show that with non-atomistic (measure zero) local governments, there exists $\psi > 0$ and $\bar{b}$ such that for all π, fiscal rules can improve upon the outcome without fiscal rules.

Proposition 8. *There exists $\psi > 0$ and $\bar{b}$ such that for all π there exists an equilibrium outcome with fiscal rules that can improve upon the equilibrium outcome without fiscal rules.*

The key intuition for Proposition 7 is that with non-atomistic local governments, there are no costs for the central government to enforce the penalty for a violation of the fiscal rule by an *individual* local government that has measure zero. Hence, if one local government expects that other local governments will respect the fiscal rule, it is optimal for it to respect the rule as well and so there is an equilibrium in which fiscal rules can curb indebtedness and where local governments internalize the free-rider problem. This result is fragile: there is always an equilibrium where the rule is ignored and not enforced. In particular, if a government expects the other governments to violate the rule, it will find it optimal to violate the rule as well since it anticipates that the rule will not be enforced ex-post. This type of multiplicity is similar to the one in Farhi and Tirole (2012) and Chari and Kehoe (2015). Note that as in Proposition 2, the central government will not find it optimal to enforce rules for π close to zero and β small enough.

[11] Other than for period 0, this is the Ramsey outcome with commitment.

Non-atomistic local governments are critical for the validity of Proposition 8. When the local governments are large the forces emphasized in our baseline environment still operate and we can prove the analog of Proposition 2 for this monetary economy. The intuition is the same as before: if local governments violate the fiscal rule and the reputation of the central government is low, ex-post the no-commitment type central government will not enforce the fiscal rule because the reputational gains are small relative to the cost of imposing the penalty for a violation. To simplify the argument, we consider a case in which the monetary authority cannot inflate in period 1[12] (or it is too costly to do so) and $I = \{N, S\}$.

Absent rules, since there is no decision by the central government in period 1, the equilibrium outcome has no revelation of uncertainty. This is beneficial because such uncertainty restrains debt issuance in period 1 and makes the path of indebtedness closer to the cooperative solution. With binding rules, if the reputation of the central government is sufficiently low, an equilibrium in pure strategies must have early revelation of uncertainty because the central government does not have incentive to enforce the sanction ex-post.

The next proposition is the analog of Proposition 2 confirming that the forces we emphasize in our baseline model extend to this monetary economy.

Proposition 9. *For all $\psi > 0$, there exist π and β small enough such that if fiscal rules are binding then the no-commitment type central government does not enforce the fiscal constitution in period 1.*

8 Conclusion

Fiscal rules are often thought to be useful in federal states when the central government cannot commit to no-bailout clauses. In this paper, we ask if this is indeed the case when the central government also cannot commit to imposing these rules. In such an environment, we show that outcomes with rules can attain lower welfare than outcomes without rules. Moreover, the outcomes associated with fiscal rules are worse exactly when there is a high probability that the central government cannot commit. Our results also shed light on the multitude of examples throughout history when fiscal rules have been instituted but not enforced. Our analysis of the equilibrium constitution suggests that stringent fiscal rules arise when the reputation of the government is low even though they are not optimal under the veil of ignorance.

[12] By doing so we do not have to check the global optimality condition when constructing equilibria and the local optimality condition is enough.

One interesting extension we do not pursue in this paper would be to study the infinite horizon dynamic game. This would be particularly interesting in the context of an environment where the local governments cannot commit to repay debt to study the joint dynamics of debt, central government reputation and interest rate spreads on local government debt. This may help to understand the dynamics of interest rates during the European debt crisis where according to several commentators much of the dynamics of spreads was attributable to political risk or the willingness of the European institutions to bail out members in crisis.

This paper does not provide a meaningful theory of the instances in which fiscal rules have been effective in reducing debt. One such example is the United States. A simplistic answer, which would be consistent with our theory, would be to say that the US central government has a high reputation. However, we believe that differences in institutional features might help account for the differences in the efficacy of fiscal rules and should be an important avenue for future research.

On a related note, it is worth considering what kinds policies can prevent over-borrowing even when the central government's reputation is very low. Our results suggest that policies which constrain the actions of the central government are more likely to work than those which constrain the actions of local governments (and are sustained by punishments). For example, if there was a cap on the amount of tax revenues the central government could access, this would reduce the underlying free-rider problem. See Rodden (2006) for a similar argument. However, this would also reduce the amount of consumption insurance possible and as a result the optimal cap would trade off the costs of consumption smoothing with the benefits of lowering debt. We leave this and similar extensions to future work.

Finally, we assumed that the central government is benevolent and maximizes utility of local governments. Another possibility is to study institutional settings where representatives from local governments vote to impose sanctions on local governments that violate the rule. This is left for future research.

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A Omitted Proofs

Proof of Lemma 1

Proof. In period 1, if there are no transfers the optimal b_2 solves

$$qu'\left(Y - b_{i1} + qb_{i2}\right) = \frac{\beta}{2}u'\left(Y - \frac{b_{i2} + b_{-i2}}{2}\right) \quad i = n, s$$

which is the same first order condition as in the equilibrium with a transfer in period 1. Hence consumption levels must be the same at $t = 1, 2$ for each states. □

Proof of Lemma 2

Proof. We first establish that $\mathbf{b}_{i2}\left(b, \pi\right)$ defined as

$$qu'\left(Y - b_{i1} + q\mathbf{b}_{i2}\right) = \beta\pi u'\left(Y - \mathbf{b}_{i2}\right) + \beta\left(1-\pi\right)\frac{u'\left(Y - \frac{\sum_j \mathbf{b}_{i2}}{2}\right)}{2} \quad \text{for } i = N, S. \tag{35}$$

is increasing in inherited debt. From (35), applying the implicit function theorem/totally differentiating we obtain

$$-qu''\left(G_{i1}\right)db_{i1} + qu''\left(G_{i1}\right)d\mathbf{b}_{i2} = -\beta\pi u''\left(G_{i2}^c\right)d\mathbf{b}_{i2} - \frac{\beta\left(1-\pi\right)}{4}u''\left(G_{i2}\right)d\mathbf{b}_{i2} - \frac{\beta\left(1-\pi\right)}{4}u''\left(G_{i2}\right)d\mathbf{b}_{-i2}$$

where G_{i2}^c (G_{i2})denotes the consumption of the local public good in period 2 when local governments know that they are facing the commitment type (no-commitment type). Rearranging

$$\left[-\beta\pi u''\left(G_{i2}^c\right) - \frac{\beta\left(1-\pi\right)}{4}u''\left(G_{i2}\right) - qu''\left(G_{i1}\right)\right]d\mathbf{b}_{i2} = -qu''\left(G_{i1}\right)db_{i1} + \frac{\beta\left(1-\pi\right)}{4}u''\left(G_{i2}\right)d\mathbf{b}_{-i2}$$

so, letting

$$A_i = \left[-\beta\pi u''\left(G_{i2}^c\right) - \frac{\beta\left(1-\pi\right)}{4} u''\left(G_{i2}\right) - q u''\left(G_{i1}\right)\right] > 0$$
$$a_i = \frac{4}{\beta\left(1-\pi\right)} A_i > 0$$

we obtain

$$\frac{\partial \mathbf{b}_{i2}}{\partial b_{i1}} = \frac{-q u''\left(G_{i1}\right)}{A_i} + \frac{u''\left(G_{i2}\right)}{a_i} \frac{d b_{-i2}}{d b_{i1}}$$

With similar manipulations we get

$$\frac{\partial \mathbf{b}_{-i2}}{\partial b_{i1}} = \frac{u''\left(G_{-i2}\right)}{a_{-i}} \frac{d b_{i2}}{d b_{i1}}$$

(as before but without direct effect). Hence, solving the system:

$$\frac{\partial \mathbf{b}_{i2}}{\partial b_{i1}} = \frac{-q u''\left(G_{i1}\right)}{A_i} + \frac{u''\left(G_{i2}\right)}{a_i} \frac{u''\left(G_{-i2}\right)}{a_{-i}} \frac{d b_{i2}}{d b_{i1}}$$
$$= \frac{1}{1 - \frac{u''\left(G_{i2}\right)}{a_i} \frac{u''\left(G_{-i2}\right)}{a_{-i}}} \frac{-q u''\left(G_{i1}\right)}{A_i} > 0$$

$$\frac{\partial \mathbf{b}_{i2}}{\partial b_{-i1}} = \frac{u''\left(G_{-i2}\right)}{a_{-i}} \frac{1}{1 - \frac{u''\left(G_{i2}\right)}{a_i} \frac{u''\left(G_{-i2}\right)}{a_{-i}}} \frac{-q u''\left(G_{i1}\right)}{A_i} < 0$$

where the sign of the two derivatives come from the fact that $u'' < 0$ and

$$\frac{u''\left(G_{i2}\right)}{a_i} \frac{u''\left(G_{-i2}\right)}{a_{-i}} = \prod_{i=N,S} \frac{\frac{\beta(1-\pi)}{4} u_i''}{A_i} = \prod_{i=N,S} \frac{-\frac{\beta(1-\pi)}{4} u''\left(G_{i2}\right)}{\left[-\beta\pi u''\left(G_{i2r}\right) - \frac{\beta(1-\pi)}{4} u''\left(G_{i2}\right) - q u''\left(G_{i1}\right)\right]}$$
$$< 1$$

Moreover, note for later that

$$\left|\frac{\partial \mathbf{b}_{i2}}{\partial b_{-i1}}\right| = \frac{-u''\left(G_{-i2}\right)}{a_{-i}} \frac{1}{1 - \frac{u''\left(G_{i2}\right)}{a_i} \frac{u''\left(G_{-i2}\right)}{a_{-i}}} \frac{-q u''\left(G_{i1}\right)}{A_i} < \frac{1}{1 - \frac{u''\left(G_{i2}\right)}{a_i} \frac{u''\left(G_{-i2}\right)}{a_{-i}}} \frac{-q u''\left(G_{i1}\right)}{A_i} = \frac{\partial \mathbf{b}_{i2}}{\partial b_{i1}}$$

so outcomes are more responsive to own debt. So an increase in b_{i1} leads to an increase in total indebtedness $B_2 = \sum_i b_{i2}$.

We now turn to show how $\mathbf{b}_{i2}\left(b_1, \pi\right)$ varies with π. First define

$$\Delta MU_i \equiv \beta \left[u'\left(Y - b_{i2}\right) - \frac{u'\left(Y - \frac{\sum_j b_{j2}}{2}\right)}{2}\right]$$

Since $b_{S2} > \frac{\sum_i b_{i2}}{2} > b_{N2}$, $\Delta MU_S > 0$. The term ΔMU_N is in general ambiguous but it is positive for π close to zero. [13]

Also notice that

$$\Delta MU_S - \Delta MU_N = \beta \left[u' \left(Y - b_{S2} \right) - u' \left(Y - b_{S2} \right) \right] > 0$$

Using the implicit function theorem we have that

$$A_i db_{i2} = \frac{\beta \left(1 - \pi \right)}{4} u'' \left(G_{i2} \right) db_{-i2} - \Delta MU_i d\pi$$

we obtain

$$\begin{aligned} \frac{\partial b_{i2}}{\partial \pi} &= \frac{-\Delta MU_i}{A_i} + \frac{u'' \left(G_{i2} \right)}{a_i} \frac{\partial b_{-i2}}{\partial \pi} \\ &= \frac{-\Delta MU_i}{A_i} + \frac{u'' \left(G_{i2} \right)}{a_i} \left(\frac{-\Delta MU_{-i}}{A_i} + \frac{u'' \left(G_{-i2} \right)}{a_{-i}} \frac{\partial b_{i2}}{\partial \pi} \right) \end{aligned} \tag{36}$$

so

$$\begin{aligned} \frac{\partial \mathbf{b}_{i2}}{\partial \pi} &= \frac{1}{1 - \frac{u''(G_{i2})}{a_i} \frac{u''(G_{-i2})}{a_{-i}}} \frac{-\Delta MU_i}{A_i} + \frac{u'' \left(G_{i2} \right)}{a_i} \frac{-\Delta MU_{-i}}{A_i}. \\ &= -\frac{1}{A_i} \left[\frac{\Delta MU_i}{1 - \frac{u''(G_{i2})}{a_i} \frac{u''(G_{-i2})}{a_{-i}}} + \frac{u'' \left(G_{i2} \right) \Delta MU_{-i}}{a_i} \right] \\ &< -\frac{1}{A_i} \left[\Delta MU_i + \frac{u'' \left(G_{i2} \right)}{a_i} \Delta MU_{-i} \right] \\ &< \frac{1}{A_i} \left[-\Delta MU_i + \Delta MU_{-i} \right] \end{aligned}$$

since $\frac{u''(G_{i2})}{a_i} > -1$. Therefore, $\frac{\partial \mathbf{b}_{S2}}{\partial \pi} < 0$. Next, we have

[13] In fact, from the foc (35) we can write

$$\left[qu' \left(Y - b_{N1} + qb_{N2} \right) - \beta \frac{u' \left(Y - \frac{\sum_j b_{j2}}{2} \right)}{2} \right] = \pi \Delta MU_N = \pi \left[\beta u' \left(Y - b_{i2} \right) - \beta \frac{u' \left(Y - \frac{\sum_j b_{j2}}{2} \right)}{2} \right]$$

Note that for $\pi \downarrow 0$ we have that the LHS equals zero and so

$$\lim_{\pi \downarrow 0} \left[\beta u' \left(Y - b_{i2} \right) - \beta \frac{u' \left(Y - \frac{\sum_j b_{j2}}{2} \right)}{2} \right] = \lim_{\pi \downarrow 0} \left[\beta u' \left(Y - b_{i2} \right) - qu' \left(Y - b_{N1} + qb_{N2} \right) \right] > 0$$

$$\frac{\partial \mathbf{B}_2}{\partial \pi} = \frac{1}{1-\frac{u''(G_{S2})}{a_S}\frac{u''(G_{N2})}{a_N}} \frac{-\Delta MU_S}{A_S} + \frac{u''(G_{S2})}{a_S}\frac{-\Delta MU_N}{A_S}$$
$$+ \frac{1}{1-\frac{u''(G_{S2})}{a_S}\frac{u''(G_{N2})}{a_N}} \frac{-\Delta MU_N}{A_N} + \frac{u''(G_{N2})}{a_N}\frac{-\Delta MU_S}{A_N}$$

At $\pi = 0$

$$A_i = \left[-\frac{\beta}{4}u''(G_{i2}) - qu''(G_{i1})\right] = A > 0$$
$$a_i = \frac{4}{\beta}A_i > 0$$

Therefore evaluating $\frac{\partial \mathbf{B}_2}{\partial \pi}$ at $\pi = 0$, we obtain

$$\frac{\mathrm{d}\mathbf{B}_2}{\mathrm{d}\pi} = \left[-\frac{1}{1-\frac{u''(G_{S2})}{a}\frac{u''(G_{N2})}{a}} - \frac{u''(G_{N2})}{a}\right]\frac{1}{A}\left[\Delta MU_S + \Delta MU_N\right]$$

We know that

$$\frac{1}{1-\frac{u''(G_{S2})}{a}\frac{u''(G_{N2})}{a}} > 1$$

and

$$\frac{u''(G_{N2})}{a} = \frac{u''(G_{N2})}{\left[-u''(G_{N2}) - q\frac{4}{\beta}u''(G_{N1})\right]} > -1$$

Therefore

$$-\frac{1}{1-\frac{u''(G_{S2})}{a}\frac{u''(G_{N2})}{a}} - \frac{u''(G_{N2})}{a} < -1+1 = 0$$

Next, notice that

$$\Delta MU_S + \Delta MU_N = \beta\left[u'(Y-b_{S2}) + u'(Y-b_{N2}) - u'\left(Y - \frac{B_2}{2}\right)\right]$$

If, $u''' > 0$, then

$$\Delta MU_S + \Delta MU_N > u'(Y-b_{S2}) + u'(Y-b_{N2}) - \frac{1}{2}\left[u'(Y-b_{S2}) + u'(Y-b_{N2})\right] > 0$$

Therefore, for π close to zero, $\frac{\partial \mathbf{B}_2}{\partial \pi} \leq 0$. □

Proof of Lemma 3

Proof. Part i). For convenience, rewrite (11):

$$W_1^{nb}(b,\pi) = \sum_i \left[u\left(Y - b_i + q\mathbf{b}_{i2}(b,\pi)\right) + \beta u\left(Y - \frac{\mathbf{b}_{i2}(b,\pi)}{2}\right)\right]$$

The fact that W_1^{nb} is continuous and differentiable in b follows from continuity and differentiability of u and $\mathbf{b_2}$.

To show that W^{nb} is decreasing in b, note that

$$\begin{aligned} \frac{\partial W_1^{nb}(b,\pi)}{\partial b_j} &= -u'\left(G_{j1}\right) + \sum_i \left[qu'\left(G_{i1}\right)\frac{\partial \mathbf{b}_{i2}}{\partial b_j} - \beta \frac{u'\left(G_{i2}\right)}{2}\left(\frac{\partial \mathbf{b}_{i2}}{\partial b_j} + \frac{\partial \mathbf{b}_{-i2}}{\partial b_j}\right)\right] \\ &= -u'\left(G_{j1}\right) + \sum_i \left[qu'\left(G_{i1}\right)\frac{\partial \mathbf{b}_{i2}}{\partial b_j} - \beta \frac{u'\left(G_{i2}\right)}{2}\frac{\partial \mathbf{B}_2}{\partial b_j}\right] \\ &= -u'\left(G_{j1}\right) + \sum_i \left[qu'\left(G_{i1}\right) - \beta u'\left(G_{i2}\right)\right]\frac{\partial \mathbf{B}_2}{\partial b_j} + \sum_i \left[qu'\left(G_{i1}\right)\frac{\partial \mathbf{b}_{i2}}{\partial b_j}\right] + \sum_i \beta \frac{u'\left(G_{i2}\right)}{2}\frac{\partial \mathbf{B}_2}{\partial b_j} \end{aligned} \tag{37}$$

Now, summing the focs (6) we obtain

$$\begin{aligned} \sum_i qu'\left(G_{i1}\right) &= \beta \sum_i \left\{ (1-\pi)\frac{u'\left(Y - \frac{\sum_i \mathbf{b}_{i2}}{2}\right)}{2} + \pi u'\left(Y - \mathbf{b}_{i2}\right)\right\} \\ &= \beta 2 \frac{u'\left(Y - \frac{\sum_i \mathbf{b}_{i2}}{2}\right)}{2} + \beta\pi \sum_i \left[u'\left(Y - \mathbf{b}_{i2}\right) - \frac{u'\left(Y - \frac{\sum_i \mathbf{b}_{i2}}{2}\right)}{2}\right] \\ &= \beta 2 u'\left(Y - \frac{\sum_i \mathbf{b}_{i2}}{2}\right) + \beta\pi \sum_i \left[u'\left(Y - \mathbf{b}_{i2}\right) - \frac{u'\left(Y - \frac{\sum_i \mathbf{b}_{i2}}{2}\right)}{2}\right] - u'\left(Y - \frac{\sum_i \mathbf{b}_{i2}}{2}\right) \end{aligned}$$

Now, if we can show that

$$\begin{aligned} &\beta\pi \sum_i \left[u'\left(Y - \mathbf{b}_{i2}\right) - \frac{u'\left(Y - \frac{\sum_i \mathbf{b}_{i2}}{2}\right)}{2}\right] - u'\left(Y - \frac{\sum_i \mathbf{b}_{i2}}{2}\right) < 0 \\ \iff &\frac{(1+\beta\pi/2)}{\beta\pi/2} u'\left(Y - \frac{\sum_i \mathbf{b}_{i2}}{2}\right) > \sum_i \frac{1}{2} u'\left(Y - \mathbf{b}_{i2}\right) \end{aligned} \tag{38}$$

then the equation above implies that the second term in the last line of (37) is negative. Note that (38) holds for u quadratic. So we are left to deal with the last term

$\sum_i \left[qu'(G_{i1})\frac{\partial \mathbf{b}_{-i2}}{\partial b_j}\right]$. Consider two cases. First, if $j = S$ then

$$\sum_i \left[qu'(G_{i1})\frac{\partial \mathbf{b}_{-i2}}{\partial b_{S1}}\right] = qu'(G_{S1})\frac{\partial \mathbf{b}_{N2}}{\partial b_{S1}} + qu'(G_{N1})\frac{\partial \mathbf{b}_{S2}}{\partial b_{S1}} < 0$$

For $j = N$

$$\sum_i \left[qu'(G_{i1})\frac{\partial \mathbf{b}_{-i2}}{\partial b_{N1}}\right] = qu'(G_{S1})\frac{\partial \mathbf{b}_{N2}}{\partial b_{N1}} + qu'(G_{N1})\frac{\partial \mathbf{b}_{S2}}{\partial b_{N1}} < 0$$

Part ii). Consider the derivative with respect to π:

$$\frac{\partial W_1^{nb}(b,\pi)}{\partial \pi} = \sum_i \left[qu'(G_{i1})\frac{\partial \mathbf{b}_{i2}}{\partial \pi} - \beta\frac{u'(G_{i2})}{2}\left(\frac{\partial \mathbf{b}_{2i}}{\partial \pi} + \frac{\partial \mathbf{b}_{-i2}}{\partial \pi}\right)\right]$$

While we cannot sign this term in general, at $\pi = 0$ since $qu'(G_{i1}) = \frac{\beta}{2}u'(G_{i2})$, we have

$$\frac{\partial W_1^{nb}(b,\pi)}{\partial \pi} = -\beta\frac{u'(G_{i2})}{2}\frac{\partial \mathbf{B}_2}{\partial \pi} > 0$$

since we have established earlier that $\frac{\partial \mathbf{B}_2}{\partial \pi} < 0$ at $\pi = 0$. So for π close to zero W_1^{nb} is increasing in π.

Part iii). Let $b_1 = (b_{N1}, b_{S1})$ with $b_{N1} \leq b_{S1}$ and $\Delta > 0$ small so

$$\frac{1}{\Delta}\left[W_1^{nb}(b_{N1}, b_{S1}, \pi) - W_1(b_{N1} + \Delta, b_{S1} - \Delta, \pi)\right] \approx$$

$$\begin{aligned}
&\approx \left[-u'(G_{N1}) + \sum_i \left[qu'(G_{i1})\frac{\partial b_{i2}}{\partial b_{N1}} - \beta\frac{u'(G_{i2})}{2}\frac{\partial \mathbf{B}_2}{\partial b_{N1}}\right]\right] \\
&- \left[-u'(G_{S1}) + \sum_i \left[qu'(G_{i1})\frac{\partial \mathbf{b}_{i2}}{\partial b_{S1}} - \beta\frac{u'(G_{i2})}{2}\frac{\partial \mathbf{B}_2}{\partial b_{S1}}\right]\right] \\
&= \left[u'(G_{S1}) - u'(G_{N1})\right] + +qu'(G_{S1})\left[\frac{\partial \mathbf{b}_{S2}}{\partial b_{N1}} - \frac{\partial \mathbf{b}_{S2}}{\partial b_{S1}}\right] + qu'(G_{N1})\left[\frac{\partial \mathbf{b}_{N2}}{\partial b_{N1}} - \frac{\partial \mathbf{b}_{N2}}{\partial b_{S1}}\right] \\
&+ u'(G_2)\left(\frac{\partial \mathbf{B}_2}{\partial b_{S1}} - \frac{\partial \mathbf{B}_2}{\partial b_{N1}}\right)
\end{aligned}$$

$$\begin{aligned}
&> \left[u'(G_{S1}) - u'(G_{N1})\right] + qu'(G_{N1})\left[\frac{\partial \mathbf{b}_{S2}}{\partial b_{N1}} - \frac{\partial \mathbf{b}_{S2}}{\partial b_{S1}} + \frac{\partial \mathbf{b}_{N2}}{\partial b_{N1}} - \frac{\partial \mathbf{b}_{N2}}{\partial b_{S1}}\right] + \beta u'(G_2)\left(\frac{\partial \mathbf{B}_2}{\partial b_{S1}} - \frac{\partial \mathbf{B}_2}{\partial b_{N1}}\right) \\
&= \left[u'(G_{S1}) - u'(G_{N1})\right] + \left[\beta u'(G_2) - qu'(G_{N1})\right]\left(\frac{\partial \mathbf{B}_2}{\partial b_{S1}} - \frac{\partial \mathbf{B}_2}{\partial b_{N1}}\right) \qquad (39)
\end{aligned}$$

The first term in (39) is positive since $G_{S1} < G_{N1}$. To sign the second term, from Lemma

35, we know that

$$\frac{\partial \mathbf{B}_2}{\partial b_i} = \frac{-qu''(G_{i1})}{A_i} \frac{1}{1 - \frac{u''(G_{i2})}{a_i}\frac{u''(G_{-i2})}{a_{-i}}} \left(1 + \frac{u''(G_{-i2})}{a_{-i}}\right) = \frac{\partial \mathbf{b}_{i2}}{\partial b_i}\left(1 - \frac{-u''(G_{-i2})}{a_{-i}}\right)$$

$$\begin{aligned} \frac{\partial \mathbf{B}_2}{\partial b_S} - \frac{\partial \mathbf{B}_2}{\partial b_N} &= \frac{\partial \mathbf{b}_{S2}}{\partial b_S}\left(1 - \frac{-u''(G_{N2})}{a_S}\right) - \frac{\partial \mathbf{b}_{N2}}{\partial b_N}\left(1 - \frac{-u''(G_{S2})}{a_N}\right) \\ &= \frac{\partial \mathbf{b}_{S2}}{\partial b_S}\left(1 - \frac{-u''(G_{N2})}{a_S}\right) - \frac{\partial \mathbf{b}_{N2}}{\partial b_N}\left(1 - \frac{-u''(G_{S2})}{a_N}\right) \end{aligned} \tag{40}$$

where recall $G_{N2} = G_{S2} = G_2$ and

$$\begin{aligned} A_i &= \left[-\beta\pi u''(G^c_{i2}) - \frac{\beta(1-\pi)}{4}u''(G_{i2}) - qu''(G_{i1})\right] > 0 \\ a_i &= \frac{4}{\beta(1-\pi)}A_i > 0 \end{aligned}$$

so assuming prudence, $u''' \geq 0$,

$$\frac{\partial \mathbf{b}_{S2}}{\partial b_{S1}} = \frac{-qu''(G_{S1})}{A_S} \frac{1}{1 - \frac{u''(G_{S2})}{a_S}\frac{u''(G_{N2})}{a_N}} > \frac{\partial \mathbf{b}_{N2}}{\partial b_{N1}}$$

$$A_S \geq A_N \quad \text{and } a_S \geq a_N$$

With quadratic utility, we know that $A_S = A_N$ and so

$$\text{if } u \text{ is quadratic} \Rightarrow \frac{\partial \mathbf{b}_{S2}}{\partial b_S} > \frac{\partial \mathbf{b}_{N2}}{\partial b_N}$$

so from (40)

$$\frac{\partial \mathbf{B}_2}{\partial b_{S1}} - \frac{\partial \mathbf{B}_2}{\partial b_{N1}} > 0$$

and from (6) for the North we have

$$\begin{aligned} qu'(G_{N1}) &= \frac{\beta(1-\pi)}{2}u'(G_{N2}) + \beta\pi u'(G^c_{N2}) \\ &< \beta(1-\pi)u'(G_{N2}) + \beta\pi u'(G_{N2}) = \beta u'(G_{N2}) \end{aligned}$$

and so $[\beta u'(G_2) - qu'(G_{N1})] > 0$. Using the above facts in (39) gives

$$W_1^{nb}(b_{1N}, b_{1S}, \pi) > W_1(b_{1N} + \Delta, b_{1S} - \Delta, \pi)$$

as wanted. □

Proof of Proposition 1

Proof. The first part of the Proposition follows from Lemma 1.

Consider now the second part. Define the following objects:

$$\begin{aligned}\mathcal{W}(\pi) &\equiv W^{nb}\left(b_1^{nb}(\pi),\pi\right) - W^{b}\left(b_1^{nb}(\pi),\pi\right)\\ &= W^{nb}\left(b_1^{nb}(\pi),\pi\right) - W^{nb}\left(b_1^{nb}(\pi),0\right)\end{aligned}$$

Note that since W^{nb} and b_1^{nb} are continuous, then $\mathcal{W}$ is continuous.

STEP 1. $\mathcal{W}(0) = 0$ and $\mathcal{W}'(0) > 0$. Hence $\mathcal{W}(\pi) > 0$ for $\pi > 0$ sufficiently close to zero.

Proof of Step 1. $\mathcal{W}(0) = 0$ follows from Lemma 1. Differentiating $\mathcal{W}$ we obtain:

$$\mathcal{W}'(\pi) = \sum_i \left[\frac{W^{nb}\left(b_1^{nb}(\pi),\pi\right)}{\partial b_{1i}} - \frac{\partial W^{nb}\left(b_1^{nb}(\pi),0\right)}{\partial b_{1i}}\right]\frac{\partial b_{i1}^{nb}(\pi)}{\partial \pi} + \frac{\partial W^{nb}\left(b_1^{nb}(\pi),\pi\right)}{\partial \pi}$$

Evaluating the expression above at $\pi = 0$ we obtain

$$\mathcal{W}'(0) = \frac{\partial W^{nb}\left(b_1^{nb}(\pi),\pi\right)}{\partial \pi} > 0$$

as wanted. That W^{nb} is increasing in π for π close to zero is established in Lemma 3 part ii).

STEP 2. For $\pi > 0$ but sufficiently close to zero there exists an NB1 equilibrium.

Proof of Step 2. From Step 1 we know that $\mathcal{W}(\pi) > 0$ in a neighborhood of zero. So it is not optimal for the central government to deviate.

We are left to check that local governments don't want to deviate. The relevant deviation is for the South. Let $v(\pi)$ be the value along the conjectured equilibrium path for the South:

$$v(\pi) = u\left(Y_{0S} + qb_{S1}^{nb}(\pi)\right) + \beta(1-\pi)V_{S1}^{nb}\left(b_{S1}^{nb}(\pi), b_{N1}^{nb}(\pi),\pi\right) + \beta\pi V_{S1}^{nb}\left(b_{S1}^{nb}(\pi), b_{N1}^{nb}(\pi),\pi\right)$$

Let $\hat{v}$ be the value of the deviation by the South if it issues a lot of debt to induce the central government to mutualize debt in period 1:

$$\hat{v}(\pi) = \sup_{b_S} u\left(Y_{0S} + qb_S\right) + \beta(1-\pi)V_{S1}^{nb}\left(b_S, b_{N1}^{nb}(\pi),0\right) + \beta\pi V_{S1}^{nb}\left(b_S, b_{N1}^{nb}(\pi),1\right)$$

subject to

$$W^{b}\left(b_S + b_{N1}^{nb}(\pi)\right) > W^{nb}\left(b_S, b_{N1}^{nb}(\pi),1\right) \iff b_S > \hat{b}_S(\pi) = \hat{b}_S\left(b_{N1}^{nb}(\pi)\right)$$

which equals

$$\hat{v}(\pi) = \max_{b \geq \hat{b}_S(\pi)} u(Y_{0S} + qb_S) + \beta(1-\pi) V_{S1}^{nb}\left(b_S, b_{N1}^{nb}(\pi), 0\right) + \beta\pi V_{S1}^{nb}\left(b_S, b_{N1}^{nb}(\pi), 1\right) \tag{41}$$

Let $\Delta V_S(\pi) \equiv v(\pi) - \hat{v}(\pi)$. It then suffices to show that $\Delta V_S(\pi) \geq 0$ for π close to zero. Inspection of the two programming problems gives that $\Delta V_S(0) \geq 0$ where the inequality is strict whenever the constraint $b_S \geq \hat{b}_S(\pi)$ is binding at the optimal solution for the unconstrained problem. This is true if

$$W^b\left(b_{S1}^{nb}(0) + b_{N1}^{nb}(0)\right) < W^{nb}\left(b_{S1}^{nb}(0), b_{N1}^{nb}(0), 1\right) \tag{42}$$

However, in proposition 2 we assume conditions that would imply that the above equation is not true. Hence we want to consider the space of parameters where

$$W^b\left(b_{S1}^{nb}(0) + b_{N1}^{nb}(0)\right) > W^{nb}\left(b_{S1}^{nb}(0), b_{N1}^{nb}(0), 1\right) \tag{43}$$

then $\Delta V_S(0) = 0$ and want to show that $\Delta V_S(\pi) \geq 0$ for π close to zero. Since we know that $\pi = 0$, $b_1^{nb}(0) = b_1^b(0)$, the above condition also holds if we substitute $b_1^{nb}(0)$ with $b_1^b(0)$ in the above equation. Therefore, since $b_1^b(\pi)$ and W^b, W^{nb} are continuous functions of π, the strict inequality will still hold for π positive but small enough. In particular, for such π, the constraint constraint in the programming problem (41) is slack. To show that the South does not want to deviate for π sufficiently close to zero, it is sufficient to show that $\Delta V_S'(0) > 0$. We have

$$v'(\pi) = \beta \frac{V_{S1}^{nb}\left(b_{S1}^{nb}(\pi), b_{N1}^{nb}(\pi), \pi\right)}{\partial b_{N1}} \frac{\partial b_{N1}^{nb}}{\partial \pi} + \beta \frac{V_{S1}^{nb}\left(b_{S1}^{nb}(\pi), b_{N1}^{nb}(\pi), \pi\right)}{\partial \pi}.$$

Moreover,

$$\begin{aligned} \hat{v}'(\pi) = & -\beta\left[V_{S1}^{nb}\left(b_S, b_{N1}^{nb}(\pi), 0\right) - V_{S1}^{nb}\left(b_S, b_{N1}^{nb}(\pi), 1\right)\right] \\ & + \beta(1-\pi)\frac{\partial V_{S1}^{nb}\left(b_S, b_{N1}^{nb}(\pi), 0\right)}{\partial b_{N1}} \frac{\partial b_{N1}^{nb}(\pi)}{\partial \pi} \end{aligned}$$

Then

$$\Delta V_S'(\pi) = \beta \frac{\partial V_{S1}^{nb}\left(b_{S1}^{nb}(\pi), b_{N1}^{nb}(\pi), 0\right)}{\partial b_{N1}} \frac{\partial b_{N1}^{nb}}{\partial \pi} + \beta \frac{\partial V_{S1}^{nb}\left(b_{S1}^{nb}(\pi), b_{N1}^{nb}(\pi), 0\right)}{\partial \pi}$$
$$- \left\{ -\beta \left[V_{S1}^{nb}\left(b_S, b_{N1}^{nb}(\pi), 0\right) - V_{S1}^{nb}\left(b_S, b_{N1}^{nb}(\pi), 1\right) \right] \right.$$
$$\left. + \beta (1-\pi) \frac{\partial V_{S1}^{nb}\left(b_S, b_{N1}^{nb}(\pi), 0\right)}{\partial b_{N1}} \frac{\partial b_{N1}^{nb}(\pi)}{\partial \pi} \right\}$$

Rearranging

$$\Delta V_S'(\pi) = \beta \left[\frac{V_{S1}^{nb}\left(b_{S1}^{nb}(\pi), b_{N1}^{nb}(\pi), 0\right)}{\partial b_{N1}} - (1-\pi) \frac{\partial V_{S1}^{nb}\left(b_S(\pi), b_{N1}^{nb}(\pi), 0\right)}{\partial b_{N1}} \right] \frac{\partial b_{N1}^{nb}}{\partial \pi}$$
$$+ \beta \left[V_{S1}^{nb}\left(b_S(\pi), b_{N1}^{nb}(\pi), 0\right) - V_{S1}^{nb}\left(b_S(\pi), b_{N1}^{nb}(\pi), 1\right) \right]$$
$$+ \beta \frac{V_{S1}^{nb}\left(b_{S1}^{nb}(\pi), b_{N1}^{nb}(\pi), 0\right)}{\partial \pi}$$

where $b_S(\pi)$ is the policy function for the programming problem (41) that defines $\hat{v}$. As $\pi \to 0$, when condition (43) holds, $b_S(\pi) \to b_{S1}^{nb}(0)$ and the first term goes to zero. Hence:

$$\lim_{\pi \to 0} \Delta V_S'(\pi) = \beta \left[V_{S1}^{nb}\left(b_{S1}^{nb}(0), b_{N1}^{nb}(0), 0\right) - V_{S1}^{nb}\left(b_{S1}^{nb}(0), b_{N1}^{nb}(0), 1\right) \right] \tag{44}$$
$$+ \beta \frac{V_{S1}^{nb}\left(b_{S1}^{nb}(0), b_{N1}^{nb}(0), 0\right)}{\partial \pi}$$

We now analyze each terms. Consider first

$$\beta \left[V_{S1}^{nb}\left(b_{S1}^{nb}(0), b_{N1}^{nb}(0), 0\right) - V_{S1}^{nb}\left(b_{S1}^{nb}(0), b_{N1}^{nb}(0), 1\right) \right]$$

The term in square brackets can be written as

$$\left[u\left(G_1^{nb}(0)\right) + \beta u \left(Y - \frac{\sum_i \mathbf{b}_{2i}\left(b_1^{nb}, 0\right)}{2} \right) - u\left(G_1^{nb}(1)\right) - \beta u\left(Y - \mathbf{b}_{S2}\left(b_1^{nb}, 1\right)\right) \right]$$

where we used the shorthand

$$G_1^{nb}(0) = Y - b_{S1}^{nb}(0) + q\mathbf{b}_{S2}\left(b_1^{nb}(0), 0\right)$$
$$G_1^{nb}(1) = Y - b_{S1}^{nb}(0) + q\mathbf{b}_{S2}\left(b_1^{nb}(0), 1\right) < G_1^{nb}(0)$$

Moreover, from (5), we can write

$$\frac{V_{S1}^{nb}(b_1,\pi)}{\partial\pi}=\beta\left[u(Y-b_{2S})-u\left(Y-\frac{b_{2S}+b_{2N}}{2}\right)\right]-\frac{\beta(1-\pi)}{2}u'\left(Y-\frac{b_{2S}+b_{2N}}{2}\right)\frac{\partial\mathbf{b}_{N2}}{\partial\pi}$$

So the limit as $\pi\downarrow 0$ of the sum of the fist two terms in (44) is

$$\begin{aligned}&\left[u\left(G_1^{nb}(0)\right)+\beta u\left(Y-\frac{\sum_i\mathbf{b}_{2i}\left(b_1^{nb},0\right)}{2}\right)-u\left(G_1^{nb}(1)\right)-\beta u\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},1\right)\right)\right]+\\&+\beta\left[u\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)-u\left(Y-\frac{\sum_i\mathbf{b}_{2i}\left(b_1^{nb},0\right)}{2}\right)\right]-\frac{\beta}{2}u'\left(Y-\frac{\sum_i\mathbf{b}_{2i}\left(b_1^{nb},0\right)}{2}\right)\frac{\partial\mathbf{b}_{N2}}{\partial\pi}\\&=u\left(G_1^{nb}(0)\right)-u\left(G_1^{nb}(1)\right)+\beta\left[u\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)-u\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},1\right)\right)\right]\\&\qquad-\frac{\beta}{2}u'\left(Y-\frac{\sum_i\mathbf{b}_{2i}\left(b_1^{nb},0\right)}{2}\right)\frac{\partial\mathbf{b}_{N2}}{\partial\pi}\end{aligned}$$

Rearranging we have

$$\begin{aligned}&u\left(G_1^{nb}(0)\right)+\beta u\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)-\left[u\left(G_1^{nb}(1)\right)+\beta u\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},1\right)\right)\right]\\&\qquad-\frac{\beta}{2}u'\left(Y-\frac{\sum_i\mathbf{b}_{2i}\left(b_1^{nb},0\right)}{2}\right)\frac{\partial\mathbf{b}_{N2}}{\partial\pi}\end{aligned}\tag{45}$$

Now since $\mathbf{b}_{S2}\left(b_1^{nb},1\right)=\arg\max_{b_{S2}}u\left(Y-b_{S1}^{nb}+qb_{S2}\right)+\beta u\left(Y-b_{S2}\right)$ the first line of the above equation is negative. We will show that under our sufficient conditions, $\frac{\partial\mathbf{b}_{N2}}{\partial\pi}$ is positive and last term is larger in absolute value than the terms on the first line.

Consider the terms on the first line of (45). Since u is concave we have,

$$\begin{aligned}&u\left(G_1^{nb}(0)\right)-u\left(G_1^{nb}(1)\right)+\beta u\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)-\beta u\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},1\right)\right)\\&\geq u'\left(G_1^{nb}(0)\right)q\left[\mathbf{b}_{S2}\left(b_1^{nb},0\right)-\mathbf{b}_{S2}\left(b_1^{nb},1\right)\right]+\beta u'\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)\left[\mathbf{b}_{S2}\left(b_1^{nb},1\right)-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right]\\&=\left[\mathbf{b}_{S2}\left(b_1^{nb},0\right)-\mathbf{b}_{S2}\left(b_1^{nb},1\right)\right]\left[u'\left(G_1^{nb}(0)\right)q-\beta u'\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)\right]\\&=\left[\mathbf{b}_{S2}\left(b_1^{nb},0\right)-\mathbf{b}_{S2}\left(b_1^{nb},1\right)\right]\left[\frac{\beta}{2}u'\left(Y-\frac{\mathbf{B}_2\left(b_1^{nb},0\right)}{2}\right)-\beta u'\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)\right]\end{aligned}$$

Suppose that $Y_{N0}=Y_{S0}$. Then, the above equals

$$-\left[\mathbf{b}_{S2}\left(b_1^{nb},0\right)-\mathbf{b}_{S2}\left(b_1^{nb},1\right)\right]\left[\frac{\beta}{2}u'\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)\right]$$

Substituting this into (45) yields,

$$
\begin{aligned}
&\frac{\beta}{2}u'\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)\left[-\frac{\partial \mathbf{b}_{N2}}{\partial \pi}-\left[\mathbf{b}_{S2}\left(b_1^{nb},0\right)-\mathbf{b}_{S2}\left(b_1^{nb},1\right)\right]\right]\\
=&\frac{\beta}{2}u'\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)\left[-\frac{\partial \mathbf{b}_{N2}}{\partial \pi}-\left[\mathbf{b}_{S2}\left(b_1^{nb},0\right)-\mathbf{b}_{S2}\left(b_1^{nb},1\right)\right]\right]\\
=&\frac{\beta}{2}u'\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)\left[\frac{\Delta MU_i}{A_i}\left[\frac{1}{1-\left(\frac{u''(G_{i2})}{a_i}\right)^2}+\frac{u''\left(G_{i2}\right)}{a_i}\right]-\left[\mathbf{b}_{S2}\left(b_1^{nb},0\right)-\mathbf{b}_{S2}\left(b_1^{nb},1\right)\right]\right]
\end{aligned}
$$

since we from earlier that

$$
\begin{aligned}
\frac{\partial \mathbf{b}_{i2}}{\partial \pi}&=\frac{1}{1-\frac{u''(G_{i2})}{a_i}\frac{u''(G_{-i2})}{a_{-i}}}\frac{-\Delta MU_i}{A_i}+\frac{u''\left(G_{i2}\right)}{a_i}\frac{-\Delta MU_{-i}}{A_i}.\\
&=-\frac{1}{A_i}\left[\frac{\Delta MU_i}{1-\frac{u''(G_{i2})}{a_i}\frac{u''(G_{-i2})}{a_{-i}}}+\frac{u''\left(G_{i2}\right)\Delta MU_{-i}}{a_i}\right]
\end{aligned}
$$

Recall that (at $\pi=0$),

$$
\begin{aligned}
A_i&=\left[-\frac{\beta}{4}u''\left(G_{i2}\right)-qu''\left(G_{i1}\right)\right]\\
a_i&=\frac{4}{\beta}A_i
\end{aligned}
$$

and (since $Y_{N0}=Y_{S0}$)

$$
\Delta MU_i=\frac{\beta}{2}u'\left(Y-b_{i2}\right)
$$

If $\frac{\beta}{q}\leq 1$, then $u'\left(G_{i1}\right)\leq u'\left(G_{i2}\right)\implies G_{i1}\geq G_{i2}$. Therefore,

$$
A_i=\left[-\frac{\beta}{4}u''\left(G_{i2}\right)-qu''\left(G_{i1}\right)\right]\leq -u''\left(G_{i2}\right)\left[\frac{\beta}{4}+q\right]
$$

and $\frac{1}{A_i}\geq\frac{1}{-u''(G_{i2})\left[\frac{\beta}{4}+q\right]}$, $\frac{1}{a_i}\geq\frac{1}{-u''(G_{i2})\left[1+\frac{4q}{\beta}\right]}$ and $\frac{1}{1-\frac{(u''(G_{i2}))^2}{a_i^2}}\geq\frac{1}{1-\frac{(u''(G_{i2}))^2}{\left(-u''(G_{i2})\left[1+\frac{4q}{\beta}\right]\right)^2}}$.

Substituting these back into (45), we obtain

$$\frac{\beta}{2}u'\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)\left[\frac{\Delta MU_i}{A_i}\left[\frac{1}{1-\left(\frac{u''(G_{i2})}{a_i}\right)^2}+\frac{u''\left(G_{i2}\right)}{a_i}\right]-\left[\mathbf{b}_{S2}\left(b_1^{nb},0\right)-\mathbf{b}_{S2}\left(b_1^{nb},1\right)\right]\right]$$

$$\geq\frac{\beta}{2}u'\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)\times$$

$$\times\left[\frac{\frac{\beta}{2}u'\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)}{-u''\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)\left[\frac{\beta}{4}+q\right]}\left[\frac{1}{1-\frac{1}{\left[1+\frac{4q}{\beta}\right]^2}}-\frac{1}{\left[1+\frac{4q}{\beta}\right]}\right]-\left[\mathbf{b}_{S2}\left(b_1^{nb},0\right)-\mathbf{b}_{S2}\left(b_1^{nb},1\right)\right]\right]$$

Let $x=1+\frac{4q}{\beta}$. Then $\left[\frac{\beta}{4}+q\right]=\frac{\beta}{4}x$. Then the above is

$$\frac{\beta}{2}u'\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)\left[\frac{2u'\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)}{-u''\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)x}\left[\frac{x^2}{x^2-1}-\frac{1}{x}\right]-\left[\mathbf{b}_{S2}\left(b_1^{nb},0\right)-\mathbf{b}_{S2}\left(b_1^{nb},1\right)\right]\right]$$

$$=\frac{\beta}{2}u'\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)\left[\frac{2u'\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)}{-u''\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)}\left[\frac{x}{x^2-1}-\frac{1}{x^2}\right]-\left[\mathbf{b}_{S2}\left(b_1^{nb},0\right)-\mathbf{b}_{S2}\left(b_1^{nb},1\right)\right]\right]$$

Consider the term inside the square brackets. Since $x>1$, $\frac{x}{x^2-1}-\frac{1}{x^2}\geq 0$. Moreover, the term $\mathbf{b}_{S2}\left(b_1^{nb},0\right)-\mathbf{b}_{S2}\left(b_1^{nb},1\right)$ is bounded from above by Y. Therefore if the coefficient of absolute risk aversion $\frac{-u''}{u'}$ is small enough, the equation above is strictly positive which in turn implies that (45) is strictly positive and hence the South does not want to deviate and induce a bailout.

STEP 3. For $\pi>0$ but sufficiently close to zero, the equilibrium is unique.

Proof of Step 3. To establish step 3 we show that an equilibrium with debt mutualization cannot exist for π sufficiently small since the North will always choose to deviate and induce an equilibrium with no debt-mutualization. Similar to step 2, let $v(\pi)$ be the value along the conjectured equilibrium path (i.e. one with debt-mutualization) for the North:

$$v(\pi)=u\left(Y_{N0}+qb_{N1}^b(\pi)\right)+\beta(1-\pi)V_{N1}^{nb}\left(b_{S1}^b(\pi),b_{N1}^b(\pi),0\right)+\beta\pi V_{N1}^{nb}\left(b_{S1}^b(\pi),b_{N1}^b(\pi),1\right)$$

Let $\hat{v}$ be the value of the deviation by the North if it issues a lot of debt to induce the central government to not bail out in period 1:

$$\hat{v}(\pi)=\sup_{b_N}u\left(Y_{N0}+qb_N\right)+\beta(1-\pi)V_{N1}^{nb}\left(b_S^b(\pi),b_{N1},\pi\right)+\beta\pi V_{N1}^{nb}\left(b_S^b(\pi),b_{N1},\pi\right)$$

subject to

$$W^b\left(b_S^b(\pi)+b_{N1}\right)<W^{nb}\left(b_S^b(\pi),b_{N1},\pi\right)\iff b_N>\hat{b}_N(\pi)=\hat{b}_N\left(b_{S1}^b(\pi),\pi\right)$$

which equals

$$\hat{v}(\pi)=\max_{b\geq\hat{b}_N\left(b_{S1}^b(\pi),\pi\right)}u\left(Y_{N0}+qb_N\right)+\beta(1-\pi)V_{N1}^{nb}\left(b_S^b(\pi),b_{N1},\pi\right)+\beta\pi V_{N1}^{nb}\left(b_S^b(\pi),b_{N1},\pi\right)\tag{46}$$

Let $\Delta V_N(\pi)\equiv v(\pi)-\hat{v}(\pi)$. Since $\Delta V_N(0)=0$, it then suffices to show that $\Delta V_N'(0)<0$.

Notice that, the constraint in (46) is slack at $\pi=0$ and hence is also slack for π sufficiently close to zero. Then, using the foc for the optimal amount of debt, we have that

$$\begin{aligned}v'(\pi)=&\beta\left[V_{N1}^{nb}\left(b_{S1}^b(\pi),b_{N1}^b(\pi),1\right)-V_{N1}^{nb}\left(b_{S1}^b(\pi),b_{N1}^b(\pi),0\right)\right]\\&+\beta(1-\pi)\frac{V_{N1}^{nb}\left(b_{S1}^b(\pi),b_{N1}^b(\pi),0\right)}{\partial b_{S1}}\frac{\partial b_{S1}^b}{\partial\pi}\end{aligned}$$

Moreover,

$$\hat{v}'(\pi)=\beta\frac{\partial V_{N1}^{nb}\left(b_S^b(\pi),b_{N1},\pi\right)}{\partial b_{S1}}\frac{\partial b_{S1}^b(\pi)}{\partial\pi}+\beta\frac{\partial V_{N1}^{nb}\left(b_S^b(\pi),b_{N1},\pi\right)}{\partial\pi}$$

Then

$$\begin{aligned}\Delta V_N'(0)=&\beta\left[V_{N1}^{nb}\left(b_{S1}^b(0),b_{N1}^b(\pi),1\right)-V_{N1}^{nb}\left(b_{S1}^b(0),b_{N1}^b(0),0\right)\right]\\&-\beta\frac{\partial V_{N1}^{nb}\left(b_S^b(0),b_{N1},0\right)}{\partial\pi}\end{aligned}\tag{47}$$

Consider the two terms on the RHS of (47). We have

$$\begin{aligned}&V_{N1}^{nb}\left(b_{S1}^b(0),b_{N1}^b(\pi),1\right)-V_{N1}^{nb}\left(b_{S1}^b(0),b_{N1}^b(0),0\right)\\=&u\left(Y-b_{N1}^b(\pi)+q\mathbf{b}_{N2}\left(b_1^b,1\right)\right)+\beta u\left(Y-\mathbf{b}_{N2}\left(b_1^b,1\right)\right)\\&-\left[u\left(Y-b_{N1}^b(\pi)+q\mathbf{b}_{N2}\left(b_1^b,0\right)\right)+\beta u\left(Y-\frac{\mathbf{B}_2\left(b_1^b,0\right)}{2}\right)\right]\end{aligned}$$

and

$$\frac{V_{N1}^{nb}\left(b_1,0\right)}{\partial\pi}=\beta\left[u\left(Y-\mathbf{b}_{N2}\left(b_1^b,0\right)\right)-u\left(Y-\frac{\mathbf{B}_2\left(b_1^b,0\right)}{2}\right)\right]-\frac{\beta}{2}u'\left(Y-\frac{\mathbf{B}_2\left(b_1^b,0\right)}{2}\right)\frac{\partial\mathbf{b}_{S2}}{\partial\pi}$$

Then (47) becomes

$$\begin{aligned}&u\left(G_{N1}\left(1\right)\right)+\beta u\left(G_{N2}\left(1\right)\right)-\left[u\left(G_{N1}\left(0\right)\right)+\beta u\left(G_{N2}\left(0\right)\right)\right]\\&+\frac{\beta}{2}u'\left(Y-\frac{\mathbf{B}_2\left(b_1^b,0\right)}{2}\right)\frac{\partial \mathbf{b}_{S2}}{\partial \pi}\end{aligned}$$

Consider the first term. Suppose $b_{N2}\left(1\right)-b_{N2}\left(0\right)\geq 0$.

$$\begin{aligned}&u\left(G_{N1}\left(1\right)\right)-u\left(G_{N1}\left(0\right)\right)+\beta u\left(G_{N2}\left(1\right)\right)-\beta u\left(G_{N2}\left(0\right)\right)\\&\leq u'\left(G_{N1}\left(0\right)\right)q\left[b_{N2}\left(1\right)-b_{N2}\left(0\right)\right]+\beta u'\left(G_{N2}\left(0\right)\right)\left[b_{N2}\left(0\right)-b_{N2}\left(1\right)\right]\\&=\left[b_{N2}\left(1\right)-b_{N2}\left(0\right)\right]\left[u'\left(G_{N1}\left(0\right)\right)q-\beta u'\left(G_{N2}\left(0\right)\right)\right]\\&=\left[b_{N2}\left(1\right)-b_{N2}\left(0\right)\right]\left[\frac{\beta}{2}u'\left(Y-\frac{\mathbf{B}_2\left(b_1^b,0\right)}{2}\right)-\beta u'\left(G_{N2}\left(0\right)\right)\right]\end{aligned}$$

Substituting back yields

$$\frac{\beta}{2}u'\left(Y-\frac{\mathbf{B}_2\left(b_1^b,0\right)}{2}\right)\left[b_{N1}\left(1\right)-b_{N1}\left(0\right)+\frac{\partial \mathbf{b}_{S2}}{\partial \pi}\right]-\left[b_{N2}\left(1\right)-b_{N2}\left(0\right)\right]\beta u'\left(G_{N2}\left(0\right)\right) \tag{48}$$

We know that

$$\frac{\partial \mathbf{b}_{S2}}{\partial \pi}=-\frac{1}{A}\left[\frac{\Delta MU_S}{1-\left(\frac{u''\left(G_{S2}\right)}{a}\right)^2}+\frac{u''\left(G_{S2}\right)\Delta MU_N}{a}\right]$$

$$\Delta MU_i\equiv\beta\left[u'\left(Y-b_{i2}\right)-\frac{u'\left(Y-\frac{\sum_j b_{j2}}{2}\right)}{2}\right]$$

and since $\frac{\beta}{q}\leq 1$,

$$A_i=\left[-\frac{\beta}{4}u''\left(G_{i2}\right)-qu''\left(G_{i1}\right)\right]\leq -u''\left(G_{i2}\right)\left[\frac{\beta}{4}+q\right]$$

and $\frac{1}{A_i} \geq \frac{1}{-u''(G_{i2})\left[\frac{\beta}{4}+q\right]}$, $\frac{1}{a_i} \geq \frac{1}{-u''(G_{i2})\left[1+\frac{4q}{\beta}\right]}$ and $\frac{1}{1-\frac{(u''(G_{i2}))^2}{a_i^2}} \geq \frac{1}{1-\frac{(u''(G_{i2}))^2}{\left(-u''(G_{i2})\left[1+\frac{4q}{\beta}\right]\right)^2}}$.

$$\begin{aligned}
\frac{\partial \mathbf{b}_{S2}}{\partial \pi} &= -\frac{1}{A}\left[\frac{\Delta MU_S}{1-\left(\frac{u''(G_{S2})}{a}\right)^2} + \frac{u''\left(G_{S2}\right)\Delta MU_N}{a}\right] \\
&\leq -\frac{1}{-u''\left(G_{i2}\right)\left[\frac{\beta}{4}+q\right]}\left[\frac{\Delta MU_S}{1-\frac{1}{\left(\left[1+\frac{4q}{\beta}\right]\right)^2}} - \frac{\Delta MU_N}{\left[1+\frac{4q}{\beta}\right]}\right]
\end{aligned}$$

Let $x = 1+\frac{4q}{\beta}$. Then $\left[\frac{\beta}{4}+q\right] = \frac{\beta}{4}x$. Consider the terms in the first square bracket of (48). We have

$$\begin{aligned}
& b_{N1}(1) - b_{N1}(0) + \frac{\partial \mathbf{b}_{S2}}{\partial \pi} \\
\leq & -\frac{1}{-u''\left(G_{i2}\right)\frac{\beta}{4}}\left[\frac{x\Delta MU_S}{x^2-1} - \frac{\Delta MU_N}{x^2}\right] + b_{N1}(1) - b_{N1}(0) \\
& -\frac{4}{-u''\left(G_{i2}\right)}\left[\frac{x\left[\left[u'\left(Y-b_{S2}\right) - \frac{u'\left(Y-\frac{\sum_j b_{j2}}{2}\right)}{2}\right]\right]}{x^2-1} - \frac{\left[u'\left(Y-b_{N2}\right) - \frac{u'\left(Y-\frac{\sum_j b_{j2}}{2}\right)}{2}\right]}{x^2}\right] \\
& + b_{N1}(1) - b_{N1}(0) \\
= & -4\left[\frac{x\left[\left[\frac{u'(Y-b_{S2})}{-u''\left(Y-\frac{\sum_j b_{j2}}{2}\right)} - \frac{u'\left(Y-\frac{\sum_j b_{j2}}{2}\right)}{-2u''\left(Y-\frac{\sum_j b_{j2}}{2}\right)}\right]\right]}{x^2-1} - \frac{\left[\frac{u'(Y-b_{N2})}{-u''\left(Y-\frac{\sum_j b_{j2}}{2}\right)} - \frac{u'\left(Y-\frac{\sum_j b_{j2}}{2}\right)}{-2u''\left(Y-\frac{\sum_j b_{j2}}{2}\right)}\right]}{x^2}\right] \\
& + b_{N1}(1) - b_{N1}(0) \\
\leq & -2\frac{u'\left(Y-\frac{\sum_j b_{j2}}{2}\right)}{-u''\left(Y-\frac{\sum_j b_{j2}}{2}\right)}\left[\frac{x}{x^2-1} - \frac{1}{x^2}\right] + b_{N1}(1) - b_{N1}(0)
\end{aligned}$$

Therefore, as in the case with step 2, for a coefficient of absolute risk-aversion small enough, (48) is less than 0.

Finally, suppose that $b_{N2}(0) - b_{N2}(1) > 0$. Then,

$$
\begin{aligned}
& u\left(G_{N1}(1)\right) - u\left(G_{N1}(0)\right) + \beta u\left(G_{N2}(1)\right) - \beta u\left(G_{N2}(0)\right) \\
\leq & u'\left(G_{N1}(0)\right) q \left[b_{N2}(1) - b_{N2}(0)\right] + \beta u'\left(G_{N2}(0)\right)\left[b_{N2}(0) - b_{N2}(1)\right] \\
= & \left[b_{N2}(0) - b_{N2}(1)\right]\left[-u'\left(G_{N1}(0)\right) q + \beta u'\left(G_{N2}(0)\right)\right] \\
\leq & \left[b_{N2}(1) - b_{N2}(0)\right]\left[-u'\left(G_{N1}(0)\right) q + \beta u'\left(Y - \frac{\mathbf{B}_2\left(b_1^b, 0\right)}{2}\right)\right]
\end{aligned}
$$

Substituting this into (47), yields

$$
\begin{aligned}
& u\left(G_{N1}(1)\right) + \beta u\left(G_{N2}(1)\right) - \left[u\left(G_{N1}(0)\right) + \beta u\left(G_{N2}(0)\right)\right] \\
& + \frac{\beta}{2} u'\left(Y - \frac{\mathbf{B}_2\left(b_1^b, 0\right)}{2}\right) \frac{\partial \mathbf{b}_{S2}}{\partial \pi} \\
\leq & \frac{\beta}{2} u'\left(Y - \frac{\mathbf{B}_2\left(b_1^b, 0\right)}{2}\right)\left[\frac{\partial \mathbf{b}_{S2}}{\partial \pi} + 2\left[b_{N2}(1) - b_{N2}(0)\right]\right] - u'\left(G_{N1}(0)\right) q \left[b_{N2}(1) - b_{N2}(0)\right]
\end{aligned}
$$

Then a similar argument to the previous case gives us the desired result. □

Examples:

If u is quadratic then $u'(c) = 1 - \alpha c$ and $u''(c) = -\alpha$. Therefore, (45) becomes

$$
\begin{aligned}
& \frac{2u'\left(Y - \mathbf{b}_{S2}\left(b_1^{nb}, 0\right)\right)}{-u''\left(Y - \mathbf{b}_{S2}\left(b_1^{nb}, 0\right)\right) x}\left[\frac{x^2}{x^2 - 1} - \frac{1}{x}\right] - \left[\mathbf{b}_{S2}\left(b_1^{nb}, 0\right) - \mathbf{b}_{S2}\left(b_1^{nb}, 1\right)\right] \\
= & \frac{\left[2 - \alpha 2Y + \alpha 2\mathbf{b}_{S2}\left(b_1^{nb}, 0\right)\right]}{\alpha x}\left[\frac{x^2}{x^2 - 1} - \frac{1}{x}\right] - \left[\mathbf{b}_{S2}\left(b_1^{nb}, 0\right) - \mathbf{b}_{S2}\left(b_1^{nb}, 1\right)\right] \\
= & \frac{2\left[1 - \alpha Y\right]}{\alpha x}\left[\frac{x^2}{x^2 - 1} - \frac{1}{x}\right] + \left[\frac{2}{x}\left[\frac{x^2}{x^2 - 1} - \frac{1}{x}\right] - 1\right] \mathbf{b}_{S2}\left(b_1^{nb}, 0\right) + \mathbf{b}_{S2}\left(b_1^{nb}, 1\right)
\end{aligned}
$$

If $\frac{2}{x}\left[\frac{x^2}{x^2-1} - \frac{1}{x}\right] - 1 \geq 0$ then we are done if $1 - \alpha Y \geq 0$. Suppose not then the above is $\geq$

$$
\frac{2}{\alpha x}\left[\frac{x^2}{x^2 - 1} - \frac{1}{x}\right] - \frac{2}{x}\left[\frac{x^2}{x^2 - 1} - \frac{1}{x}\right] Y + \left[\frac{2}{x}\left[\frac{x^2}{x^2 - 1} - \frac{1}{x}\right] - 1\right] Y + \mathbf{b}_{S2}\left(b_1^{nb}, 1\right)
$$

$$
= \frac{2}{\alpha x}\left[\frac{x^2}{x^2 - 1} - \frac{1}{x}\right] - Y + \mathbf{b}_{S2}\left(b_1^{nb}, 1\right)
$$

which is positive for α sufficiently small.

With CRRA

$$\frac{2u'\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)}{-u''\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)x}\left[\frac{x^2}{x^2-1}-\frac{1}{x}\right]-\left[\mathbf{b}_{S2}\left(b_1^{nb},0\right)-\mathbf{b}_{S2}\left(b_1^{nb},1\right)\right]$$
$$=\frac{2\left(Y-\mathbf{b}_{S2}\left(b_1^{nb},0\right)\right)}{\sigma}\left[\frac{x}{x^2-1}-\frac{1}{x^2}\right]-\left[\mathbf{b}_{S2}\left(b_1^{nb},0\right)-\mathbf{b}_{S2}\left(b_1^{nb},1\right)\right]$$

Proof of Proposition 2

Proof. Suppose by way of contradiction that we have equilibrium in which $b_{N1}^{nb}<\bar{b}$, $b_{S1}^{nb}\geq\bar{b}$ and there is no bailout in period 1. Suppose first that $b_{S1}^{nb}>\bar{b}$. We know that at $\pi=0$, $W_1^{nb}\left(b_{N1}^{nb},b_{S1}^{nb},0\right)<W_1^{b}\left(b_{N1}^{nb},b_{S1}^{nb}\right)$. Since $b_{i1}^{nb}(\pi)$ is continuous in π, for π positive but small we have that $W_1^{nb}\left(b_{N1}^{nb}(\pi),b_{S1}^{nb}(\pi),\pi\right)<W_1^{b}\left(b_{N1}^{nb}(\pi),b_{S1}^{nb}(\pi)\right)$. In particular, for π small, this allocation will induce a bailout in period 1. Therefore we cannot have a no-bailout equilibrium corresponding to such an allocation for π small. If $b_{S1}^{nb}=\bar{b}$ then there exists a deviating strategy $\tilde{b}_{S2}$, which will induce bailout in in period 1 for π sufficiently small and so is profitable for the South. In particular, at $\pi=0$, it is never optimal for the South to set $b_{S1}=\bar{b}$ if the constraint is binding since at the privately optimal level, the central authority does not enforce the punishment. Thus, this is also true for π sufficiently small.

To show that such a bailout equilibrium can exist, we have to show that

1. The North does not have an incentive to try and prevent a bailout. To see this, first notice that $\pi=0$, there does not exist a deviating strategy that the north can undertake which would make $W_1^{nb}\left(\tilde{b}_{N1},b_{S1}^{b},\pi,\psi\right)\geq W_1^{b}\left(\tilde{b}_{N1},b_{S1}^{b}\right)=W_1^{nb}\left(\tilde{b}_{N1},b_{S1}^{b},0,0\right)$. Similarly for π small and positive, no such strategy exists. In particular even if the North sets $\tilde{b}_{N1}=b_{S1}^{b}$ so that the spread is zero, the central government still strictly prefers to bailout since both regions now have to pay a fixed cost.

3. The central authority, does indeed want to bail out, i.e. $W_1^{nb}\left(b_{N1}^{b},b_{S1}^{b},1\right)<W_1^{b}\left(b_{N1}^{b},b_{S1}^{b}\right)$ or

$$\sum_i u\left(Y-b_{i1}^{b}(\pi,\psi)+q\mathbf{b}_{i2}\left(b_1^{b},0,\psi\right)\right)+\beta u\left(Y-\frac{\mathbf{B_2}\left(b_1^{nb},0,\psi\right)}{2}\right)$$
$$\geq\sum_i u\left(Y-\psi Y\mathbf{1}_{b_{i1}>\bar{b}}-b_{i1}^{b}(\pi,\psi)+q\mathbf{b}_{i2}\left(b_1^{b},1,\psi\right)\right)+\beta u\left(Y-\frac{\mathbf{B_2}\left(b_1^{nb},1,\psi\right)}{2}\right)$$

At $\pi = 0$,

$$\sum_i u\left(Y - b_{i1}^b(0,\psi) + q\mathbf{b}_{i2}\left(b_1^b, 0, \psi\right)\right) + \beta u\left(Y - \frac{\mathbf{B_2}\left(b_1^{nb}, 0, \psi\right)}{2}\right)$$
$$\geq \sum_i u\left(Y - \psi Y \mathbf{1}_{b_{i1} > \bar{b}} - b_{i1}^b(0,\psi) + q\mathbf{b}_{i2}\left(b_1^b, 1, \psi\right)\right) + \beta u\left(Y - \frac{\mathbf{B_2}\left(b_1^{nb}, 1, \psi\right)}{2}\right)$$

This is technically a condition on primitives. One sufficient condition for the above to hold is if β is small enough. Define

$$\bar{\beta}^{nc} \equiv \frac{\sum_i \left[u\left(Y - b_{i1}^b(0,\psi) + q\mathbf{b}_{i2}\left(b_1^b, 0, \psi\right)\right) - u\left(Y - \psi Y \mathbf{1}_{b_{i1} > \bar{b}} - b_{i1}^b(0,\psi) + q\mathbf{b}_{i2}\left(b_1^b, 1, \psi\right)\right)\right]}{2\left[u\left(Y - \frac{\mathbf{B_2}\left(b_1^{nb}, 1, \psi\right)}{2}\right) - u\left(Y - \frac{\mathbf{B_2}\left(b_1^{nb}, 0, \psi\right)}{2}\right)\right]}$$

Then for any $\beta < \bar{\beta}^{nc}$, the inequality holds. Finally, by continuity, the inequality holds for π close to zero. Therefore either the fiscal rule is violated and we have a unique bailout equilibrium or the fiscal rule does not bind and we equilibrium outcome is the one described in Proposition 1.

We now turn to show that the debt level for the South in the bailout equilibrium can be larger than in the case without rules (and the corresponding no-bailout equilibrium). Let $b_{i1}^b(\pi,\psi)$ denote the debt holdings in period 1 for local government in the bailout allocation given π and some fiscal rule $\left(\bar{b}, \psi\right)$ and $b_{i1}^b(\pi,\psi)$, the debt holdings in the no-bailout equilibrium with no rules. The foc for $b_{S1}^b(\pi,\psi)$ is

$$\begin{aligned} qu'\left(Y_{S0} + qb_{S1}^b(\pi,\psi)\right) =& \beta\pi u'\left(Y - \psi - b_{S1}^b(\pi,\psi) + q\mathbf{b}_{S2}\left(b_1^b, 1, \psi\right)\right) \\ &+ \beta(1-\pi)u'\left(Y - b_{S1}^b(\pi,\psi) + q\mathbf{b}_{S2}\left(b_1^b, 0, \psi\right)\right) \\ &+ \frac{\beta^2}{2}(1-\pi)u'\left(Y - \frac{\mathbf{B}_2\left(b_1^b, 0, \psi\right)}{2}\right)\frac{\partial \mathbf{b}_{N2}\left(b_1^b, 0, \psi\right)}{\partial b_{S1}} \end{aligned} \tag{49}$$

or, using the foc in period 1,

$$qu'\left(Y - b_{S1}^b + q\mathbf{b}_{S2}\left(b_1^b, 0, \psi\right)\right) = \frac{\beta}{2}u'\left(Y - \frac{\mathbf{B}_2\left(b_1^b, 0, \psi\right)}{2}\right)$$
$$qu'\left(Y - b_{S1}^b + q\mathbf{b}_{S2}\left(b_1^b, 1, \psi\right)\right) = \beta u'\left(Y - \mathbf{b}_{S2}\left(b_1^b, 1, \psi\right)\right)$$

we can write

$$qu'\left(Y_{S0}+qb_{S1}^{b}\left(\pi,\psi\right)\right)=\frac{\beta^{2}}{q}\pi u'\left(Y-\mathbf{b}_{S2}\left(b_{S1}^{b},0,\psi\right)\right)+\frac{\beta^{2}}{q}\left(1-\pi\right)u'\left(Y-\frac{\mathbf{B}_{2}\left(b_{1}^{b},0,\psi\right)}{2}\right)+\frac{\beta^{2}}{2}\left(1-\pi\right)u'\left(Y-\frac{\mathbf{B}_{2}\left(b_{1}^{b},0,\psi\right)}{2}\right)\frac{\partial\mathbf{b}_{N2}\left(b_{1},0,\psi\right)}{\partial b_{S1}} \tag{50}$$

The foc for $b_{S1}^{nb}\left(\pi,0\right)$ is

$$qu'\left(Y_{S0}+qb_{S1}^{nb}\left(\pi,0\right)\right)=\beta u'\left(Y-b_{S1}\left(\pi,0\right)+q\mathbf{b}_{S2}\left(b_{1}^{nb},\pi,0\right)\right)+\frac{\beta^{2}}{2}\left(1-\pi\right)u'\left(Y-\frac{\mathbf{B}_{S2}\left(b_{1}^{nb},\pi,0\right)}{2}\right)\frac{\partial\mathbf{b}_{N2}\left(b_{1}^{nb},\pi,0\right)}{\partial b_{S1}} \tag{51}$$

or, using the foc in period 1,

$$qu'\left(Y-b_{i1}^{nb}+q\mathbf{b}_{i2}\left(b_{1}^{nb},\pi,0\right)\right)=\beta\pi u'\left(Y-\mathbf{b}_{i2}\left(b_{1}^{nb},\pi,0\right)\right)+\left(1-\pi\right)\frac{\beta}{2}u'\left(Y-\frac{\mathbf{B}_{2}\left(b_{1}^{nb},\pi,0\right)}{2}\right)$$

we can write

$$qu'\left(Y_{S0}+qb_{S1}^{nb}\left(\pi,0\right)\right)=\frac{\beta^{2}}{q}\pi u'\left(Y-\mathbf{b}_{S2}\left(b_{1}^{nb},\pi,0\right)\right)+\frac{\beta^{2}}{q}\left(1-\pi\right)u'\left(Y-\frac{\mathbf{B}_{2}\left(b_{1}^{nb},\pi,0\right)}{2}\right)+\frac{\beta^{2}}{2}\left(1-\pi\right)u'\left(Y-\frac{\mathbf{B}_{2}\left(b_{1}^{nb},\pi,0\right)}{2}\right)\frac{\partial\mathbf{b}_{N2}\left(b_{1}^{nb},\pi,0\right)}{\partial b_{S1}} \tag{52}$$

Define the following functions:

$$F_i^b(\pi, b_1) = \pi u'(Y - \mathbf{b}_{i2}(b_1, 1)) + (1-\pi)\frac{u'\left(Y - \frac{\sum_j \mathbf{b}_{j2}(b_1,0)}{2}\right)}{2}$$
$$+ q(1-\pi)\frac{u'\left(Y - \frac{\frac{\sum_j \mathbf{b}_{j2}(b_1,0)}{2}}{2}\right)}{2}\frac{\partial \mathbf{b}_{N2}(b_1, 0)}{\partial b_{S1}}$$
$$F_i^{nb}(\pi, b_1) = \pi u'(Y - \mathbf{b}_{i2}(b_1, \pi)) + (1-\pi)\frac{u'\left(Y - \frac{\sum_j \mathbf{b}_{j2}(b_1,\pi)}{2}\right)}{2}$$
$$+ q(1-\pi)\frac{u'\left(Y - \frac{\frac{\sum_j \mathbf{b}_{j2}(b_1,\pi)}{2}}{2}\right)}{2}\frac{\partial \mathbf{b}_{N2}(b_1, \pi)}{\partial b_{S1}}$$

which are proportional to the RHS of (50) and (52) respectively. Let

$$H(\pi, b_1) = F_S^{nb}(\pi, b) - F_S^b(\pi, b_1).$$

We know that $H(0, b_1) = 0$ and we want to find conditions such that $\partial H / \partial \pi > 0$ as $\pi \downarrow 0$ so that the expected marginal utility of debt is smaller when there is resolution of uncertainty in period 1 (and so debt must be higher). Consider

$$\frac{\partial H(b_1, \pi)}{\partial \pi} = \left[u'(Y - \mathbf{b}_{i2}(b_1, \pi)) - \frac{u'\left(Y - \frac{\sum_j \mathbf{b}_{j2}(b_1,\pi)}{2}\right)}{2}\right] +$$
$$- \pi u''(Y - \mathbf{b}_{i2}(b_1, \pi))\frac{\partial \mathbf{b}_{i2}(b_1, \pi)}{\partial \pi} - (1-\pi)\sum_j \frac{u''\left(Y - \frac{\sum_j \mathbf{b}_{j2}(b_1,\pi)}{2}\right)}{4}\frac{\partial \mathbf{b}_{j2}(b_1, \pi)}{\partial \pi}$$
$$- \left[u'(Y - \mathbf{b}_{i2}(b_1, 1)) - \frac{u'\left(Y - \frac{\sum_j \mathbf{b}_{j2}(b_1,0)}{2}\right)}{2}\right]$$
$$- q\left[u'\left(Y - \frac{\sum_j \mathbf{b}_{j2}(b_1, \pi)}{2}\right)\frac{\partial \mathbf{b}_{N2}(\pi, 0)}{\partial b_{S1}} - u'\left(Y - \frac{\sum_j \mathbf{b}_{j2}(b_1, 0)}{2}\right)\frac{\partial b_{N2}^{nb}(0, 0)}{\partial b_{S1}}\right]$$
$$+ q(1-\pi)\left[-\frac{u''\left(Y - \frac{\sum_j \mathbf{b}_{j2}(b_1,\pi)}{2}\right)}{4}\sum_j \frac{\partial \mathbf{b}_{j2}(b_1, \pi)}{\partial \pi}\frac{\partial b_{N2}^{nb}(\pi, 0)}{\partial b_{S1}}\right.$$
$$\left. + \frac{u'\left(Y - \frac{\sum_j \mathbf{b}_{j2}(b_1,\pi)}{2}\right)}{2}\frac{\partial b_{N2}^{nb}(\pi, 0)}{\partial \pi \partial b_{S1}}\right]$$

Taking limit as π goes to zero we obtain

$$\lim_{\pi\downarrow 0}\frac{\partial H\left(b_1,\pi\right)}{\partial\pi}=\underbrace{\left[u'\left(Y-\mathbf{b}_{S2}\left(b_1,0\right)\right)-u'\left(Y-\mathbf{b}_{S2}\left(b_1,1\right)\right)\right]}_{>0}+\underbrace{\left[-\frac{u''\left(Y-\frac{\mathbf{B}_2\left(b_1,0\right)}{2}\right)}{4}\right]}_{>0}\underbrace{\frac{\partial\mathbf{B}_2\left(b_1,0\right)}{\partial\pi}}_{<0}$$
$$+q\left[-\frac{u''\left(Y-\frac{\sum_j\mathbf{b}_{j2}\left(b_1,0\right)}{2}\right)}{4}\sum_j\frac{\partial\mathbf{b}_{j2}\left(b_1,0\right)}{\partial\pi}\frac{\partial\mathbf{b}_{N2}\left(b_1,0\right)}{\partial b_{S1}}+\frac{u'\left(Y-\frac{\sum_j\mathbf{b}_{j2}\left(b_1,0\right)}{2}\right)}{2}\frac{\partial\mathbf{b}_{N2}\left(b_1,0\right)}{\partial\pi\partial b_{S1}}\right]$$
$$=\underbrace{\left[u'\left(Y-\mathbf{b}_{S2}\left(b_1,0\right)\right)-u'\left(Y-\mathbf{b}_{S2}\left(b_1,1\right)\right)\right]}_{>0}+$$
$$+\underbrace{\left[-\frac{u''\left(Y-\frac{\mathbf{B}_2\left(b_1,0\right)}{2}\right)}{4}\right]}_{>0}\underbrace{\frac{\partial\mathbf{B}_2\left(b_1,0\right)}{\partial\pi}}_{<0}\left(1-q\frac{\partial\mathbf{b}_{N2}\left(b_1,0\right)}{\partial b_{S1}}\right)$$
$$+q\frac{u'\left(Y-\frac{\sum_j\mathbf{b}_{2j}\left(b_1,0\right)}{2}\right)}{2}\frac{\partial\mathbf{b}_{N2}\left(b_1,0\right)}{\partial\pi\partial b_{S1}}$$

where the first term is positive since $\mathbf{b}_{2S}\left(b_1,0\right)>\mathbf{b}_{2S}\left(b_1,1\right)$ the first term is positive and the second term is negative by concavity of u and the fact that aggregate debt is decreasing in π.

We can interpret the first two terms: The first term captures the fact that knowing the type of the central government in period 1 allows the government to adjust its debt going forward so in the absence of a bailout marginal utility is not so large.The second term captures the fact that an increase in π reduces debt issued from period 1 to 2 if there is no resolution of uncertainty in period 1. This effect is absent when there is separation in period 1 since continuation equilibrium on each branch of the game tree does not depend on π.

We can find conditions on primitives such that the limit above is positive. For example, for quadratic utility, as we show in Appendix B, we can write the decision rules in period 1 as

$$\mathbf{b}_{S2}\left(b_1,\pi,\psi\right)=\gamma_1\left(\pi\right)b_{S1}+\gamma_2\left(\pi\right)b_{N1}+\gamma_3\left(\pi\right)Y+\gamma_4\left(\pi\right)\psi\mathbb{I}_S+\gamma_5\left(\pi\right)$$
$$\mathbf{b}_{N2}\left(b_1,\pi,\psi\right)=\gamma_1\left(\pi\right)b_{N1}+\gamma_2\left(\pi\right)b_{S1}+\gamma_3\left(\pi\right)Y+\gamma_4\left(\pi\right)\psi\mathbb{I}_N+\gamma_5\left(\pi\right)$$

and

$$\mathbf{B}_2\left(b_1,\pi,\psi\right)=\left(\gamma_1\left(\pi\right)+\gamma_2\left(\pi\right)\right)\left(b_{N1}+b_{S1}\right)+\gamma_3\left(\pi\right)2Y+\sum_i\gamma_4\left(\pi\right)\psi\mathbb{I}_i+2\gamma_5\left(\pi\right)$$

where the coefficient $\gamma_n(\pi)$ are defined in Appendix B. Then

$$
\begin{aligned}
H(b,\pi) &= \alpha\pi\left[\mathbf{b}_{S2}(b,\pi) - \mathbf{b}_{S2}(b,1)\right] + \alpha\frac{1-\pi}{4}\left[\sum_i \mathbf{b}_{i2}(b,\pi) - \sum_i \mathbf{b}_{i2}(b,0)\right] \\
&+ \frac{1-\pi}{2}\left\{\left[1-\alpha\left(Y - \frac{\sum_i \mathbf{b}_{i2}(b,\pi)}{2}\right)\right]\gamma_2(\pi) - \left[1-\alpha\left(Y-\frac{\sum_i \mathbf{b}_{i2}(b,0)}{2}\right)\right]\gamma_2(0)\right\} \\
&= \alpha\left\{\left[\pi\mathbf{b}_{S2}(b,\pi) + \frac{(1-\pi)}{4}\sum_i \mathbf{b}_{i2}(b,\pi)\right] - \left[\pi\mathbf{b}_{S2}(b,1) + \frac{(1-\pi)}{4}\sum_i \mathbf{b}_{i2}(b,0)\right]\right\} \\
&+ \frac{1-\pi}{2}\left\{\left[1-\alpha\left(Y - \frac{\sum_i \mathbf{b}_{i2}(b,\pi)}{2}\right)\right]\gamma_2(\pi) - \left[1-\alpha\left(Y-\frac{\sum_i \mathbf{b}_{i2}(b,0)}{2}\right)\right]\gamma_2(0)\right\}
\end{aligned}
$$

Taking the limit as $\alpha \downarrow 0$ we obtain

$$
\lim_{\alpha\downarrow 0} H(b,\pi) = \frac{1-\pi}{2}\left[\gamma_2(\pi) - \gamma_2(0)\right] > 0
$$

since $\gamma_2(\pi)$ is increasing in π. □

Proof of Proposition 3

Proof. First consider the case without rules. We will first show that for π close to zero, the ex-ante utilitarian welfare on the NB allocation is strictly larger than the B allocation. We can define

$$
\begin{aligned}
W_0^{nb}(\pi) &\equiv \sum_i u\left(G_{i0}^{nb}\right) + \beta W_1\left(b_1^{nb},\pi\right) \\
W_0^{b}(\pi) &\equiv \sum_i u\left(G_{i0}^{b}\right) + \beta\pi W_1\left(b_1^{nb},1\right) + \beta(1-\pi)W_1\left(b_1^{nb},0\right)
\end{aligned}
$$

We know that $W_0^{nb}(0) - W_0^{b}(0) = 0$. We have

$$
\begin{aligned}
&\frac{d}{d\pi}\left[W_0^{nb}(\pi) - W_0^{b}(\pi)\right] \\
=&\frac{d}{d\pi}\left(\sum_i\left[u\left(G_{i0}^{nb}\right) - u\left(G_{i0}^{b}\right)\right]\right) + \beta\frac{\mathrm{d}W_1\left(b_1^{nb},\pi\right)}{\mathrm{d}\pi} - \beta(1-\pi)\frac{\mathrm{d}W_1\left(b_1^{b},0\right)}{\mathrm{d}\pi} \\
&-\beta\pi\frac{\mathrm{d}W_1\left(b_1^{b},1\right)}{\mathrm{d}\pi} - \beta\left[W_1\left(b_1^{nb},1\right) - W_1\left(b_1^{nb},0\right)\right]
\end{aligned}
$$

At $\pi = 0$ this is

$$\frac{d}{d\pi}\left(\sum_i \left[u\left(G_{i0}^{nb}\right) - u\left(G_{i0}^{b}\right)\right]\right) + \beta\frac{\partial W_1\left(b_1^{nb},0\right)}{\partial\pi} - \beta\left[W_1\left(b_1^{nb},1\right) - W_1\left(b_1^{nb},0\right)\right] \quad (53)$$

The first term is

$$\begin{aligned}&\sum_i \left[u'\left(G_{i0}^{nb}\right) q\frac{db_{i1}^{nb}}{d\pi} - u'\left(G_{i0}^{b}\right) q\frac{db_{i1}^{b}}{d\pi}\right]\\ =&\sum_i u'\left(G_{i0}^{nb}\right) q\left[\frac{db_{i1}^{nb}}{d\pi} - \frac{db_{i1}^{b}}{d\pi}\right]\end{aligned}$$

$$\frac{db_{N1}^{nb}}{d\pi} - \frac{db_{N1}^{nb}}{d\pi} = -\frac{\beta\left[u'\left(Y - b_{N1}^{b} + q\mathbf{b}_{N2}\left(b_1^{b},1\right)\right) - u'\left(Y - b_{N1}^{b} + q\mathbf{b}_{N2}\left(b_1^{b},0\right)\right)\right]}{\left[u''\left(Y_{N0} + qb_{N1}^{nb}\right)q^2 + \beta u''\left(Y - b_{N1}^{nb} + q\mathbf{b}_{N2}\right)\right]}$$

With quadratic utility,

$$\sum_i\left(\frac{db_{i1}^{nb}}{d\pi} - \frac{db_{i1}^{nb}}{d\pi}\right) = \beta q\frac{\mathbf{B}_2\left(b_1^{b},1\right) - \mathbf{B}_2\left(b_1^{b},0\right)}{\left[q^2+\beta\right]}$$

Next from the proof of Proposition 3 we know that the second term,

$$\frac{\partial W_1\left(b_1^{nb},0\right)}{\partial\pi} = -\beta\frac{u'\left(G_{i2}\right)}{2}\frac{\partial\mathbf{B}_2}{\partial\pi}$$

Finally,

$$\begin{aligned}&W_1\left(b_1^{nb},0\right) - W_1\left(b_1^{nb},1\right)\\ =&\sum_i\left[u\left(Y - b_{i1}^{nb} + q\mathbf{b}_{i2}\left(b_1,0\right)\right) + \beta u\left(Y - \frac{\mathbf{B}_2\left(b_1,0\right)}{2}\right)\right]\\ &-\sum_i\left[u\left(Y - b_{i1}^{nb} + q\mathbf{b}_{i2}\left(b_1,1\right)\right) + \beta u\left(Y - \frac{\mathbf{B}_2\left(b_1,1\right)}{2}\right)\right]\\ \geq&\sum u'\left(Y - b_{i1}^{nb} + q\mathbf{b}_{i2}\left(b_1,0\right)\right) q\left[\mathbf{b}_{i2}\left(b_1,0\right) - \mathbf{b}_{i2}\left(b_1,1\right)\right]\\ &+\beta u'\left(Y - \frac{\mathbf{B}_2\left(b_1,0\right)}{2}\right)\left[\mathbf{B}_2\left(b_1,1\right) - \mathbf{B}_2\left(b_1,0\right)\right]\\ =&\left[\mathbf{B}_2\left(b_1,0\right) - \mathbf{B}_2\left(b_1,1\right)\right]\left[u'\left(Y - b_{i1}^{nb} + q\mathbf{b}_{i2}\left(b_1,0\right)\right)q - \beta u'\left(Y - \frac{\mathbf{B}_2\left(b_1,0\right)}{2}\right)\right]\\ =&-\left[\mathbf{B}_2\left(b_1,0\right) - \mathbf{B}_2\left(b_1,1\right)\right]\frac{\beta}{2}u'\left(Y - \frac{\mathbf{B}_2\left(b_1,0\right)}{2}\right)\end{aligned}$$

Substituting these into (53) yields

$$
\begin{aligned}
&-u'\left(G_{i0}^{nb}\right)\beta q^2\frac{\mathbf{B}_2\left(b_1^b,0\right)-\mathbf{B}_2\left(b_1^b,1\right)}{[q^2+\beta]}-\beta\frac{u'\left(G_{i2}\right)}{2}\frac{\partial \mathbf{B}_2}{\partial \pi}-\left[\mathbf{B}_2\left(b_1,0\right)-\mathbf{B}_2\left(b_1,1\right)\right]\frac{\beta}{2}u'\left(Y-\frac{\mathbf{B}_2\left(b_1,0\right)}{2}\right)\\
\geq &-u'\left(Y_{i0}\right)\beta q^2\frac{\eta Y}{[q^2+\beta]}-\beta\frac{u'\left(G_{i2}\right)}{2}\frac{\mathrm{d}B_2}{\mathrm{d}\pi}-\left[\mathbf{B}_2\left(b_1,0\right)-\mathbf{B}_2\left(b_1,1\right)\right]\frac{\beta}{2}u'\left(Y-\frac{\mathbf{B}_2\left(b_1,0\right)}{2}\right)
\end{aligned}
$$

With quadratic utility this is

$$
-\left[1-\alpha Y_{i0}\right]\beta q^2\frac{\eta Y}{[q^2+\beta]}-\beta\frac{[1-\alpha G_{i2}]}{2}\frac{\partial \mathbf{B}_2}{\partial \pi}-\left[\mathbf{B}_2\left(b_1,0\right)-\mathbf{B}_2\left(b_1,1\right)\right]\frac{\beta}{2}\left[1-\alpha G_{i2}\right]
$$

We know from the proof of Lemma 2 that

$$
\frac{\partial \mathbf{B}_2}{\partial \pi}=\left[-\frac{1}{1-\frac{u''(G_{S2})}{a}\frac{u''(G_{N2})}{a}}-\frac{u''\left(G_{N2}\right)}{a}\right]\frac{1}{A}\left[\Delta MU_S+\Delta MU_N\right]
$$

When u is quadratic, one can show that

$$
-\frac{\partial \mathbf{B}_2}{\partial \pi}=4\left[\frac{x}{x^2-1}-\frac{1}{x^2}\right]\left[\frac{1}{\alpha}-Y+\frac{B_2}{2}\right]
$$

where $x=1+\frac{4q}{\beta}$. As a result this can be made arbitrarily large by sending $\alpha\to 0$. Finally, consider the case with rules and define $W_0^{nb}\left(\pi,\psi\right), W_0^{b}\left(\pi,\psi\right)$ analogously to before. Since

$$
\begin{aligned}
\frac{d}{d\pi}\left[W_0^{nb}\left(0,0\right)-W_0^{b}\left(0,\psi\right)\right]&=\frac{d}{d\pi}\left(\sum_i\left[u\left(G_{i0}^{nb}\right)-u\left(G_{i0}^{b}\right)\right]\right)+\beta\frac{\partial W_1\left(b_1^{nb},0\right)}{\partial \pi}\\
&\quad-\beta\left[W_1\left(b_1^{nb},1,\psi\right)-W_1\left(b_1^{nb},0,\psi\right)\right]\\
&>\frac{d}{d\pi}\left(\sum_i\left[u\left(G_{i0}^{nb}\right)-u\left(G_{i0}^{b}\right)\right]\right)+\beta\frac{\partial W_1\left(b_1^{nb},0\right)}{\partial \pi}\\
&\quad-\beta\left[W_1\left(b_1^{nb},1,0\right)-W_1\left(b_1^{nb},0,0\right)\right]
\end{aligned}
$$

we also have that $W_0^{nb}\left(\pi,0\right)>W_0^{b}\left(\pi,\psi\right)$ for π sufficiently small. □

Proof of Proposition 4

Proof. Given the punishment ψ we know that for π small enough that the only two possible equilibria are i) the debt limit is never binding and ii) there is separation in period 1 and early resolution of uncertainty. To prove the first part of the proposition and check whether $W_0^{r,\text{sep}} < W_0^{r,\text{pool}}$, it suffices to check that equilibrium i) gives the commitment type higher ex-ante welfare than equilibrium ii).

Let $\Delta(\pi,\psi) = W_0^{r,\text{sep}} - W_0^{r,\text{pool}}$. Then

$$\begin{aligned}\Delta(\pi,\psi) = &\sum_i \Big[u\left(Y_{i0} + qb_i^b(\pi,\psi)\right) + \beta u\left(Y - \psi Y \mathbf{1}_{b_{i1}>\bar{b}} - b_{i1}^b(\pi,\psi) + q\mathbf{b}_{i2}\left(b_{i1}^b(\pi,\psi),1,\psi\right)\right) + \\ &+\beta^2 u\left(Y - \mathbf{b}_{i2}\left(b_{i1}^b(\pi,\psi),1,\psi\right)\right)\Big] \\ &-\sum_i \Big[u\left(Y_{i0} + qb_{i1}^{nb}(\pi,0)\right) + \beta u\left(Y - b_{i1}^{nb}(\pi,0) + q\mathbf{b}_{i2}\left(b_{i1}^{nb}(\pi,0),\pi,0\right)\right) + \\ &+\beta^2 u\left(Y - \mathbf{b}_{i2}\left(b_{i1}^{nb}(\pi,0),\pi,0\right)\right)\Big]\end{aligned}$$

As $\pi \to 0$, $\Delta(\pi,\psi) \to$

$$\begin{aligned}&\beta\sum_i \Big[u\left(Y - \psi Y \mathbf{1}_{b_{i1}>\bar{b}} - b_{i1}^b(0,\psi) + q\mathbf{b}_{i2}\left(b_{i1}^b(0,\psi),1,\psi\right)\right) + \beta u\left(Y - \mathbf{b}_{i2}\left(b_{i1}^b(0,\psi),1,\psi\right)\right)\Big] \\ &-\beta\sum_i \Big[u\left(Y - b_{i1}^b(0,\psi) + q\mathbf{b}_{i2}\left(b_{i1}^b(0,\psi),0,0\right)\right) + \beta u\left(Y - \mathbf{b}_{i2}\left(b_{i1}^b(0,\psi),0,0\right)\right)\Big]\end{aligned}$$

since $b_{i1}^b(0,\psi) = b_{i1}^{nb}(0,0) = b_{i1}^{nb}(0,\psi)$. Notice that if the $\Delta(0,\psi) < 0$, then for π small $W_0^{r,\text{sep}} < W_0^{r,\text{pool}}$. This implies that the commitment type will optimally choose to set $\psi = 0$ and no separation in period 1.

Define

$$\underline{\beta} \equiv \frac{\sum_i \left[u\left(Y - b_{i1}^b(0,\psi) + q\mathbf{b}_{i2}\left(b_{i1}^b(0,\psi),0,0\right)\right) - u\left(Y - \psi Y \mathbf{1}_{b_{i1}>\bar{b}} - b_{i1}^b(0,\psi) + q\mathbf{b}_{i2}\left(b_{i1}^b(0,\psi),1,\psi\right)\right)\right]}{\sum_i \left[u\left(Y - \mathbf{b}_{i2}\left(b_{i1}^b(0,\psi),1,\psi\right)\right) - u\left(Y - \mathbf{b}_{i2}\left(b_{i1}^b(0,\psi),0,0\right)\right)\right]}$$

Then clearly for $\beta < \underline{\beta}$, the unique constitution will feature no fiscal rules.

To prove the next part, notice that if $\beta > \underline{\beta}$, then for π close to zero, $W_0^{r,\text{sep}} > W_0^{r,\text{pool}}$. To show that this is an equilibrium, we need to show that the no-commitment type does indeed want to separate for $\beta > \underline{\beta}$. Define

$$\bar{\beta} \equiv \frac{\sum_i \left[u\left(Y - b_{i1}^b(0,\psi) + q\mathbf{b}_{i2}\left(b_1^b,0,\psi\right)\right) - u\left(Y - \psi Y \mathbf{1}_{b_{i1}>\bar{b}} - b_{i1}^b(0,\psi) + q\mathbf{b}_{i2}\left(b_1^b,1,\psi\right)\right)\right]}{2\left[u\left(Y - \frac{\mathbf{B_2}\left(b_1^{nb},1,\psi\right)}{2}\right) - u\left(Y - \frac{\mathbf{B_2}\left(b_1^{nb},0,\psi\right)}{2}\right)\right]}$$

We know from earlier that if $\beta < \bar{\beta}$, then for π close to zero, the no-commitment will strictly prefer to not enforce the rule at $t = 1$. To show that this a well defined interval we need to show that $\bar{\beta} > \underline{\beta}$. This is true if $2\left[u\left(Y - \frac{\mathbf{B_2}\left(b_1^{nb},1,\psi\right)}{2}\right) - u\left(Y - \frac{\mathbf{B_2}\left(b_1^{nb},0,\psi\right)}{2}\right)\right] <$

$\sum_i \left[u\left(Y - \mathbf{b}_{i2}\left(b^b_{i1}\left(0,\psi\right),1,\psi\right)\right) - u\left(Y - \mathbf{b}_{i2}\left(b^b_{i1}\left(0,\psi\right),0,0\right)\right)\right]$, or

$$\begin{aligned}0 > 2u\left(Y - \frac{\mathbf{B_2}\left(b^b_1,1,\psi\right)}{2}\right) - \sum_i u\left(Y - \mathbf{b}_{i2}\left(b^b_{i1}\left(0,\psi\right),1,\psi\right)\right) \\ - \left[2u\left(Y - \frac{\mathbf{B_2}\left(b^b_1,0,\psi\right)}{2}\right) - \sum_i u\left(Y - \mathbf{b}_{i2}\left(b^b_{i1}\left(0,\psi\right),0,0\right)\right)\right].\end{aligned}$$

For this to be true we need $\mathbf{b}_{i2}\left(b^b_1,1,\psi\right) - \mathbf{b}_{-i2}\left(b^b_1,1,\psi\right) < \mathbf{b}_{i2}\left(b^b_1,0,\psi\right) - \mathbf{b}_{-i2}\left(b^b_1,0,\psi\right)$. From the first order conditions for $\mathbf{b}_{i2}\left(b^b_1,0,\psi\right)$ we have

$$u'\left(Y - b^b_{i1}\left(0,\psi\right) + q\mathbf{b}_{i2}\left(b^b_1,0,\psi\right)\right)q = \frac{\beta}{2}u'\left(Y - \frac{\mathbf{B}_{i2}\left(b^b_1,0,\psi\right)}{2}\right)$$

This implies that

$$\mathbf{b}_{S2}\left(b^b_1,0,\psi\right) - \mathbf{b}_{N2}\left(b^b_1,0,\psi\right) = \frac{b^b_{S1}\left(0,\psi\right) - b^b_{N1}\left(0,\psi\right)}{q}$$

Next from the first order conditions for $\mathbf{b}_{i2}\left(b^b_1,1,\psi\right)$ we have

$$u'\left(Y - \psi Y \mathbf{1}_{b_{i1}>\bar{b}} - b^b_{i1}\left(0,\psi\right) + q\mathbf{b}_{i2}\left(b^b_1,1,\psi\right)\right)q = \beta u'\left(Y - q\mathbf{b}_{i2}\left(b^b_1,1,\psi\right)\right)$$

Then

$$\begin{aligned}&u'\left(Y - \psi Y - b^b_{S1}\left(0,\psi\right) + q\mathbf{b}_{S2}\left(b^b_1,1,\psi\right)\right) - u'\left(Y - b^b_{S1}\left(0,\psi\right) + q\mathbf{b}_{S2}\left(b^b_1,1,\psi\right)\right) \\ &= \beta u'\left(Y - q\mathbf{b}_{S2}\left(b^b_1,1,\psi\right)\right) - \beta u'\left(Y - q\mathbf{b}_{N2}\left(b^b_1,1,\psi\right)\right) > 0\end{aligned}$$

and so

$$\mathbf{b}_{S2}\left(b^b_1,1,\psi\right) - \mathbf{b}_{N2}\left(b^b_1,1,\psi\right) < \frac{\psi Y + b^b_{S1}\left(0,\psi\right) - b^b_{N1}\left(0,\psi\right)}{q}$$

Note that if $Y = 0$, then $\mathbf{b}_{S2}\left(b^b_1,0,\psi\right) - \mathbf{b}_{N2}\left(b^b_1,0,\psi\right) > \mathbf{b}_{S2}\left(b^b_1,1,\psi\right) - \mathbf{b}_{N2}\left(b^b_1,1,\psi\right)$ and so $\bar{\beta} > \underline{\beta}$ for Y small enough. Finally, we need to show that the no-commitment type will mimic the commitment type in period 0 and announce the same rule. The value of mimicking is given by

$$\begin{aligned}W^m_0\left(\pi\right) &= \sum_i u\left(Y_{i0} + qb^b_{i1}\left(\pi\right)\right) + \beta W^b_1\left(B^b_1\left(\pi\right)\right) \\ &= \sum_i \left[u\left(Y_{i0} + qb_{i1}\left(\pi\right)\right) + \beta u\left(Y - b_{i1}\left(\pi\right) + qb_{i2}\left(b_1,0\right)\right) + \beta^2 u\left(Y - \frac{b_{i2}\left(b_1,0\right) + b_{-i2}\left(b_1,0\right)}{2}\right)\right]\end{aligned}$$

while the value of not mimicking is just $W_0^m(0)$. We will establish that $\frac{\partial}{\partial\pi}W_0^m(0) > 0$, which in turn implies that if π is close to 0, the no-commitment type will always find it optimal to mimic.

Differentiating $W_0^m(\pi)$ wrt π yields

$$\frac{\partial}{\partial\pi}W_0^m(0) = \sum_i \Big[u'(G_{i0})\, q b_{i1}'(\pi) - \beta u(G_{i1})\, b_{i1}'(\pi) + \\ + \sum_j u'(G_{i1})\, q \frac{\partial}{\partial b_{j1}} b_{i2}(b_1,0)\, b_{j1}'(\pi) - \frac{\beta^2}{2}\sum_j u'(G_{i2}) \frac{\partial}{\partial b_{j1}} B_2(b_1,0)\, b_{j1}'(\pi)\Big]$$

Recall the first order conditions for the fiscal authority in periods 1 and 2

$$u'(G_{i0})\, q = \beta u'(G_{i1}) + \frac{\beta^2}{2}u'(G_{i2})\frac{\partial}{\partial b_{i1}}b_{-i2}$$
$$u'(G_{i1})\, q = \frac{\beta}{2}u'(G_{i2})$$

Substituting these into the previous equation yields

$$\frac{\partial}{\partial\pi}W_0^m(0) = \sum_i u'(G_{i1})\, q \frac{\partial}{\partial b_{-i1}} b_{i2}(b_1,0)\, b_{-i1}'(0)$$
$$= u(G_{i1})\, q \frac{\partial}{\partial b_{-i1}} b_{i2}(b_1,0)\, B_1'(0) > 0$$

since at $\pi = 0$, $\frac{\partial}{\partial b_{N1}} b_{S2}(b_1,0) = \frac{\partial}{\partial b_{N1}} b_{S2}(b_1,0) < 0$ and $B_1'(0) < 0$. □

Proof of Proposition 5

We prove this analogously to proposition 1. First consider step 1. As before, we have that

$$\mathcal{W}'(0) = \frac{\partial W^{nb}\left(b_1^{nb}(0),0\right)}{\partial\pi}$$
$$= \sum_i \left(\begin{array}{c} u'(G_{i1})\left[\frac{\partial}{\partial\pi}Q_2\left(\mathbf{b}_2\left(b_1^{nb},0\right),0\right)\mathbf{b}_{i2} + \sum_j \frac{\partial}{\partial b_{j2}}Q_2\left(\mathbf{b}_2\left(b_1^{nb},0\right),0\right)\frac{\partial \mathbf{b}_{j2}}{\partial\pi} + \right. \\ \left. +Q_2\left(\mathbf{b}_2\left(b_1^{nb},0\right),0\right)\frac{\partial \mathbf{b}_{i2}}{\partial\pi}\right] - \frac{\beta}{2}u'(G_{i2})\left[\frac{\partial \mathbf{b}_{i2}}{\partial\pi} + \frac{\partial \mathbf{b}_{-i2}}{\partial\pi}\right] \end{array} \right)$$

From the focs at $\pi = 0$ we have

$$u'(G_{i1})\, q = \frac{\beta}{2}u'(G_{i2}) + \mu_{i2}$$

where μ_{i2} is the multiplier on the constraint $b_{i2} \leq 2\phi_2 - b_{-i2}$. Then,

$$\begin{aligned}\mathcal{W}'(0) = \sum_i \Bigg(& u'(G_{i1}) \left[\frac{\partial}{\partial \pi} Q_2 \left(b_2^{nb}(0), 0 \right) b_{i2}(0) + \frac{\partial}{\partial b_{-i2}} Q_2 \left(b_2^{nb}(0), 0 \right) \frac{\partial b_{-i2}}{\partial \pi} \right] \\ & - \frac{\beta}{2} u'(G_{i2}) \left[\frac{\partial b_{-i2}}{\partial \pi} \right] + \mu_{i2} \frac{\partial b_{i2}}{\partial \pi} \Bigg) \\ = {} & u'(G_{i1}) \frac{\partial}{\partial \pi} Q_2 \left(b_2^{nb}(0), 0 \right) B_2(0) + u'(G_{i1}) \frac{\partial}{\partial b_{-i2}} Q_2 \left(b_2^{nb}(0), 0 \right) \frac{\partial B_2}{\partial \pi} \\ & - \frac{\beta}{2} u'(G_{i2}) \frac{\partial B_2}{\partial \pi} + \sum_i \mu_{i2} \frac{\partial b_{i2}}{\partial \pi}\end{aligned}$$

Suppose that $B_2(0) < 2\phi(0)$. Then $\mu_{i2} = 0$ and the above is

$$\begin{aligned} & - q u'(G_{i1}) B_2(0) - \frac{\beta}{2} u'(G_{i2}) \frac{\partial B_2}{\partial \pi} \\ = & \frac{\beta}{2} u'(G_{i2}) \left[- \frac{\partial B_2}{\partial \pi} - B_2(0) \right] \\ = & \frac{\beta}{2} u'(G_{i2}) \left[- \frac{\partial B_2}{\partial \pi} - B_2(0) \right] \\ \geq & \frac{\beta}{2} u'(G_{i2}) \left[- \frac{\partial B_2}{\partial \pi} - 2Y \right]\end{aligned}$$

We know from the proof of Proposition 1 that the term $-\frac{\partial B_2}{\partial \pi}$ can be made arbitrarily large (and positive) by lowering the coefficient of absolute risk aversion. Next, notice that step 2 is trivially true since any such deviation is not possible given that the South is already at the borrowing constraint.

Details for the monetary economy model

Proof of Proposition 7

Proof. We will prove the statement for $\pi = 0$. A similar argument can be used for $\pi > 0$. Consider the solution of the cooperative problem in which fiscal policy is chosen cooperatively. The problem solves

$$W_t(B_t) = \max_{\Pi_t, B_{t+1}} u \left(Y - \frac{B_t}{\Pi_t} + Q_{t+1}(B_{t+1}) B_{t+1} \right) - \tau(\Pi_t) + \beta W_{t+1}(B_{t+1})$$

The solution to this problem satisfies

$$
\begin{aligned}
u'\left(Y-\frac{B_t}{\Pi_t}+Q_{t+1}B_{t+1}\right)\frac{B_t}{\Pi_t^2} &= \tau'\left(\Pi_t\right) \\
u'\left(Y-\frac{B_t}{\Pi_t}+Q_{t+1}B_{t+1}\right)\left[Q_{t+1}+\frac{\partial Q_{t+1}\left(B_{t+1}\right)}{B_{t+1}}\right] &= -\beta\frac{\partial W_{t+1}\left(B_{t+1}\right)}{\partial B_{t+1}} \qquad (54)\\
&= \beta u'\left(Y-\frac{B_{t+1}}{\Pi_{t+1}}+Q_{t+2}B_{t+2}\right)\frac{1}{\Pi_{t+1}}
\end{aligned}
$$

Denote the solution to this problem as $\left\{B_1^{coop}, B_2^{coop}, \Pi_0^{coop}, \Pi_1^{coop}, \Pi_2^{coop}, Q_1^{coop}, Q_2^{coop}\right\}$. Clearly the solution to this problem attains a higher value than the equilibrium outcome when fiscal policy is chosen in a non-cooperative fashion where the equilibrium outcome $\{B_1, B_2, \Pi_0, \Pi_1, \Pi_2, Q_1,$ solves

$$
\begin{aligned}
u'\left(Y-\frac{B_0}{\Pi_0}+Q_1B_1\right)Q_1 &= \beta u'\left(Y-\frac{B_1}{\Pi_1}+Q_2B_2\right)\frac{1}{\Pi_1} \qquad (55)\\
u'\left(Y-\frac{B_1}{\Pi_2}+Q_2B_2\right)Q_2 &= \beta u'\left(Y-\frac{B_2}{\Pi_2}\right)\frac{1}{\Pi_2} \qquad (56)\\
u'\left(Y-\frac{B_1}{\Pi_1}+Q_2B_2\right)\left[\frac{B_1}{\Pi_1^2}-\frac{q}{\Pi_2}B_2\frac{\partial \Pi_2}{\partial B_2}\frac{\partial B_2}{\partial \Pi_1}\right] &\leq \tau,\ \forall t\\
u'\left(Y-\frac{B_2}{\Pi_2}\right)\frac{B_2}{\Pi_2^2} = u'\left(Y-\frac{B_2}{\Pi_2}\right)\frac{B_2}{\Pi_2}\frac{1}{\Pi_2} &\leq \tau,\ \forall t\\
Q_t &= \frac{q}{\Pi_t},\ \forall t
\end{aligned}
$$

Note that the Euler equations (55) and (56) differ from their analog in the cooperative solution, (54), because measure zero local governments do not internalize the effect of their debt issuances on the price of debt. Mechanically, the term $\frac{\partial Q_{t+1}(B_{t+1})}{B_{t+1}} < 0$ is missing from (55) and (56). The central authority can internalize such effect by imposing a rule $b_{it} \leq B_t^{coop}$. Since

$$
\begin{aligned}
u'\left(Y-\frac{B_t^{coop}}{\Pi_t^{coop}}+Q_t^{coop}B_{t+1}^{coop}\right)Q_t^{coop} &> u'\left(Y-\frac{B_t^{coop}}{\Pi_t^{coop}}+Q_t^{coop}B_{t+1}^{coop}\right)\left[Q_t^{coop}+\frac{\partial Q_{t+1}\left(B_{t+1}^{coop}\right)}{B_{t+1}^{coop}}\right]\\
&= \beta u'\left(Y-\frac{B_{t+1}^{coop}}{\Pi_{t+1}^{coop}}+Q_{t+2}^{coop}B_{t+2}^{coop}\right)\frac{1}{\Pi_{t+1}^{coop}}
\end{aligned}
$$

each local government has an incentive to violate the rule if it anticipates that it won't be enforced next period or if the penalty is too low. Clearly, we can always find a sufficiently severe penalty ψ so that the rule will be satisfied if a country anticipates enforcement. To show that we can support the cooperative solution with rules we are left to show that it is ex-post optimal for the central government to enforce the rule if one country deviates (assuming all other countries satisfies the rule). Of course, since an individual country is measure zero, the government is willing to enforce the penalty. □

Proof of Proposition 8

Proof. Without rules, A symmetric equilibrium outcome is $\{b_{i1}, b_{i2}, Q_2, \Pi_2\}$ such that

$$u'\left(Y+qb_{i1}\right)=\beta u'\left(Y-b_{i1}+Q_2b_{i2}\right)-\beta\left[u'\left(Y-b_{i1}+Q_2b_{i2}\right)\frac{\partial Q_2}{\partial b_{-i2}}b_{i2}\right]\frac{\partial \mathbf{b}_{-i2}\left(b_1,\pi\right)}{\partial b_{i1}} \tag{57}$$

$$u'\left(Y-b_1+Q_2b_{i2}\right)\left[Q_2+\frac{\partial Q_2}{\partial b_{i2}}b_{i2}\right]=\beta\pi u'\left(Y-b_{i2}\right)+\beta\left(1-\pi\right)u'\left(Y-\frac{b_{i2}}{\Pi_2\left(b_{i2}\right)}\right)\frac{1}{\Pi_2\left(b_{i2}\right)} \tag{58}$$

where $Q_2 = q\left(\pi+(1-\pi)\frac{1}{\Pi_2}\right)$ and the optimal inflation decision in the last period satisfies

$$\sum_i u'\left(Y-\frac{b_{i2}}{\Pi_2}\right)\frac{b_{i2}}{\Pi_2{}^2}=2\tau'\left(\Pi_2\right) \tag{59}$$

Along the equilibrium outcome, relative to the cooperative equilibrium in which fiscal policy is chosen by the monetary authority, there is too much debt because of the free-rider problem.[14] Consider now imposing a fiscal rule, $\bar{b}_1 < b_1$ in period 1 so that debt in period 2 is lower. We now check if such a rule is credible. Suppose that one country follows the rule and borrows $\bar{b}$ and the other local government chooses $b_1 > \bar{b}$. The central government/monetary authority enforces the rule if

$$W\left(\left(\bar{b},b_1-\psi\right),\pi\right)>W\left(\left(\bar{b},b_1\right),0\right) \tag{60}$$

where

$$W\left(b,\pi\right)=\sum_i\left\{u\left(Y-b_i+Q_2\left(\mathbf{b}_2\left(b,\pi\right)\right)\mathbf{b}_2\left(b,\pi\right)\right)+\beta W_2\left(\mathbf{b}_2\left(b,\pi\right)\right)\right\}$$

with $W_2\left(b_2\right)=\max_{\Pi_2}\sum_i\left[u\left(Y-\frac{b_{i2}}{\Pi_2}\right)-\tau\left(\Pi_2\right)\right]$. If π is sufficiently close to zero then (60) does not hold and so there cannot exist an equilibrium with late revelation of uncertainty. Therefore, an equilibrium in pure strategies must have early revelation of uncertainty. Since the fiscal rule constrains each local government to issue debt below what is individually optimal, both local governments will issue debt above the rule for π low enough

[14]Relative to the case with non-atomistic local governments, the free-rider problem with 2 local government is smaller because each government internalize the effect of its debt issuances on the price of the debt it issues but still it does not internalize the impact on the price of the other local government.

and the equilibrium outcome $\left\{b_{i1}, b_{i2}, b_{i2}^c, Q_2, \Pi_2\right\}$ solves

$$u'\left(Y+qb_1\right) = \beta\pi u'\left(Y-b_1-\psi+qb_2^c\right)+\beta\left(1-\pi\right)u'\left(Y-b_1+Q_2b_2\right) \tag{61}$$
$$-\beta\left(1-\pi\right)u'\left(Y-b_1+Q_2b_2\right)\frac{\partial Q_2}{\partial b_{-i2}}b_{i2}\frac{\partial \mathbf{b}_{-i2}\left(b_1,0\right)}{\partial b_{i1}}$$

$$u'\left(Y-b_1+Q_2b_2\right)\left[Q_2+\frac{\partial Q_2}{\partial b_{i2}}b_2\right] = \beta u'\left(Y-\frac{b_2}{\Pi_2}\right)\frac{1}{\Pi_2} \tag{62}$$

$$u'\left(Y-b_1-\psi+qb_2^c\right)q = \beta u'\left(Y-b_2^c\right) \tag{63}$$

with $Q_2 = q\frac{1}{\Pi_2}$ and Π_2 given by (59). This is an equilibrium if it is optimal ex-post not to enforce it one country deviates:

$$W\left(\left(\bar{b}, b_1\right), 0\right) > W\left(\left(\bar{b}, b_1-\psi\right), 1\right) \tag{64}$$

$$W\left(\left(\bar{b}, b_1\right), 0\right) > W\left(\left(\bar{b}, b_1-\psi\right), 1\right) \tag{65}$$

for $b_1 > \bar{b}$. Using arguments similar to the one used in the proof of Proposition 2, if β is sufficiently small then (65) holds and so there exists an equilibrium where rules are violated in period 0 and not enforced by the no-commitment central government in period 1. □

B Quadratic Utility

Consider a special case in which $\beta = q = 1$ and local governments have quadratic utility

$$u\left(c\right) = c - \frac{\alpha}{2}c^2.$$

The system of focs (6) reduces to

$$\left\{1-\alpha\left[Y-b_{N1}-\psi+qb_{N2}\right]\right\} = \pi\left\{1-\alpha\left(Y-b_{N2}\right)\right\}+\frac{\left(1-\pi\right)}{2}\left\{1-\alpha\left(Y-\frac{b_{N2}+b_{S2}}{2}\right)\right\}$$
$$\left\{1-\alpha\left[Y-b_{S1}-\psi+qb_{S2}\right]\right\} = \pi\left\{1-\alpha\left(Y-b_{S2}\right)\right\}+\frac{\left(1-\pi\right)}{2}\left\{1-\alpha\left(Y-\frac{b_{N2}+b_{S2}}{2}\right)\right\}$$

or

$$\left[Y-b_{S1}-\psi+qb_{S2}\right] = \frac{1}{\alpha}\left[\frac{1-\pi}{2}\right]+\pi\left(Y-b_{S2}\right)+\frac{\left(1-\pi\right)}{2}\left(Y-\frac{b_{S2}+b_{N2}}{2}\right)$$

$$
\begin{aligned}
b_{S2} &= \frac{b_{S1}}{\left[q+\frac{1-\pi}{4}+\pi\right]}+\frac{\left[\pi+\frac{1-\pi}{2}-1\right]}{\left[q+\frac{1-\pi}{4}+\pi\right]}Y+\frac{1}{\left[q+\frac{1-\pi}{4}+\pi\right]}\psi+\frac{\frac{1}{\alpha}\left[\frac{1-\pi}{2}\right]}{\left[q+\frac{1-\pi}{4}+\pi\right]}-\frac{1-\pi}{4\left[q+\frac{1-\pi}{4}+\pi\right]}b_{N2} \\
&= \frac{b_{S1}}{\left[q+\frac{1-\pi}{4}+\pi\right]}+\frac{\left[\pi+\frac{1-\pi}{2}-1\right]}{\left[q+\frac{1-\pi}{4}+\pi\right]}Y+\frac{1}{\left[q+\frac{1-\pi}{4}+\pi\right]}\psi+\frac{\frac{1}{\alpha}\left[\frac{1-\pi}{2}\right]}{\left[q+\frac{1-\pi}{4}+\pi\right]}-\frac{1-\pi}{4\left[q+\frac{1-\pi}{4}+\pi\right]}\times \\
&\times\left[\frac{b_{N1}}{\left[q+\frac{1-\pi}{4}+\pi\right]}+\frac{\left[\pi+\frac{1-\pi}{2}-1\right]}{\left[q+\frac{1-\pi}{4}+\pi\right]}Y+\frac{\frac{1}{\alpha}\left[\frac{1-\pi}{2}\right]}{\left[q+\frac{1-\pi}{4}+\pi\right]}-\frac{1-\pi}{4\left[q+\frac{1-\pi}{4}+\pi\right]}b_{S2}\right]
\end{aligned}
$$

$$
\begin{aligned}
& b_{S2}\left\{\frac{\left[q+\frac{1-\pi}{4}+\pi\right]^2-\left((1-\pi)/4\right)^2}{\left[q+\frac{1-\pi}{4}+\pi\right]}\right\} \\
&= b_{S1}+\left[\pi+\frac{1-\pi}{2}-1\right]Y+\psi+\frac{1}{\alpha}\left[\frac{1-\pi}{2}\right]-\frac{1-\pi}{4}\left[b_{N1}+\left[\pi+\frac{1-\pi}{2}-1\right]Y+\psi+\frac{1}{\alpha}\left[\frac{1-\pi}{2}\right]\right]
\end{aligned}
$$

$$
\begin{aligned}
b_{S2} &= \frac{\left[q+\frac{1-\pi}{4}+\pi\right]}{\left[q+\frac{1-\pi}{4}+\pi\right]^2-\left((1-\pi)/4\right)^2} \\
&\times\left\{b_{S1}-\frac{1-\pi}{4}b_{N1}+\left[\pi+\frac{1-\pi}{2}-1\right]\left(1-\frac{1-\pi}{4}\right)Y+\psi+\frac{1}{\alpha}\left[\frac{1-\pi}{2}\right]\left(1-\frac{1-\pi}{4}\right)\right\}
\end{aligned}
$$

which can be solved to get

$$
\begin{aligned}
\mathbf{b}_{S2}\left(b_1,\pi,\psi\right) &= \frac{\left[q+\frac{1-\pi}{4}+\pi\right]}{\left[q+\frac{1-\pi}{4}+\pi\right]^2-\left((1-\pi)/4\right)^2}b_{S1}-\frac{\left(\frac{1-\pi}{4}\right)\left[q+\frac{1-\pi}{4}+\pi\right]}{\left[q+\frac{1-\pi}{4}+\pi\right]^2-\left((1-\pi)/4\right)^2}b_{N1} \\
&+\frac{\left[q+\frac{1-\pi}{4}+\pi\right]^2}{\left[q+\frac{1-\pi}{4}+\pi\right]^2-\left((1-\pi)/4\right)^2}\left(1-\frac{1-\pi}{4}\right)Y+\frac{\left[q+\frac{1-\pi}{4}+\pi\right]}{\left[q+\frac{1-\pi}{4}+\pi\right]^2-\left((1-\pi)/4\right)^2}\psi \\
&+\frac{\left[q+\frac{1-\pi}{4}+\pi\right]\frac{1}{\alpha}\left[\frac{1-\pi}{2}\right]\left(1-\frac{1-\pi}{4}\right)}{\left[q+\frac{1-\pi}{4}+\pi\right]^2-\left((1-\pi)/4\right)^2} \\
&= \gamma_1(\pi)\,b_{S1}+\gamma_2(\pi)\,b_{N1}+\gamma_3(\pi)\,Y+\gamma_4(\pi)\,\psi+\gamma_5(\pi)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{b}_{N2}\left(b_1,\pi,\psi\right) &= \frac{\left[q+\frac{1-\pi}{4}+\pi\right]}{\left[q+\frac{1-\pi}{4}+\pi\right]^2-\left(\left(1-\pi\right)/4\right)^2}b_{N1}-\frac{\left(\frac{1-\pi}{4}\right)\left[q+\frac{1-\pi}{4}+\pi\right]}{\left[q+\frac{1-\pi}{4}+\pi\right]^2-\left(\left(1-\pi\right)/4\right)^2}b_{S1} \\
&+\frac{\left[q+\frac{1-\pi}{4}+\pi\right]^2}{\left[q+\frac{1-\pi}{4}+\pi\right]^2-\left(\left(1-\pi\right)/4\right)^2}\left(1-\frac{1-\pi}{4}\right)Y+\frac{\left[q+\frac{1-\pi}{4}+\pi\right]}{\left[q+\frac{1-\pi}{4}+\pi\right]^2-\left(\left(1-\pi\right)/4\right)^2}\psi \\
&+\frac{\left[q+\frac{1-\pi}{4}+\pi\right]\frac{1}{\alpha}\left[\frac{1-\pi}{2}\right]\left(1-\frac{1-\pi}{4}\right)}{\left[q+\frac{1-\pi}{4}+\pi\right]^2-\left(\left(1-\pi\right)/4\right)^2} \\
&=\gamma_1\left(\pi\right)b_{N1}+\gamma_2\left(\pi\right)b_{S1}+\gamma_3\left(\pi\right)Y+\gamma_4\left(\pi\right)\psi+\gamma_5\left(\pi\right)
\end{aligned}
$$

where

$$
\begin{aligned}
\gamma_1\left(\pi\right) &= \frac{\left[q+\frac{1-\pi}{4}+\pi\right]}{\left[q+\frac{1-\pi}{4}+\pi\right]^2-\left(\left(1-\pi\right)/4\right)^2} \\
\gamma_2\left(\pi\right) &= -\left(\frac{1-\pi}{4}\right)\frac{\left[q+\frac{1-\pi}{4}+\pi\right]}{\left[q+\frac{1-\pi}{4}+\pi\right]^2-\left(\left(1-\pi\right)/4\right)^2} \\
\gamma_3\left(\pi\right) &= \frac{\left[q+\frac{1-\pi}{4}+\pi\right]^2}{\left[q+\frac{1-\pi}{4}+\pi\right]^2-\left(\left(1-\pi\right)/4\right)^2}\left(1-\frac{1-\pi}{4}\right) \\
\gamma_4\left(\pi\right) &= \gamma_1\left(\pi\right) \\
\gamma_5\left(\pi\right) &= \frac{\left[q+\frac{1-\pi}{4}+\pi\right]\frac{1}{\alpha}\left[\frac{1-\pi}{2}\right]\left(1-\frac{1-\pi}{4}\right)}{\left[q+\frac{1-\pi}{4}+\pi\right]^2-\left(\left(1-\pi\right)/4\right)^2}
\end{aligned}
$$